



Tsinghua Sanya International Mathematics Forum

(清华三亚国际数学论坛)

Hilbert Modules in Analytic Function Spaces Workshop

May 22-26, 2017

No.100 Tsinghua Rd., Tianya District, Sanya, Hainan.

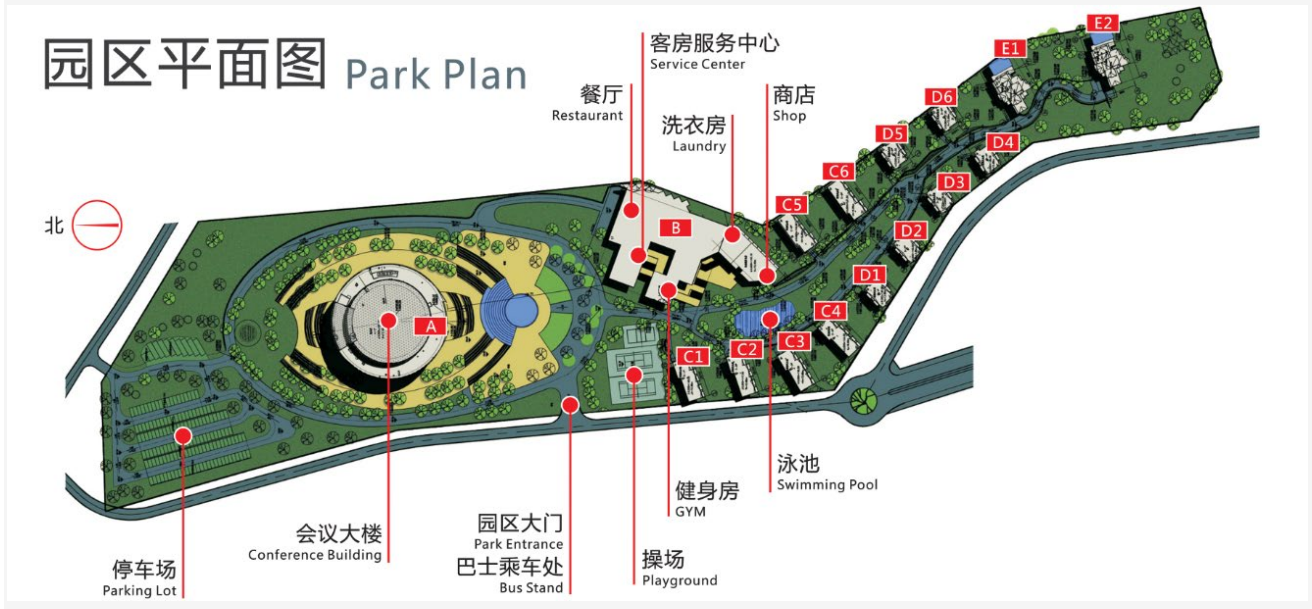
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Welcome to TSIMF

The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops simultaneously, two libraries, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, gym and sports court and other recreational facilities.

Mathematical Sciences Center (MSC) of Tsinghua University, assisted by TSIMF's International Advisory Committee and Scientific Committee, will take charge of the academic and administrative operation of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other relating disciplines, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.

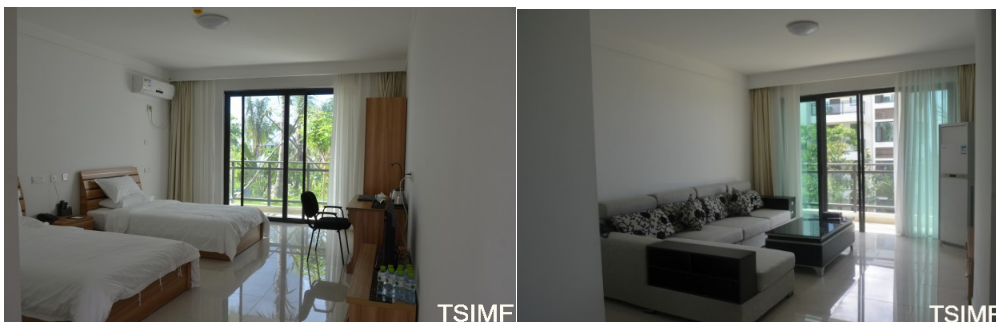
About Facilities



Registration

Conference booklets, room keys and name badges for all participants will be distributed at the Registry. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.

Guest Room



Conference Center can receive about 378 people having both single and double rooms, and 42 family rooms.

All the rooms are equipped with: free Wi-Fi, TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.

Library

Opening Hours: 08:00am-22:00pm

TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.

Restaurant



All the meals are provided in the Chinese Restaurant (Building B1) according to the time schedule.

