

Talk Titles and Abstracts

A Fast Alternating Minimization Algorithm for Total Variation Deblurring without Boundary Artifacts

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Abstract

Recently, a fast alternating minimization algorithm for total variation image deblurring (FTVd) has been presented by Wang, Yang, Yin, and Zhang [SIAM J. Imaging Sci., 1 (2008), pp. 248–272]. This is consists of a discrete Fourier transform-based alternating minimization algorithm with periodic boundary conditions and in which two fast Fourier transforms (FFTs) are required per iteration. In this paper, we propose a continuous alternating minimization algorithm for the total variation image deblurring problem and establish its convergence. The continuous setting is very useful to have a unifying representation of the algorithm, independently of the discrete approximation of the deconvolution problem, in particular concerning the strategies for dealing with boundary artifacts. A discrete version of our continuous alternating minimization algorithm is obtained following two different strategies: the imposition of appropriate boundary conditions and the enlargement of the domain. The first one is computationally useful in the case of a symmetric blur, while the second one can be efficiently applied for a nonsymmetric blur. Numerical tests show that our algorithm generates higher quality images in comparable running times with respect to the Fast Total Variation deconvolution algorithm.

On Preconditioned Iteration Methods for Complex Linear Systems

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Abstract

Complex system of linear equations arises in many important applications. We further explore algebraic and convergence properties and present analytical and numerical comparisons among several available iteration methods such as C-to-R and PMHSS for solving such a class of linear systems. Theoretical analyses and computational results show that reformulating the complex linear system into an equivalent real form is a feasible and effective approach, for which we can construct, analyze and implement accurate, efficient and robust preconditioned iteration methods.

Point-Spread Function Reconstruction in Ground-Based Astronomy

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Abstract

In ground-based astronomy, images of objects in outer space are acquired via ground-based telescopes. However, the imaging system is generally interfered by atmospheric turbulence and hence images so acquired are blurred with unknown point spread function (PSF). To restore the observed images, the abbreviation of wavefront at the telescope's aperture, i.e., the phase, is utilized to derive the PSF. However, the phase is not readily available. Instead its gradients can be collected by wavefront sensors. Thus the usual approach is to use regularization methods to reconstruct high-resolution phase gradients and then use them to recover the phase in high accuracy. Here, we develop a model that reconstructs the phase directly. The proposed model uses the tight frame regularization and it can be solved efficiently by the Douglas-Rachford alternating direction method of multiplier whose convergence has been well established. Numerical results illustrate that our new model is efficient and give more accurate estimation for the PSF.

This work is joint with Xiao-Ming Yuan and Wen-Xing Zhang.

AOR Inner-Iteration GMRES Method for Least Squares Problems

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Abstract

We study the accelerated overrelaxation (AOR) inner iteration in combination with GMRES method for overdetermined least squares problems. The AOR inner iteration, a two-parameter generalization of the successive overrelaxation (SOR) method, is efficient and also serves as a powerful preconditioner for ill-conditioned problems. Necessary and sufficient conditions for the proposed inner-iteration GMRES method are presented. Finally, numerical experiments on overdetermined sparse least squares problems are given to show the superiority of the new method.

This work is joint with Lu Liu, Keiichi Morikuni and Jun-Feng Yin.

Preconditioning and Iterative Methods in Image Restoration

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Abstract

Image restoration is a fundamental problem in image processing. The key issue in image restoration is to construct fast and efficient algorithms to restore degraded images. This talk mainly focus on image restoration algorithms and theories by using structured preconditioning and iterative methods in numerical linear algebra. There are many different kinds of noises in different image applications, we

consider additive Gaussian white noise and multiplicative noise removal. A half-quadratic regularization method is applied to remove additive Gaussian white noise. And Newton method is used to solve it. A linear system arises in each Newton iteration step. We construct modified SSOR preconditioners and decomposition-based preconditioners which are used in preconditioned conjugate gradient method to solve the linear system. Theoretical analyses and numerical results show the effectiveness of the proposed method. For multiplicative noise, firstly, we consider pure multiplicative noise removal. Based on maximum a posterior method, we propose a new method which comes from the idea of dictionary image restoration method and total variation regularization. The proposed method can remove multiplicative noise well. Then, we introduce a fast two-step method for restoring degraded images corrupted by multiplicative noise and blur simultaneously. In the first step, the multiplicative noise is removed by nonlocal filtering method, then the recovered image is obtained by solving a minimization problem which consists of a data-fidelity term and a total variation term. We design an alternating iterative algorithm to solve this minimization problem. Numerical results show that the proposed two-step method is effective and it is superior to the existing methods in terms of both image restoring quality and computing speed.

