

**Program of Workshop on Harmonic Analysis and Applications,  
Sanya, China, January 6-10, 2020**

Vasilis Chousionis, University of Connecticut, USA

**Title:** Singular integrals on sub-Riemannian curves and surfaces

**Abstract:** Since the pioneering work of Calderón in the late 70s, the study of singular integral operators on lower dimensional subsets of Euclidean space has evolved into a rich theory which has been one of the highlights of the interface between harmonic analysis and geometric measure theory. In this talk I will discuss recent progress on establishing an analogous theory in the setting of sub-Riemannian Carnot groups. As in the Euclidean case, the topic has deep connections with removability for Lipschitz harmonic functions and quantitative rectifiability.

Seng-Kee Chua\*, National University of Singapore, Singapore

**Title:** Embedding theorems for weighted fractional Sobolev spaces and their applications

**Abstract:** Let  $\Omega$  be a reasonable nice open set (bounded or unbounded) in  $\mathbb{R}^n$ . Let  $\rho$  be a positive locally integrable function on  $\Omega$  such that it is essentially constant on each ball that is deep inside  $\Omega \setminus F$  where  $F$  is a finite collection of points or hyperplanes. Let  $\mu, \omega, \sigma$  be Borel measures on  $\Omega$ . We study embedding theorems of weighted fractional Sobolev spaces  $W_{\omega}^{s,p}(\Omega)$  to  $L_{\mu\rho}^q(\Omega)$  where  $(0 < s < 1)$

$$W_{\omega}^{s,p}(\Omega) = \{f \in L_{loc,\omega}^1(\Omega) : \|f\|_{W_{\omega}^{s,p}(\Omega)} < \infty\}$$

$$\|f\|_{W_{\omega}^{s,p}(\Omega)} = \left( \int_{\Omega} \int_{\Omega} \frac{|f(x) - f(y)|^p}{|x - y|^{n+sp}} \rho(x)\rho(y) d\omega(x) d\omega(y) \right)^{1/p}.$$

\*partially supported by Singapore Ministry of Education Academic Research Fund R-146 - 000 - 206 - 112.

Wei Ding, Nantong University, China

**Title:** Multi-parameter local Hardy spaces and applications

**Abstract:** Using multi-parameter local reproducing formula, multi-parameter local Hardy spaces  $h^p(\mathbb{R}^n \times \mathbb{R}^n)$  are introduced. This space can also be characterized by discrete Littlewood-Paley-Stein function. We also establish the

boundedness of multi-parameter pseudodifferential operators on this space. Such boundedness also holds for more general singular integral operators.

Hongjie Dong, Brown University, USA

**Title:** Mixed boundary value problem in nonsmooth domains

**Abstract:** I will discuss some recent results about elliptic equations in a nonsmooth domain, with the Dirichlet boundary condition on part of the boundary and the conormal boundary condition in the complement. We obtained the  $W^{1,p}$  estimate for equations with homogeneous boundary conditions, and a non-tangential maximal function estimate for equations with inhomogeneous boundary conditions. This is based on joint work with Jongkeun Choi (KIAS) and Zongyuan Li (Brown).

Mengxia Dong, Shantou University, China

**Title:** Stability for Hardy-Sobolev inequalities and weighted Sobolev inequalities

**Abstract:** Consider a geometric or functional inequality for which optimizers are known, the stability property tells us for functions almost attain the equality, they must be close (in some suitable sense) to the manifold of all extremal functions. In this talk, we established suitable deficits and distances for Hardy-Sobolev inequalities and weighted Sobolev inequalities, and studied stability for these inequalities.

Xuan Thinh Duong, Macquarie University, Australia

**Title:** Spectral multipliers of self-adjoint operators on Besov and Triebel-Lizorkin spaces associated to operators

**Abstract:** Let  $X$  be a space of homogeneous type and let  $L$  be a non-negative self-adjoint operator on  $L^2(X)$  which satisfies a Gaussian estimate on its heat kernel. In this talk, we prove a Hörmander type spectral multiplier theorem for  $L$  on the Besov and Triebel-Lizorkin spaces associated to  $L$ . Our result not only recovers the boundedness of the spectral multipliers on  $L^p$  spaces and Hardy spaces associated to  $L$ , but also is the first one which proves the boundedness of a general spectral multiplier theorem on Besov and Triebel-Lizorkin spaces.

This is joint work with The Anh Bui, to appear in International Mathematics Research Notices.