Breakfast	07:30-08:30
Lunch	12:00-13:30
Dinner	17:30-19:00

Laundry

Opening Hours: 24 hours

The self-service laundry room is located in the Building 1 (B1), next to the shop.

Gym

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables and etc.

Playground

Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

Swimming Pool



Please note that there are no lifeguards. We will not be responsible for any accidents or injuries. In case of any injury or any other emergency, please call the reception hall at +86-898-38882828.

Outside Shuttle Service:

We have shuttle bus to take participants to the airport for your departure service. Also, we would provide transportation at the Haipo Square (海坡广场) of Howard Johnson for the participants who will stay outside TSIMF. If you have any questions about transportation arrangement, please feel free to contact Ms. Li Ye (叶莉), her cell phone number is (0086)139-7679-8300.

Free Shuttle Bus Service at TSIMF:

We provide free shuttle bus for all participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.

Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



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Schedule for the Hilbert Modules in Analytic Function Spaces Workshop, May 22-26, 2017

Time & Date	Monday (May 22)	Tuesday (May 23)	Wednesday (May 24)	Thursday (May 25)	Friday (May 26)
7:30-8:30	<i>Breakfast</i>				
<i>Chair</i>	—	<i>Kehe Zhu</i>	<i>Gadadhar Misra</i>	<i>Jingbo Xia</i>	<i>Laszlo Kerchy</i>
8:30-9:10	Registration	Harald Ernst Otto Upmeyer	Kunyu Guo	Keji Izuchi	Jingbo Xia
<i>Chair</i>	<i>Kunyu Guo</i>	—	—	—	—
9:20-10:00	Ronald G. Douglas	Xiang Fang	Laszlo Kerchy	Dechao Zheng	Gadadhar Misra
10:00-10:30	<i>Tea Break</i>				
10:30-11:10	Conni Liaw	Kouhei Izuchi	Shibananda Biswas	Miroslav Englis	Hansong Huang
11:20-12:00	Jaydeb Sarkar	Sameer Chavan	Wing-Suet Li & Photo	Marek Ptak	Penghui Wang
12:00-13:30	<i>Lunch</i>				
<i>Chair</i>	<i>Wing-Suet Li</i>	<i>Marek Ptak</i>	—	<i>Harald Ernst Otto Upmeyer</i>	<i>Miroslav Englis</i>
13:30-14:10	Quanlei Fang	Sujit Kumar	<i>Free Discussion</i> 13:30-17:00	Dinesh Kumar Keshari	Sommath Hazra
14:20-15:00	Yueshi Qin	Yixin Yang		Mohammed Ramiz Reza	Soumitra Ghara
15:00-15:30	<i>Tea Break</i>				
15:30-16:10	Yue Wu	Fateme Azarkey	<i>Tea break</i>		
18:00-19:30	<i>Dinner</i>		<i>Dinner</i>		<i>Departure</i>
			<i>Banquet 18:00-20:00</i>		

Titles and Abstracts

1. Fatemeh Azarikey, Dalian University of Technology, China

Title: Numerical Invariants of Homogeneous Submodules

Abstract: The core operator C^M is a bounded self-adjoint integral operator defined on submodules M of $H^2(\mathbb{D}^2)$. It has played important roles in our study of submodules. Its eigenvalues are useful numerical invariants for the submodules with respect to unitary equivalence. However, the eigenvalues are very difficult to compute in general. In this talk we will see that for principal homogeneous submodules the eigenvalues can be completely determined through Toeplitz determinants. This is a joint work with Y. Lu and R. Yang.

2. Shibabanda Biswas, Indian Institute of Science Education Research, Kolkata

Title: On Inequivalence of Submodules of Weighted Bergman Modules Under the Action of Symmetric Group

Abstract: We would see that a weighted Bergman space on the polydisc, $\mathbb{A}^{(\lambda)}(\mathbb{D}^n)$, for $\lambda > 0$ splits up to orthogonal direct sum of subspaces $\mathbb{P}_{\mathbf{p}}(\mathbb{A}^{(\lambda)}(\mathbb{D}^n))$ indexed by partitions \mathbf{p} of n corresponding to an irreducible representation of the symmetric group. We would then discuss the inequivalence of the submodules $\mathbb{P}_{\mathbf{p}}(\mathbb{A}^{(\lambda)}(\mathbb{D}^n))$ for different partitions \mathbf{p} of n and for different parameters λ . This is joint work with Gargi Ghosh, Gadadhar Misra and Subrata Shyam Roy.