On Eigenvalue Perturbation Bounds for Hermitian Block Tridiagonal Matrices

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Abstract

In this paper, we give some structured perturbation bounds for generalized saddle point matrices and Hermitian block tridiagonal matrices. Our bounds improve some existing ones. In particular, the proposed bounds reveal the sensitivity of the eigenvalues with respect to perturbations of different blocks. Numerical examples confirm the theoretical results.

This work is joint with SW Vong, XF Peng.

An Extragradient-Based Alternating Direction Method for Convex Minimization

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Abstract

We consider the problem of minimizing the sum of two convex functions subject to linear linking constraints. The classical alternating direction type methods usually assume that the two convex functions have relatively easy proximal mappings. However, many problems arising from statistics, image processing and other fields have the structure that only one of the two functions has easy proximal mapping, and the other one is smoothly convex but does not have an easy proximal mapping. Therefore, the classical alternating direction methods cannot be applied. For solving this kind of problems, we propose in this paper an alternating direction method based on extragredients. Under the assumption that the smooth function has a Lipschitz continuous gradient, we prove that the proposed method returns an ϵ -optimal solution within $O(1/\epsilon)$ iterations. We test the performance of different

variants of the proposed method through solving the basis pursuit problem arising from compressed sensing. We then apply the proposed method to solve a new statistical model called fused logistic regression. Our numerical experiments show that the proposed method performs very well when solving the test problems.

Solving Sparse Linear Systems in Large-Scale Scientific Problems

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Abstract

The U.S. Department of Energy's Scientific Discovery through Advanced Computing (SciDAC) program is a multidisciplinary and diverse R&D program that brings together domain scientists, applied mathematicians, and computer scientists to tackle challenging problems in several scientific domains, such as accelerator science, astrophysics, climate, fusion energy, materials science, and nuclear physics. Most, if not all, of these problems are modeled using differential equations. After discretization, the innermost kernels are often the solution of sparse systems of linear equations. In many cases, the level of realism required in the modeling results in simulations that have to be performed at high resolutions. Consequently, the solution of these sparse linear systems can be extremely challenging, in terms of the size and/or the conditioning of the matrices. In this talk, we will discuss some of the approaches we consider in solving these large sparse linear systems.

Weighted Toeplitz Regularized Least Squares Computation for Image Restoration

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Abstract

The main aim of this paper is to develop a fast algorithm for solving weighted Toeplitz regularized least squares problems arising from image restoration. Based on augmented system formulation, we develop new Hermitian and skew-Hermitian splitting (HSS) preconditioners for solving such linear systems. The advantage of the proposed preconditioner is that the blurring matrix, weighting matrix and regularization matrix can be decoupled such that the resulting preconditioner is not expensive to use. We show that for such preconditioned system that is derived from a saddle-point structure of size $(m+n) \times (m+n)$, the preconditioned matrix has an eigenvalue at 1 with multiplicity n , and the other m eigenvalues of the form $1-\lambda$ with $|\lambda| < 1$. We also study how to choose the HSS parameter to minimize the magnitude of $1-\lambda$, and therefore the Krylov subspace method applied to solving the preconditioned system converges very quickly. Experimental results for image restoration problems are reported to demonstrate that the performance of the proposed preconditioner is better than the other testing preconditioners.

Block Preconditioners for the Optimization Problems with Convection-Diffusion Equation Constraints

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Abstract

Optimization problems with constraints which require the solution of a partial differential equation arise widely in many areas of the sciences and engineering. The solution of such PDE-constrained optimization problems is usually a major computational task. In this talk, we consider the preconditioning techniques for such optimization problems with convection-diffusion equation constraints. We employ optimize-then-discretize approach to discretize the problem. The effectiveness of our proposed preconditioners is illustrated by numerical examples.