Shaoming Guo, University of Wisconsin, USA

**Title:** Polynomial Roth theorems on sets of fractional dimensions

**Abstract:** Let  $P(t)$  be a polynomial of one real variable. I will report a result on searching for patterns of the form  $(x, x+t, x+P(t))$  within Salem sets, whose Hausdorff dimension is sufficiently close to one. Joint work with Fraser and Pramanik.

Xiaoqi Huang, Johns Hopkins University, USA

**Title:** Quasimode, eigenfunction estimates for fractional schrodinger operators

**Abstract:** We prove eigenfunction and quasimode estimates on compact Riemannian manifolds for fractional Schrödinger operators,  $H_V = (-\Delta_g)^{\alpha/2} + V$ , where we assume the potential  $V$  to be in kato class  $\mathcal{K}^\alpha$  and/or  $L^{n/\alpha}$ , and  $(-\Delta_g)^{\alpha/2}$  is defined by spectral theorem. This generalizes a recent result of C. Sogge, M. Blair and Y. Sire which corresponds to  $\alpha = 2$ . Our proof is based on modifying a formula for the resolvent operators that was used to study  $L^p$  resolvent estimates on manifolds, as well as a modification of heat kernel estimates for fractional diffusion operators with perturbations. This is joint work with Y. Sire and C. Zhang.

Dong Li, Hong Kong University of Science and Technology

**Title:** On SQG and related models

**Abstract:** We will discuss some recent results on supercritical dissipative quasi-geostrophic equations and related models.

Jungang Li, Brown University, USA

**Title:** Higher order Brezis-Nirenberg problem on hyperbolic spaces

**Abstract:** In their celebrated paper, Brezis and Nirenberg studied a family of elliptic equations whose nonlinearity is associated with critical Sobolev exponent. Such equations play important roles in the study of PDE, differential geometry and physics. Due to the lack of compactness, many interesting phenomenons have been observed. In this talk, we will discuss some recent progress of higher order Brezis-Nirenberg problem on hyperbolic spaces. These results are motivated by a recent breakthrough of higher order Hardy-Sobolev-Maz'ya inequalities, which is due to Lu and Yang. Our results include the existence of solutions, nonexistence of solutions and the symmetry of positive solutions.

Xiaochun Li, University of Illinois at Urbana Champaign, USA

**Title:** Roth's type theorems on (polynomial) progressions

**Abstract:** The arithmetic progression problems were posed by Erdős-Turan, answered affirmatively by Szemerédi. However, there are still many questions remained on precise quantitative description on how large a subset shall be in order to guarantee a progression in it. Involving with Fourier analysis, considerable work had been accomplished recently. We will give a survey on those progress, and report our recent progress on quantitative version of Roth's type theorem on (polynomial) progressions of short length.

Akihiko Miyachi, Tokyo Woman's Christian University, Japan

**Title:** Basic estimates for multilinear pseudo-differential operators of  $S_{00}$ -type

**Abstract:** The multilinear pseudo-differential operator is defined by

$$\begin{aligned} & T_\sigma(f_1, \dots, f_k)(x) \\ &= \int_{(\mathbb{R}^n)^k} e^{2\pi i x \cdot (\xi_1 + \dots + \xi_k)} \sigma(x, \xi_1, \dots, \xi_k) \widehat{f}_1(\xi_1) \cdots \widehat{f}_k(\xi_k) d\xi_1 \cdots d\xi_k, \\ & x \in \mathbb{R}^n. \end{aligned}$$

We consider  $T_\sigma$  with symbols  $\sigma$  satisfying the condition

$$\left| \partial_x^\alpha \partial_{\xi_1}^{\beta_1} \cdots \partial_{\xi_k}^{\beta_k} \sigma(x, \xi_1, \dots, \xi_k) \right| \leq C_{\alpha, \beta_1, \dots, \beta_k} W(\xi_1, \dots, \xi_k),$$

where  $W$  is a given nonnegative function on  $(\mathbb{R}^n)^k$ . We shall almost characterize those  $W$  that guarantee the boundedness of  $T_\sigma$  from  $L^2 \times \cdots \times L^2$  to  $L^{2/k}$  and also prove boundedness of such  $T_\sigma$  on the product of  $L^2$ -based amalgam spaces.