3. Sameer Chavan, Indian Institute of Technology, Kanpur, India

Title: Module Tensor Product of Subnormal Modules

Abstract: Let $\kappa : \mathbb{D} \times \mathbb{D} \rightarrow \mathbb{C}$ be a diagonal positive definite kernel and let H_{κ} denote the associated reproducing kernel Hilbert space of holomorphic functions on the open unit disc \mathbb{D} . Assume that $zf \in H$ whenever $f \in H$. Then H is a Hilbert module over the polynomial ring $\mathbb{C}[z]$ with module action $p \cdot f \mapsto pf$. We say that H_{κ} is a subnormal Hilbert module if the operator M_z of multiplication by the coordinate function z on H_{κ} is subnormal. In [Oper. Theory Adv. Appl, 32: 219-241, 1988], N. Salinas asked whether the module tensor product $H_{\kappa_1} \otimes_{\mathbb{C}[z]} H_{\kappa_2}$ of subnormal Hilbert modules H_{κ_1} and H_{κ_2} is again subnormal. In this regard, we describe all subnormal module tensor products $L_a^2(\mathbb{D}, w_{s_1}) \otimes_{\mathbb{C}[z]} L_a^2(\mathbb{D}, w_{s_2})$, where $L_a^2(\mathbb{D}, w_s)$ denotes the weighted Bergman Hilbert module with radial weight

$$w_s(z) = \frac{1}{s\pi} |z|^{\frac{2(1-s)}{s}} \quad (z \in \mathbb{D}, s > 0).$$

In particular, the module tensor product $L_a^2(\mathbb{D}, w_s) \otimes_{\mathbb{C}[z]} L_a^2(\mathbb{D}, w_s)$ is never subnormal for any $s \geq 6$. Thus the answer to this question is no. This is a joint work with A. Anand.

4. Ronald G. Douglas, Texas A&M, USA

Title: Some Problems for Hilbert Modules

Abstract: Problems and results in operator theory can often be usefully framed in the language of Hilbert modules. In this talk we will consider several problems and their related contexts, attempting the focus on some questions of importance. The topics include the Lifting Theorem, the Halmos question on similarity, the corona theorem, and an abstract categorical approach related to K theory. Moreover, we will show how this approach to the Lifting Theorem leads to an interesting counterexample for the corona theorem and a positive answer to this question would lead to affirmative solution for the corona theorem on the unit ball.

5. Miroslav Engliš, Mathematics Institute, Silesian University in Opava, Czech Republic; Mathematics Institute, Czech Academy of Sciences, Prague, Czech Republic

Title: Hankel Operators and the Dixmier Trace On the Hardy Space

Abstract: We give criteria for the membership of Hankel operators on the Hardy space on the disc in the Dixmier class, and establish estimates for their Dixmier trace. In contrast to the situation in the Bergman space setting, it turns out that there exist Dixmier-class Hankel operators which are not measurable (i.e. their Dixmier trace depends on the choice of the underlying Banach limit), as well as Dixmier-class Hankel operators which do not belong to the $(1, \infty)$ Schatten-Lorentz ideal. A related question concerning logarithmic interpolation of Besov spaces is also discussed.

6. Quanlei Fang, Department of Mathematics and Computer Science, CUNY-BCC, NY USA

Title: Submodules of Reproducing Kernel Hilbert Modules Over $C[z_1, z_2, \dots, z_n]$

Abstract: With the natural multiplication of functions as the module operation, reproducing kernel Hilbert spaces such as Hardy space, Bergman space and Drury-Arveson space can be treated as Hilbert modules over the polynomial ring $\mathbb{C}[z_1, z_2, \dots, z_n]$. A submodule is a closed linear subspace that is invariant under module multiplications. In this talk we will discuss associated defect operators of submodules and essential normality of some polynomial-generated submodules. This is based on joint work with Jingbo Xia.

7. Xiang Fang, National Central University, Taiwan

Title: Random Weighted Shifts

Abstract: This is part of an effort to develop what we might call “random operator theory”. We start with two basic objects in operator theory: The unilateral shift and the Hardy space, and we try to develop their random counterparts. A sample result we found is that for any Hardy function

$$f(z) = \sum_{k=0}^{\infty} a_k z^k \in H^2(\mathbb{D}),$$

if we put random signs \pm before each coefficient, that is, to consider

$$R(f) = \sum_{k=0}^{\infty} \pm a_k z^k,$$

then almost surely $R(f) \in H^\infty$.

This is joint work with Cheng Guozheng (Wenzhou University) and Zhu Sen (Jilin University).