Gram-Schmidt Process with a Non-Standard Inner Product and its Application to Approximate Inverse Preconditioning

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Abstract

One of the most important and frequently used preconditioning techniques for solving symmetric positive definite systems is based on computing the approximate factorization of the matrix system inverse. It is also a well-known fact that such a triangular factor can be computed column by column by means of the QR-orthogonalization process applied to the unit basis vectors. This fact has been exploited to construct efficient sparse approximate inverse preconditioners. In this contribution we review the most important schemes used for orthogonalization of column vectors (stored in the matrix B) with respect to the non-standard inner product (induced by some symmetric positive definite matrix A) and give the worst-case bounds for corresponding quantities computed in finite precision arithmetic. We formulate our results on the loss of orthogonality and on the factorization error for the classical Gram-Schmidt algorithm (CGS), modified Gram-Schmidt algorithm (MGS) algorithm and for yet another variant of sequential orthogonalization, which is motivated originally by the AINV preconditioner and which uses oblique projections. Although all orthogonalization schemes are mathematically equivalent, their numerical behavior can be significantly different. It follows from our analysis that while the factorization error is quite comparable for all these schemes, the orthogonality between computed vectors can be significantly lost and it depends on the condition number $\kappa(A)$. This is the case also for the expensive implementation based on eigenvalue decomposition (EIG) and Gram-Schmidt with reorthogonalization (CGS2). The classical Gram-Schmidt algorithm and AINV orthogonalization behave very similarly and generate vectors with the orthogonality that essentially depends quadratically on the condition number of the matrix $A^{1/2}B$. Since the orthogonality in the modified Gram-Schmidt algorithm depends only linearly on $\kappa(A^{1/2}B)$, MGS appears to be a good compromise between expensive EIG or CGS2 and less accurate CGS or AINV. Indeed in the context of approximate inverse preconditioning the stabilization of AINV has led to the SAINV algorithm which uses exactly the MGS orthogonalization. We treat

separately the particular case of a diagonal A which is extremely useful in the context of weighted least squares problems. One can show then that local errors arising in the computation of a non-standard inner product do not play an important role here and that the numerical behavior of these schemes is almost identical to the behavior of the orthogonalization schemes with the standard inner product. For all these results we refer to [1], see also [2].

[1]. M. Rozloznic, J. Kopal, M. Tuma, A. Smoktunowicz: Numerical stability of orthogonalization methods with a non-standard inner product. *BIT Numerical Mathematics* (2012), 52, 1035 – 1058.

[2]. B.R. Lowery and J. Langou, Stability analysis of QR factorization in an oblique inner product, *arXiv*. 1401.5171, 2014..

Boundary Integral Methods Involving Implicit Representation of Surfaces

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Abstract

We will start the talk with discussion on a commonly used formula for computing surface integrals in the level set framework. We shall then generalize that formula for integrating open curves and surfaces with boundaries which are implicitly defined by functions in three dimensions. A simple formulation for constructing boundary integral methods to solve Helmholtz and Poisson's equations on domains of irregular shapes is then proposed. Our formulation is based on averaging a family of parameterizations of an integral equation defined on the boundary of the domain, where the integrations are carried out in the level set framework using an appropriate Jacobian. By the coarea formula, the algorithm operates in the Euclidean space and does not require any explicit parameterization of the boundaries. We present numerical results in two and three dimensions.

Multigrid Method for Nonrigid Image Registration Based on the Optimal Mass Transport

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Abstract

Image registration problem is to transform one image to align with another image. Nonrigid registration allows the deformation to vary at different pixel locations. There are parametric approaches based on splines and physical approaches based on physics such as elasticity. One recent approach is motivated by the Monge-Kantorovich mass transfer problem. The goal is to find the optimal mapping M which minimizes the Kantorovich-Wasserstein distance. The optimal mapping can be written as $M = \nabla\psi$, where ψ satisfies the following Monge-Ampere equation

$$\det(D^2\psi(x)) = \frac{I_1(x)}{I_2(\nabla\psi)},$$

where I_1 and I_2 are the given images. Here $\det(D^2\psi(x))$ denotes the determinant of the Hessian of ψ . In this talk, we will present an iterative method for solving the Monge-Ampere equation. Our

approach is to reformulate the highly nonlinear Monge-Ampere equation as a Hamilton-Jacobi-Bellman (HJB) equation. The HJB equation is still a nonlinear equation but for each fixed control, it is a linear second order PDE. We will develop a finite difference discretization such that it is monotone and hence the numerical solution will converge to the viscosity solution. We will then present a relaxation scheme which is a very slowly convergent iterative method as a standalone solver. However, it is very effective for reducing high frequency errors. We will adopt it as a smoother for multigrid and demonstrate its smoothing properties. Finally, numerical results will be presented to illustrate the effectiveness of the method.