This talk is based on joint works with Tomoya Kato (Gunma University) and Naohito Tomita (Osaka University), which are available in:

Boundedness of bilinear pseudo-differential operators of  $S_{0,0}$ -type on  $L^2 \times L^2$ , <http://arxiv.org/abs/1901.07237>

Boundedness of multilinear pseudo-differential operators of  $S_{0,0}$ -type in  $L^2$ -based amalgam spaces, <http://arxiv.org/abs/1908.11641>

Phuc Cong Nguyen, Louisiana State University, USA

**Title:** Weighted and pointwise bounds in measure datum problems with applications

**Abstract:** Muckenhoupt-Wheeden type bounds and pointwise bounds by Wolff's potentials are obtained for gradients of solutions to a class of quasilinear elliptic equations with measure data. Such results are obtained globally over sufficiently flat domains in  $\mathbb{R}^n$  in the sense of Reifenberg. The principal operator here is modeled after the  $p$ -Laplacian, where the singular case  $1 < p \leq 2 - \frac{1}{n}$  is considered. As an application, sharp existence results are obtained for a class of quasilinear Riccati type equations having a gradient source term with linear or super-linear power growth. This talk is based on joint work with Quoc-Hung Nguyen.

Joris Roos, University of Wisconsin, USA

**Title:** Spherical maximal functions and fractal dimensions

**Abstract:** In this talk we are concerned with sharp  $L^p$  improving estimates for spherical maximal functions in dimensions  $d \geq 2$  with a supremum taken over a fractal set of radii. It turns out that the  $L^p$  improving region depends on various fractal dimensions of the set of radii, such as Minkowski and Assouad dimensions and the Assouad spectrum. We characterize all convex sets that can arise as  $L^p$  improving region of such a spherical maximal operator, up to endpoints. Surprisingly, a critical segment of the boundary of such a set is given by an essentially arbitrary convex curve, which leads to non-polygonal  $L^p$  improving regions. An application of these estimates are new sparse and weighted  $L^p$  estimates for an associated global spherical maximal operator. Based on joint works with A. Seeger and with T. Anderson, K. Hughes, A. Seeger.

Lindsay Stovall (Betsy), University of Wisconsin, USA

**Title:** Fourier restriction estimates above rectangles and an application

**Abstract:** We discuss the problem of obtaining Lebesgue space inequalities for the Fourier restriction operator associated to rectangular pieces of the paraboloid and perturbations thereof. We state a conjecture for the dependence of the operator norms in these inequalities on the sidelengths of the rectangles, outline a proof of the conjecture (conditional in some cases, unconditional in others), and demonstrate how these estimates can be applied to obtain sharp restriction inequalities on some degenerate hypersurfaces. This is joint work with Jeremy Schwend.

Hanli Tang, Beijing normal university, China

**Title:** Limiting weak type behavior for multilinear fractional integrals

**Abstract:** In this talk, we establish the limiting behavior for multilinear fractional integrals  $\mathcal{I}_{\alpha,A}$  and  $I_{\alpha,A}$ , which were introduced by Kenig and Stein. Although Kenig and Stein proved that these two kinds of multilinear operators have the same weak and strong type estimates, we found that their limiting behaviors are different. This is a joint work with Dr. Guanglan Wang.

Zijiang Wang, Shantou University, China

**Title:** Iterated spherical average on Lebesgue spaces

**Abstract:** In this talk, we consider the iteration of the Laplacian operator and the spherical average operators and study their boundedness.

Yifei Wu, Tianjin University, China

**Title:** The EdEx schemes and their applications to the KdV-type equations

**Abstract:** In this talk, we introduce the EdEx low-regularity integrators for solving the KdV-type equations, and establish the optimal convergence result under rough initial data. The results are confirmed by numerical experiments and comparisons are made with other scheme like the Strang splitting scheme.

Xiangjin Xu, Binghamton University-SUNY, USA

**Title:** Characterization of Carleson measures on compact manifolds with boundary

**Abstract:** On the subspaces  $\{E_L\}_{L \geq 1} \subset L^2(M)$  generated by eigenfunctions of eigenvalues less than  $L (> 1)$  associated to the Dirichlet (and Neumann) Laplace-Beltrami operator on a compact Riemannian manifold  $(M, g)$  with boundary, the sequence of measures  $\mu = \{\mu_L\}_{L \geq 1}$  on  $\{E_L\}_{L \geq 1}$  is an  $L^2$ -Carleson sequence if  $\exists C > 0$  independent of  $L$  such that for any  $L \geq 1$ ,

$$\int_M |f_L|^2 d\mu_L \leq C \int_M |f_L|^2 dV, \quad \forall f_L \in E_L$$

We discuss some positive and negative results on the characterization of the Carleson measures for Dirichlet (and Neumann) Laplacian on  $M$ , which generalized the corresponding results of J. Ortega.