8. Soumitra Ghara, Indian Institute of Science, Bangalore

Title: Description of Certain Quotient Modules in the Tensor Product of Two Hilbert Modules

Abstract: Let $\Omega \subset \mathbb{C}^m$ be a bounded domain and $K : \Omega \times \Omega \rightarrow \mathbb{C}$ be a positive definite kernel. Let $\alpha, \beta > 0$ be such that K^α and K^β are also a positive definite kernel. Let Δ be the set $\{(z, w) \in \Omega \times \Omega : z = w\}$. Define the subspace $\mathcal{A}_k, k \geq 0$ by

$$\mathcal{A}_k := \{f \in (\mathcal{H}, K^\alpha) \otimes (\mathcal{H}, K^\beta) : (\frac{\partial}{\partial w})^I f(z, w)|_\Delta = 0, |I| \leq k\}.$$

Let $\mathcal{H}_k = \mathcal{A}_{k-1} \ominus \mathcal{A}_k, k \geq 1$ and $\mathcal{H}_0 = \mathcal{A}_0^\perp$. A Theorem of Aronszajn shows that the map $f \mapsto f|_\Delta$ defines a unitary from \mathcal{A}_0^\perp onto $(\mathcal{H}, K^{\alpha+\beta})$. If p is a complex polynomial in $2m$ variables, then this map also intertwines the operator $P_{\mathcal{H}_0} M_{p|_{\mathcal{H}_0}}$ on \mathcal{H}_0 and the multiplication operator $M_{p|_\Delta}$ on $(\mathcal{H}, K^{\alpha+\beta})$.

In this talk, we will construct a unitary from \mathcal{H}_1 onto the reproducing kernel Hilbert space $(\mathcal{H}, (K^{\alpha+\beta}(z, w) \partial_i \bar{\partial}_j \log K(z, w))_{i,j=1}^m)$ and show that it intertwines the operator $P_{\mathcal{H}_1} M_{p|_{\mathcal{H}_1}}$ and the multiplication operator $M_{p|_\Delta}$ on $(\mathcal{H}, (K^{\alpha+\beta}(z, w) \partial_i \bar{\partial}_j \log K(z, w))_{i,j=1}^m)$.

We will also show that the operator $P_{\mathcal{A}_1^\perp} (M_p)|_{\mathcal{A}_1^\perp}$ is unitarily equivalent to

$$\begin{pmatrix} M_{p|_\Delta} & 0 \\ S & M_{p|_\Delta} \end{pmatrix}$$

on $(\mathcal{H}, K^{\alpha+\beta}) \oplus (\mathcal{H}, ((K^{\alpha+\beta} \partial_i \bar{\partial}_j \log K(z, w))_{i,j=1}^m))$, where S is the operator given by $S(f) = \frac{\beta}{\sqrt{\alpha\beta(\alpha+\beta)}} \sum_{j=1}^m (\partial_i p|_\Delta - \partial_{m+i} p|_\Delta) f \otimes e_i, f \in (\mathcal{H}, K^{\alpha+\beta})$.

Finally, for $\Omega \subset \mathbb{C}$, we show that the operator $P_{\mathcal{A}_1^\perp} (M^* \otimes I)|_{\mathcal{A}_1^\perp}$ belongs to the class $\mathcal{FB}_2(\Omega^*)$, introduced by Ji, Jiang, Keshari and Misra.

9. Kunyu Guo, Fudan University, China

Title: The Hardy Module in Infinitely Many Variables

Abstract: In this talk we mainly consider Ahern-Clark's remarkable algebraic reduction for finite codimensional submodules of the Hardy module in infinitely many variables. It is shown that the infinite-variable case diverges a lot from the finite-variable case.

Also, this talk shows that the Douglas-Paulsen's Conjecture on contracted ideals fails for infinitely many variables. This is a joint work with Hui Dan and Hansong Huang.

10. Somnath Hazra, Indian Institute of Science, Bangalore

Title: Homogeneous Tuples in the Cowen-Douglas Class of the Polydisc

Abstract: Let G be either (i) the direct product of n -copies of the bi-holomorphic automorphism group of the disc or (ii) the bi-holomorphic automorphism group of the polydisc \mathbb{D}^n .

A commuting tuple of bounded operators $\mathbf{T} = (T_1, T_2, \dots, T_n)$ is said to be G -homogeneous if the joint spectrum of \mathbf{T} lies in \mathbb{D}^n and $\varphi(\mathbf{T})$, defined using the usual functional calculus, is unitarily equivalent with \mathbf{T} for all $\varphi \in G$.

We show that a commuting tuple \mathbf{T} in the Cowen-Douglas class of rank 1 is G -homogeneous if and only if it is unitarily equivalent to the tuple of the multiplication operators on either the reproducing kernel Hilbert space with reproducing kernel $\prod_{i=1}^n \frac{1}{(1-z_i\bar{w}_i)^{\lambda_i}}$ or $\prod_{i=1}^n \frac{1}{(1-z_i\bar{w}_i)^\lambda}$, where $\lambda, \lambda_i, 1 \leq i \leq n$, are positive real numbers, according as G is as in (i) or (ii).