Numerical Solvers for a Class of Saddle Point Systems Arising from PDE-Constrained Optimizations

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Abstract

The optimization problem is one of the research hot points recently. Treated with the discretize-then-optimize approach to the weak formulation of it, KKT system in the saddle point form is obtained. There are numerous literatures devoted to the numerical solvers for the subsequent saddle point problems. The aim of this report is to present and discuss several prominent solution methods for linear systems which arise from the simple model problems.

Lopsided PMHSS Iteration Method for a Class of Complex Symmetric Linear Systems

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Abstract

A lopsided PMHSS (LPMHSS) iteration method is presented for solving a broad class of complex symmetric linear systems. The convergence properties of the LPMHSS method are analyzed, which show that, under a loose restriction on parameter α , the iterative sequence produced by the LPMHSS method is convergent to the unique solution of the linear system for any initial guess. Furthermore, we derive an upper bound for the spectral radius of the LPMHSS iteration matrix, and the quasi-optimal parameter α^* which minimizes the above upper bound is also obtained. Both theoretical and numerical results indicate that the LPMHSS method outperforms the PMHSS method when the real part of the coefficient matrix is dominant.

Matrix-Free Structured Preconditioning

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Abstract

In iterative solutions, often only matrix-vector multiplications instead of the explicit matrices are available. We seek to design matrix-free preconditioners in such cases based on recent research on randomized sampling by researchers. With up to $O(\log n)$ matrix-vector products, we can construct structured preconditioners which approximate an order n matrix A up to a given accuracy. The off-diagonal blocks of A are approximated with (possibly low-accuracy) low-rank forms, and no *a priori* knowledge of the off-diagonal ranks are needed. Such preconditioners have several significant benefits:

- Only the evaluations of matrix-vector products are needed, instead of any entry of A . The cost of applying the preconditioner is about $O(n)$.
- For certain cases, the off-diagonal singular values can be aggressively truncated. This results in low-accuracy preconditioners which can be shown to greatly improve the conditioning of the original matrix.
- Preliminary results also show the improvement of the eigenvalue distribution of A .
- Unlike in existing randomized structured computations which usually need the evaluation of both A and A^T with vectors, here only the products of A with vectors are needed.
- The preconditioner can be quickly updated if the problem has few varying parameters such as diagonal shifts.

We illustrate the performance in terms of several important interface problems where GMRES fails to converge. On the other hand, GMRES with our preconditioner converges rapidly, usually in few steps nearly independent of n . For certain interface problems with incompressible flows, only one step is needed for convergence.

Part of the work is joint with Raymond Chan, Zhilin Li, and Yuanzhe Xi.

Least-Squares Symmetric Solution to the Matrix Equation $AXB=C$ with the Norm Inequality Constraint

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Abstract

In this paper, an iterative method to compute the norm-constrained least-squares solution of the matrix $AXB=C$ with the matrix having symmetric structure, is proposed. For this method, without the error of calculation, a desired solution can be obtained with finitely iterate step. Numerical experiments are performed to illustrate the efficiency and real application of the algorithm.

Hierarchical Optimizations for Certain Inverse Problems in Image and Signal Processing

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Abstract

Many tasks in image and signal processing can be formulated as certain optimization problems where the solution sets are often infinite. This means that a further strategic selection from the solution set should be of great importance. In the 1st part of this talk, motivated by recent elegant characterizations of the solution sets of convex optimization problems as the fixed point sets of computable nonexpansive mappings (e.g., Proximal Forward-Backward splitting operator, Douglas-Rachford splitting operator and Primal-Dual splitting operator), we present an algorithmic selection of an optimal point in the solution set of a convex optimization problem. The proposed approach is based on the hybrid steepest descent method and can minimize the sum of the Moreau envelopes of nonsmooth convex functions over the fixed point set of a nonexpansive mapping. In the 2nd part of this talk, as an algorithmic solution to a hierarchical nonconvex optimization problem, we introduce the minimum-variance pseudo-unbiased reduced-rank estimator (MV-PURE), together with its rank selection technique, for ill-conditioned linear inverse problems.