Lixin Yan, Sun Yat-sen (Zhongshan) University, P.R. China

**Title:** Sharp endpoint  $L^p$  estimates for Schrödinger groups

**Abstract:** Let  $L$  be a non-negative self-adjoint operator acting on  $L^2(X)$  where  $X$  is a space of homogeneous type with a dimension  $n$ . Suppose that the heat operator  $e^{-tL}$  satisfies the generalized Gaussian  $(p_0, p'_0)$ -estimates of order  $m$  for some  $1 \leq p_0 < 2$ . We prove sharp endpoint  $L^p$ -Sobolev bound for the Schrödinger group  $e^{itL}$ , that is, for every  $p \in (p_0, p'_0)$  there exists a constant  $C = C(n, p) > 0$  independent of  $t$  such that

$$\|(I + L)^{-s} e^{itL} f\|_p \leq C(1 + |t|)^s \|f\|_p, \quad t \in \mathbb{R}, \quad s \geq n \left| \frac{1}{2} - \frac{1}{p} \right|.$$

The above estimate holds for all  $1 < p < \infty$  when the heat kernel of  $L$  satisfies a Gaussian upper bound. This extends classical results due to Fefferman and Stein, and Miyachi for the Laplacian on the Euclidean spaces  $\mathbb{R}^n$ . We also give an application to obtain an endpoint estimate for  $L^p$ -boundedness of the Riesz means of the solutions of the Schrödinger equations. This is a joint work with P. Chen, X.T. Duong and L. Ji.

Dachun YANG, School of Mathematical Sciences, Beijing Normal University, P.R. China

**Title:** Multiplication Between Hardy Spaces and Their Dual Spaces

**Abstract:** It is well known that bilinear decompositions of products of Hardy spaces and their dual spaces play an important role in the study on various problems from analysis. In this talk, we present some recent progresses on such bilinear decompositions of products of Hardy spaces and their dual spaces. Some open questions are also mentioned in this talk.

Qiaohua Yang, Wuhan University, China

**Title:** Geometric inequalities on complex hyperbolic spaces

**Abstract:** In this talk, Ill give a factorization theorem for the operators on complex hyperbolic space which is closely related to Gellers operator, as well as the CR invariant differential operators on Heisenberg group and CR sphere. By using, among other things, the Kunze-Stein phenomenon on the closed linear group  $SU(1, n)$  and Fourier analysis techniques on complex hyperbolic space, we establish the Hardy-Sobolev-Mazya inequalities and Trudinger-Moser-Adams inequalities on the Siegel domain and the unite ball. The talk is based on a recent joint work with Guozhen Lu.

Hong-Wei ZHANG, Universit dOrlans, France

**Title:** Wave equation in complex symmetric spaces

**Abstract:** In this joint work with Jean-Philippe Anker, we study the dispersive properties of wave equations on complex Riemannian symmetric spaces of non-compact type and arbitrary rank. As a consequence, we obtain the Strichartz estimate and prove global well-posedness results for the corresponding semilinear equation with low regularity data as in the rank one case.

Lu Zhang, ShaanXi Normal University, P.R. China

**Title:** Factorizations and Hardy's type Identities and Inequalities

**Abstract:** Motivated and inspired by the improved Hardy inequalities in the work of Brezis and Vázquez, Brezis and Marcus, we first establish several Hardy type identities that imply many sharpened forms of the Hardy type inequalities on upper half spaces. Furthermore, some geometric Hardy inequalities on  $\mathbb{R}^n$  with the general distance to surfaces of codimension  $1 \leq k \leq n$  are also obtained. The main approach is the Bessel pairs introduced by Ghoussoub and Moradifam and the factorizations of differential operators. (Joint work with N. Lam and G. Lu).

Maochun Zhu, Jiangsu university, China

**Title:** Trace Trudinger-Moser and Adams inequalities on the half space

**Abstract:** In this talk, I will present some new results concerning Trace type Trudinger-Moser and Adams inequalities on the half space. Much work has been done with respect to the trace Trudinger-Moser and Adams inequalities on bounded domain with the smooth boundary. However, much less is known for that of the trace Trudinger-Moser on unbounded domain. Half space, as a particular unbounded domain with the smooth boundary, sharp geometrical inequalities such as Trace-Sobolev inequalities on this domain have attracted much attention due to their importance in geometrical analysis and PDEs. In our recent work, we are concerned with the border-line case of the Trace Sobolev inequalities on the half space, that is the sharp subcritical and critical trace Trudinger-Moser and Adams inequalities. We obtain the best constants of these inequalities and existence of their extremals through the harmonic extension. As an application, we study the existence of the minimum point of related functional by using the method based on Fourier rearrangement, Pohozoev identity and minimization theory of concentration functional.