Let $\mathbf{T} := (T_1, \dots, T_{n-1})$ be a G -homogeneous $(n-1)$ -tuple of rank 1 Cowen-Douglas class, where G is the the direct product of $n-1$ -copies of the bi-holomorphic automorphism group of the disc. Let \hat{T} be an irreducible homogeneous (with respect to the bi-holomorphic group of automorphisms of the disc) operator in the Cowen-Douglas class on the disc of rank 2. We show that every irreducible G -homogeneous operator, G as in (i), of rank 2 must be of the form

$$(T_1 \otimes I_{\hat{H}}, \dots, T_{n-1} \otimes I_{\hat{H}}, I_H \otimes \hat{T}).$$

We also show that if G is chosen to be the group as in (ii), then there are no irreducible G -homogeneous operators of rank 2. We intend to study the case of a G -homogeneous operator of arbitrary rank n in the future.

11. Hansong Huang, East China University of Science and Technology, China

Title: Rigidity of Polygons and Operator Theory

Abstract: In this talk, we study unitary equivalences and reducing lattices of analytic Toeplitz operators on Bergman spaces over polygons. It is shown that the geometry of polygons plays an important role in the study. This is a joint work with Professor Dechao Zheng.

12. Keiji Izuchi, Niigata University, Japan

Title: Kernels of Hankel Operators on the Hardy Space over the Bidisk

Abstract: In the Hardy space over the bidisk, a kernel of an Hankel operator is an invariant subspace, but it is known that there is an invariant subspace which is not a kernel of any Hankel operator. We shall talk about an invariant subspace which is a kernel of an Hankel operator.

13. Kouhei Izuchi, Yamaguchi University, Japan

Title: Cyclicity of Reproducing Kernels in Weighted Hardy Spaces over the Bidisk

Abstract: In general, the Beurling theorem does not hold for an invariant subspace in the Hardy space over the bidisk. In 1991, Nakazi posed a conjecture that the Beurling theorem holds for a singly generated invariant subspace. In this talk, a

relation between a singly generated invariant subspace and a weighted Hardy space over the bidisk is studied. It is showed that there exists a weighted Hardy space over the bidisk which has a non-cyclic reproducing kernel. Also a counterexample for Nakazi's conjecture is given.

14. László Kérchy, University of Szeged, Hungary

Title: Quasianalytic n -tuples of Hilbert Space Operators

Abstract: Let $\mathbf{T} = (T_1, \dots, T_n)$ be a commuting n -tuple of bounded, linear operators on a separable, complex Hilbert space \mathcal{H} . From another point of view, we may consider \mathcal{H} as a Hilbert module with the multiplication $p \cdot x := p(\mathbf{T})x = p(T_1, \dots, T_n)x$, where $x \in \mathcal{H}$ and p is any polynomial in n complex variables. Continuing and extending former research we investigate the role of unitary asymptotes (X, \mathbf{U}) in the study of \mathbf{T} . Here $\mathbf{U} = (U_1, \dots, U_n)$ is a commuting n -tuple of unitaries on a Hilbert space \mathcal{K} , and $X: \mathcal{H} \rightarrow \mathcal{K}$ is a bounded linear transformation intertwining \mathbf{T} with \mathbf{U} : $XT_i = U_iX$ ($i = 1, \dots, n$); furthermore, the pair (X, \mathbf{U}) fulfills universality conditions. Following categorical approach, orbit conditions are given for the existence of unitary asymptotes. Relying on the induced homomorphism between the commutants of \mathbf{T} and \mathbf{U} , the hyperinvariant subspaces lattice of \mathbf{T} is related to that of \mathbf{U} . Special attention is paid on the quasianalytic case, when the local spectral measures of \mathbf{U} show uniform behaviour on the range of X .

15. Dinesh Kumar Keshari, Indian Statistical Institute, Bangalore

Title: Rigidity of the Flag Structure for a Class of Cowen-Douglas Operators

Abstract: Motivated by homogeneous operators, we consider a subclass of Cowen-Douglas class of operators possessing a flag structure. We show that these operators are irreducible. We also show that the flag structure is rigid, that is, the unitary equivalence class of the operator and the flag structure determine each other. We also obtain a complete set of unitary invariants which are somewhat more tractable than those of an arbitrary operator in the Cowen-Douglas class. This is a joint work with Kui Ji, Chunlan Jiang and Gadadhar Misra.