On the Convergence of the Primal-Dual Hybrid Gradient Method

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Abstract

The primal-dual hybrid gradient method (PDHG) has been widely used in the literature, especially for some basic image processing models. There is still some gap between the PDHG's efficient performance in practice and its rigorously provable convergence in theory. We revisit the PDHG's convergence in the context of a saddle-point problem, and try to understand this gap better. More specifically, we show by an extremely simple example that such a condition that yields tiny step sizes seems necessary for theoretically deriving the rigorous convergence of the PDHG. We then show that the convergence of PDHG can be ensured if one of the functions in the saddle-point problem is uniformly strong convex, a condition that does hold for some popular applications in various fields including some basic variational models in imaging. In fact, under this additional condition, the PDHG's step sizes are not necessarily to be convergent to 0. This justifies why some practical step-size strategies in the literature work well for the PDHG. Furthermore, with this additional condition, we establish a worst-case $O(1/k)$ convergence rate measured by the iteration complexity for the PDHG, where k represents the iteration counter.

Preconditioning for Pseudo-Polar Fourier Transform and its Application to Digital Shearlet Transform

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Abstract

It's been well-known that high-dimensional data are governed by an-isotropic features embedded under the low-dimensional manifolds (points, edges, surface, etc.). Efficient and sparse representation systems that are capable of capturing such anisotropic features are undoubtedly desired for the application of high-dimensional data processing. Shearlets, as one of the directional multiscale representation systems, provide not only theoretically the optimal sparse representation of cartoon-like images, but also in practice multiscale analysis and directional sensitivity for high-dimensional data.

The pseudo-polar Fourier transform \mathcal{F}_p (PPFT) which evaluate the discrete Fourier transform (DFT) at points on a trapezoidal grid in the frequency domain, the so-called pseudo-polar grid, allows the faithful design for the digital shearlet transform (DST). The PPFT is not isometric. To achieve isometry property, weighting \mathbf{w} needs to be done on the pseudo-polar grid. We show that the design of suitable weighting \mathbf{w} provides not only nearly isometry property for the DST, but also a preconditioning for solving the system of equations: $\mathcal{F}_p \mathbf{x} = \mathbf{y}$, thereby accelerating the inverse DST using iterative methods such as the conjugate gradient (CG).

The applications of DST in image inpainting and denoising show the advantages of pseudo-polar grid for the digitization of shearlet transform.

Poster Titles and Abstracts

Parallel Solution for Multi-Group Radiation Transfer Equations

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Abstract

In inertial confinement fusion (ICF) numerical simulations, it is necessary and very important to solve multi-group radiation transfer (MGRT) equations. Usually, the cost of solution for MGRT equations is more than 90% of the whole simulation. The source iteration method is often used to solve MGRT equations, and one typical parallel solution method for MGRT equations is the spacial domain decomposition (spacial parallelism) method. In numerical simulations, the scalability of application

code is limited if only spacial parallel strategy is used.

By exploring the character of the source iteration method, a two level parallel strategy is designed for solving MGRT equations. In this strategy, first divide the equations into several sets, and in each source iteration, solve the equations in different sets in parallel. And then for solving each equation, the spacial parallel strategy is used. The scalability of application code is improved dramatically with the two level parallel strategy.

An Alternating Positive Semidefinite Splitting Preconditioner for Saddle Point Problems from Time-Harmonic Eddy Current Models

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Abstract

For the saddle point problem arising from the finite element discretization of the hybrid formulation of the time-harmonic eddy current problem, we propose an alternating positive semidefinite splitting preconditioner which is based on two positive semidefinite splittings of the saddle point matrix. It is proved that the corresponding alternating positive semidefinite splitting iteration method is unconditional convergent. We analyze that the new preconditioner is much easier to implement than the block alternating splitting implicit preconditioner proposed in [Z.-Z. Bai, Numer. Linear Algebra Appl., 19 (2012), 914--936] when they are used to accelerate the convergence rate of Krylov subspace methods such as GMRES. Numerical examples are given to show the effectiveness of our proposed preconditioner.