16. Surjit Kumar, Indian Institute of Science, Bangalore

Title: On Sum of Two Subnormal Kernels

Abstract: For any two positive definite kernels K_1 and K_2 , their sum $K_1 + K_2$ is again a positive definite kernel and therefore determines a Hilbert space $\mathcal{H}(K_1 + K_2)$ of functions. The multiplication operator M_z on $\mathcal{H}(K_1 + K_2)$ is unitarily equivalent to the operator $P_{\mathcal{M}^\perp}(M_{z,1} \oplus M_{z,2})|_{\mathcal{M}^\perp}$, where $M_{z,i}$ is the multiplication operator on $\mathcal{H}(K_i)$ and

$$\mathcal{M} = \{(g, -g) \in \mathcal{H}(K_1) \oplus \mathcal{H}(K_2) : g \in \mathcal{H}(K_1) \cap \mathcal{H}(K_2)\} \subseteq \mathcal{H}(K_1) \oplus \mathcal{H}(K_2).$$

Evidently, if $M_{z,1}$ and $M_{z,2}$ are subnormal, then so is $M_{z,1} \oplus M_{z,2}$.

Here, we discuss the subnormality of the compression $P_{\mathcal{M}^\perp}(M_{z,1} \oplus M_{z,2})|_{\mathcal{M}^\perp}$ for a class of kernels. We show, by means of a class of examples, that if K_1 and K_2 are two positive definite kernels on the unit disc such that the multiplication by the coordinate

function on the corresponding reproducing kernel Hilbert space is subnormal, then the multiplication operator on the Hilbert space determined by their sum $K_1 + K_2$ need not be subnormal. This settles a recent conjecture of Gregory T. Adams, Nathan S. Feldman and Paul J. McGuire in the negative. We also discuss some cases for which the answer is affirmative.

This is a joint work with Soumitra Ghara.

17. Wing Suet Li, Georgia Institute of Technology, USA

Title: On the Geometric Implications of the Horn Inequalities

Abstract: The Horn Conjecture, a characterization of eigenvalues of sums of self-adjoint matrices, is elementary to state (albeit rather technical in combinatorial terms). The proof of the conjecture is involved, and insights from the analysis point of view have so far not been easy to see. We will discuss what we have learned of the geometric information that can be obtained from just the relations of eigenvalues, and the difficulties in proving the elusive Danilov conjecture.

18. Constanze Liaw, CASPER and Department of Mathematics, Baylor University, USA

Title: General Clark Model for Finite Rank Perturbations

Abstract: The unitary perturbations of a given unitary operator by finite rank d operators can be parametrized by $d \times d$ unitary matrices; this generalizes the rank $d = 1$ setting, where the Clark family is parametrized by the scalars on the unit circle. For finite rank perturbations we investigate the functional model of a related class of contractions, as well as a (unitary) Clark operator that realizes such a model representation for a particular contraction. We find a universal representation of the adjoint of the Clark operator, which features a matrix-valued Cauchy integral operator. By universal we simply mean that our formula is given in the coordinate free Nikolski–Vasyunin functional model. We express the matrix-valued characteristic functions of the model (for the class of contractions). In the case of inner characteristic functions results suggest a generalization of the normalized Cauchy transform to the finite rank setting. This presentation is based on joint work with Sergei Treil.

19. Gadadhar Misra, Indian Institute of Science, Bangalore

Title: Homogeneous Hermitian Holomorphic Vector Bundles and the Cowen-Douglas Class over Bounded Symmetric Domains

Abstract: It is known that all the Homogeneous Hermitian holomorphic vector bundles can be obtained by holomorphic induction from representations of a certain group on finite-dimensional inner product spaces. The representations, and the induced bundles, have composition series with irreducible factors. We write down an equivariant constant coefficient differential operator that intertwines the bundle with the direct sum of its irreducible factors. As an application, we show that in the case of the closed unit ball, all the homogeneous n -tuples of Cowen-Douglas operators are similar to direct sums of certain basic n -tuples.

20. Marek Ptak, University of Agriculture in Krakow, Poland

Title: Asymmetric Truncated Toeplitz Operators

Abstract: Let H^2 be the Hardy space on the unit disc, identified as usual with a subspace of L^2 on the unit circle. With any nonconstant inner function θ we associate the model space K_θ^2 , defined by $K_\theta^2 = H^2 \ominus \theta H^2$.

Let us consider two nonconstant inner functions α and θ such that α divides θ . For a certain function $\varphi \in L^2$ we can define an asymmetric truncated Toeplitz operator $A_\varphi: K_\theta^2 \rightarrow K_\alpha^2$ by $A_\varphi f = P_\alpha(\varphi f)$, where $P_\alpha: L^2 \rightarrow K_\alpha^2$ is the orthogonal projection. Characterizations of bounded asymmetric truncated Toeplitz operators with L^2 symbols are given in terms of rank two operators. The relations between this characterizations and the symbol of the operator will be presented. A description of the class of symbols for which the corresponding asymmetric truncated Toeplitz operator is equal to the zero operator is also given.

Joint work with C. Câmara, K. Kliś-Garlicka, J. Blicharz.

21. Yueshi Qin, Department of Mathematics and Statistics, Chongqing University, China

Title: A Note on Rudin's Pathological Submodule

Abstract: Rudin constructed a submodule in $H^2(\mathbb{D}^2)$ that contains no nontrivial bounded functions. We show that this type of submodules is Hilbert-Schmidt by selecting a suitable power sequence. In an attempt to generalize Rudin's submodules, the paper defines *zero-based submodule* and makes a preliminary study about its Hilbert-Schmidtness. This is a joint work with Professor Rongwei Yang.

22. Md. Ramiz Reza, Department of Mathematics, Indian Institute of Science, Bangalore

Title: Curvature Inequalities for Operators in the Cowen-Douglas Class of a Planar Domain

Abstract: Fix a bounded domain Ω in the complex plane \mathbb{C} . If an operator T , in the Cowen-Douglas class $B_1(\Omega)$, admits the compact set $\bar{\Omega}$ as a spectral set, then the curvature inequality $\mathcal{K}_T(w) \leq -4\pi^2 S_\Omega(w, w)^2$, where S_Ω is the Szegő kernel of the domain Ω , is evident. Except when Ω is simply connected, the existence of an operator for which $\mathcal{K}_T(w) = 4\pi^2 S_\Omega(w, w)^2$ for all w in Ω is not known. However, one knows that if w is a fixed but arbitrary point in Ω , then there exists a bundle shift of rank 1, say S , depending on this w , such that $\mathcal{K}_{S^*}(w) = 4\pi^2 S_\Omega(w, w)^2$. We prove that these *extremal* operators are uniquely determined: If T_1 and T_2 are two operators in $B_1(\Omega)$ each of which is the adjoint of a rank 1 bundle shift and $\mathcal{K}_{T_1}(w) = -4\pi^2 S(w, w)^2 = \mathcal{K}_{T_2}(w)$ for a fixed w in Ω , then T_1 and T_2 are unitarily equivalent. A surprising consequence is that the adjoint of only some of the bundle shifts of rank 1 occur as extremal operators in domains of connectivity > 1 . These are described explicitly.

23. Jaydeb Sarkar, Indian Statistical Institute, Bangalore

Title: Variations on a Theme of Beurling and Hilbert Modules

Abstract: First we will formalize (along with a proof [1]) the classical Beurling theorem in the Language of Hilbert modules over the ring of polynomials. With this motivation, we then survey prior work on submodules, quotient modules of general Hilbert modules, both in one and several variables, and remark on some interpretation of the invariant subspace (or, representations of submodules of Hilbert modules) problem.

Finally, we will discuss the Beurling's theme for a concrete (or, more tractable) case - commuting pair of isometries/contractions (that is, contractive Hilbert modules over $\mathbb{C}[z_1, z_2]$). Along the way, we will discuss the failure and success of the classical von Neumann inequality for a general (and contractive) Hilbert module.

REFERENCES

- [1] R. G. Douglas, Variations on a theme of Beurling, *New York J. Math.* **17a**, (2011), 1–10.

24. Shailesh Trivedi, Indian Institute of Technology, Kanpur

Title: Homogeneous Hilbert Modules Associated with Locally Finite Rooted Directed Trees

Abstract: Let $\mathcal{T} = (V, \mathcal{E})$ be a leafless, locally finite rooted directed tree. We associate with \mathcal{T} a one parameter family of reproducing kernel Hilbert spaces \mathcal{H}_q ($q \in \mathbb{N}$ and $q \geq 2$) of vector-valued holomorphic functions defined on the unit disc \mathbb{D} in the complex plane. The associated reproducing kernel is given by

$$\begin{aligned} \kappa_{\mathcal{H}_q}(z, w) &= \sum_{n=0}^{\infty} \frac{(q)_n}{(1)_n} z^n \bar{w}^n P_{\langle e_{\text{root}} \rangle} \\ &+ \sum_{v \in V_{\downarrow}} \sum_{n=0}^{\infty} \frac{(n_v + q + 1)_n}{(n_v + 2)_n} z^n \bar{w}^n P_{l^2(\text{Chi}(v)) \ominus \langle \lambda^v \rangle} \quad (z, w \in \mathbb{D}). \end{aligned}$$

where V_{\downarrow} denotes the set of branching vertices of \mathcal{T} , n_v denotes the depth of $v \in V$ in \mathcal{T} , and $P_{\langle e_{\text{root}} \rangle}$, P_v ($v \in V_{\downarrow}$) are certain orthogonal projections. The spaces \mathcal{H}_q can be realized as Hilbert modules over the polynomial ring $\mathbb{C}[z]$. We show that \mathcal{H}_q is a homogeneous Hilbert module if and only if \mathcal{T} is isomorphic to \mathbb{N} .