This work is joint with Zhi-Ru Ren.

Numerical Methods for a Nonsymmetric Algebraic Riccati Equation Arising in Transport Theory

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Abstract

The computation of the minimal positive solution of a non-symmetric algebraic Riccati equation arising in transport theory is considered. This work can be done via only computing the minimal positive solution of a vector equation, which is derived from special form of the solution of the Riccati equation and by exploitation of the special structure of the coefficient matrices of the Riccati equation. In this paper, the fixed-point method, the Newton method and the structure-preserving doubling algorithm are introduced for the vector equation. Numerical examples show that these methods are feasible and effective.

An Adaptive Inpainting Algorithm Based on DCT Induced Wavelet Regularization

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Abstract

In this work, we propose an image inpainting optimization model whose objective function is a smoothed \mathbf{L}^1 norm of the weighted non-decimated discrete cosine transform (DCT) coefficients of the underlying image. By identifying the objective function of the proposed model as a sum of a differentiable term and a non-differentiable term, we give a basic algorithm inspired by Beck and Teboulle's recent work (SIIMS, Vol.2, No.1, 2009) for the model. Based on this basic algorithm, we propose an automatic way to determine the weights involved in the model and update them at each iteration. The discrete cosine transform as an orthogonal transform is used in various applications. We view the rows of a discrete cosine transform matrix as the filters associated with a multiresolution analysis. Non-decimated wavelet transforms with these filters are explored to analyze images to be inpainted. Our numerical experiments verify that under the proposed framework, the filters from a discrete cosine transform matrix demonstrate promise for the task of image inpainting.

Asymptotically Optimal Parameters for the Splitting Iteration

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Abstract

We consider the self-adaptive methods for choosing the asymptotically optimal parameters, with regard to parameters that have a significant influences on the efficiency of iteration methods. And the numerical examples show that these self adaptive schemes are more effective and robust than the classical methods with choosing the theoretical optimal parameters.

Accelerated Uzawa Methods for Convex Optimization

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Abstract

We focus on a linearly constrained convex minimization model and discuss the application of the classical Uzawa method. Our principal goal is to show that some existing acceleration schemes can be used to accelerate the Uzawa method in the sense that the worst-case convergence rate (measured by the iteration complexity) of the resulting accelerated Uzawa schemes is $O(1/k^2)$ where k represents the iteration counter. Our discussion assumes that the objective function is given by a black-box oracle; thus an inexact version of the Uzawa method with a dynamically-chosen step size is

implemented. A worst-case convergence rate of $O(1/k)$ is also shown for this inexact version. Some preliminary numerical results are reported to verify the acceleration effectiveness of the accelerated Uzawa schemes and their superiority over some existing methods.

This work is jointwith Xiao-Ming Yuan.

Quasi-Chebyshev Accelerated Iteration Methods Based on Optimization for Linear Systems

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Abstract

In this study, we present a quasi-Chebyshev accelerated iteration method for solving a system of linear equations. Compared with the Chebyshev semi-iterative method the main difference is that the parameter ω is not obtained by Chebyshev polynomial but optimization models. We prove that the quasi-Chebyshev accelerated iteration method is unconditionally convergent if the original iteration method is convergent and discuss also the convergence rate. Finally, three numerical examples indicate that our method is more efficient than the Chebyshev semi-iterative method.

We describe the quasi-Chebyshev accelerated iteration method. Let

$$A = M - N,$$

and let the iteration matrix be

$$T = M^{-1}N.$$

Method (*quasi-Chebyshev accelerated iteration method*)

Given an initial point x_0 , a precision $\epsilon > 0$. For $k = 1, 2, \dots$, until converges, do

Step 1. Let $\bar{x}_{k+1} = M^{-1}Nx_k + M^{-1}b$. Solve the system

of linear equations as follows

$$x_{k+1} = \omega_{k+1}(\bar{x}_{k+1} - x_{k-1}) + x_{k-1},$$

where

$$x_0 \in R^n, \quad x_1 = M^{-1}Nx_0 + M^{-1}b.$$

ω_{k+1} is the solution of the following optimization problems: \\\

(a) when A is a symmetric positive definite matrix, set $x = \omega(\bar{x}_{k+1} - x_{k-1}) + x_{k-1}$,

$$\min_{\omega} \frac{1}{2} x^T A x - x^T b;$$

(b) when A is not symmetric positive definite, set $r = Ax - b, x = \omega(\bar{x}_{k+1} - x_{k-1}) + x_{k-1}$,

$$\min_{\omega} r^T (\alpha I + H)^{-2} r;$$

Step 2. If $\|r_{k+1}\| < \epsilon$, stop; Otherwise, $k \leftarrow k + 1$ and go to Step 1.

A New PHSS Iteration Method for Non-Hermitian Positive-Definite Linear Systems

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Abstract

By utilizing the preconditioned Hermitian and skew-Hermitian splitting (PHSS) iteration technique, we establish a new PHSS (NPHSS) iteration method for solving large sparse non-Hermitian positive-definite linear systems. The convergence properties of the NPHSS method are analyzed, which show that, under a loose restriction on parameter α , the iterative sequence produced by NPHSS method is convergent to the unique solution of the linear system for any initial guess. Furthermore, we derive an upper bound for the spectral radius of the NPHSS iteration matrix, and the quasi-optimal parameter α^* which minimizes the above upper bound is also obtained. Both theoretical and numerical results indicate that the NPHSS method outperforms the PHSS method when the Hermitian part of the coefficient matrix is dominant.

This work is joint with Xu Li and Ai-Li Yang.

Aitken Extrapolation Iterative Method for Computing the Largest Eigenvalue of Nonnegative Tensors

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Abstract

Finding the maximum eigenvalue of a tensor is an important topic in tensor computation and multilinear algebra. Recently, when the tensor is non-negative, efficient numerical schemes have been proposed to calculate the maximum eigenvalue based on a Perron-Frobenius type theorem for non-negative tensors.

Using Aitken extrapolation technique, we propose an new iterative method to calculate the largest eigenvalue of irreducible nonnegative tensors. Numerical experiments show that this method is feasible and requires less iteration steps and CPU time compares with original method for essentially positive tensors.

An Alternating Direction Method for Deblurring Images Corrupted by Impulsive Noise

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Abstract

In this paper, we study the problem of restoring blurred images corrupted by impulsive noises. This is a difficult problem which has been considered in a series of recent papers. The two-phase approaches proposed in some of these papers are very effective. The two-phase approaches first identify the outlier candidates, the pixels that are likely to be corrupted by the impulsive noise, and then remove them from the data set. In a second phase, the image is deblurred and denoised simultaneously using essentially the outlier-free data. Using essentially the outlier-free data makes the two-phase approaches effective, they either destruct the structure of the blurring matrix, and prevent the efficient algorithms designing. Following the same line, we also restore the degraded images using essentially the outlier-free data. By using variable splitting technique, we try to preserve the structure of the blurring matrix, and we propose a fast and effective alternating direction method. Experimental results show that the quality of restored images by our proposed method is competitive with those restored by the other existing two-phase image restoration methods.

Two-Stage Multisplitting Iteration Methods using Modulus-Based Matrix Splitting as Inner Iteration for Linear Complementarity Problems

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Abstract

The matrix multisplitting iteration method is an effective tool for solving large sparse linear complementarity problems. However, at each iteration step we have to solve a sequence of linear complementarity sub-problems exactly. In this paper, we present a two-stage multisplitting iteration method, in which the modulus-based matrix splitting iteration and its relaxed variants are employed

as inner iterations to solve the linear complementarity sub-problems approximately. The convergence theorems of these two-stage multisplitting iteration methods are established. Numerical experiments show that the two-stage multisplitting relaxation methods are superior to the matrix multisplitting iteration methods in computing time, and can achieve a satisfactory parallel efficiency.

An Inexact Newton Method Based on Multi-Step Scheme for Non-Symmetric Algebraic Riccati Equations

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Abstract

Various methods are used to deal with the minimal non-negative solution of the nonsymmetric algebraic Riccati equation

$$XCX - XD - AX + B = 0.$$

In which, Newton iteration method is adopted in many cases for it has fast convergence speed. However, the cost of Newton iteration method is large for it demands to solve the involved Sylvester equation in each iteration step. To preserve its advantage in convergence speed and improve the computation speed, we apply the multi-step scheme to reduce the cost.