This is a joint work with Sameer Chavan and Deepak K. Pradhan.

25. Harald Upmeyer, University of Marburg, Germany

Title: Analysis and Reproducing Kernels on Kepler Manifolds Modules

Abstract: Kepler manifolds and varieties form an important class of complex analytic varieties which are defined via determinantal equations. They also carry Kähler metrics induced by pluri-subharmonic functions, giving rise to reproducing kernel Hilbert spaces. In joint work with M. Englis, we study Kepler manifolds in the general framework of Jordan algebras as orbits of a complex reductive Lie group. The principal results are

- Kepler varieties are normal varieties (allowing a Riemann type extension theorem)

- Peter-Weyl expansion of the reproducing kernels, for a large class of pluri-subharmonic Kähler potentials.
- Asymptotic (Tian-Yau-Zelditch) expansion of the reproducing kernel, in terms of a natural quantization parameter,
- Characterization and polar decomposition of invariant n -forms and measures on Kepler manifolds

Some consequences concerning Hankel type operators are also given.

26. Penghui Wang, Shandong University, China

Title: Essential Normality of Quotient Modules of Analytic Hilbert Modules over the Polydisc

Abstract: In this talk, I will give some recent progress on the essential normality of quotient modules over the polydisc. A polydisc version of Arveson's conjecture will be considered. The talk is based on the joint works with K. Guo, and with C. Zhao.

27. Yue Wu, Central University of Finance and Economics, China

Title: Lorentz Groups of Analytic Hilbert Modules

Abstract: Let M be a submodule and C_M be the associated core operator in $H^2(\mathbb{D}^2)$. We give a representation of M by using a Kreĭn space indefinite metric on the range of C_M . This talk will show some results about the group (called Lorentz group) of isometric self-maps of M with respect to the indefinite metric. Further, the Lorentz group contains an interesting abelian subgroup (called little Lorentz group) which turns out to be a finer invariant for M .

28. Jingbo Xia, State University of New York at Buffalo, USA

Title: On the Essential Commutant of the Toeplitz Algebra on the Bergman Space

Abstract: Let \mathcal{T} be the C^* -algebra generated by the Toeplitz operators $\{T_f : f \in L^\infty(\mathbf{B}, dv)\}$ on the Bergman space of the unit ball. We show that the essential commutant of \mathcal{T} equals $\{T_g : g \in \text{VO}_{\text{bdd}}\} + \mathcal{K}$, where VO_{bdd} is the collection of bounded functions of vanishing oscillation on \mathbf{B} and \mathcal{K} denotes the collection of compact operators on $L_a^2(\mathbf{B}, dv)$.

29. Yixin Yang, Dalian University of Technology, China

Title: Reducibility and Unitarily Equivalence for a Class of Truncated Toeplitz Operators

Abstract: Truncated Toeplitz operators (TTO) on model space have been formally introduced by Sarason in [1], mostly notably as model operators for completely nonunitary contractions with defect numbers one and their commutant. The algebraic prosperities has blossomed over the last several years. On the reducing subspace of the TTO, by using the model theory for C_0 operators, R. G. Douglas and C. Foias gave a necessary and sufficient condition of reducibility of A_{z^2} (see [2]). In this talk, we give a necessary and sufficient condition of reducibility of A_φ^θ induced by Blaschke product with two zeros. Also, we prove that if A_φ^θ is reducible, the restriction of A_φ^θ

on a reducing subspace is unitarily equivalent to $A_{\frac{\phi}{z}}$ for some inner function ϕ . This is joint work with Yufei Li and Yufeng Lu.

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- [1] D. Sarason, Algebraic properties of truncated Toeplitz operators, *Oper. Matrices* **1** (2007), no. 4, 491–526.
- [2] R. G. Douglas and C. Foias, On the structure of the square of a $C_0(1)$ operator, *Modern Operator Theory and Applications, Oper. Theory Adv. Appl.*, Birkhäuser, Basel, **170** (2007), 75–84.

30. Dechao Zheng, Vanderbilt University, USA

Title: A Theorem of Brown-Halmos type On the Bergman Space Modular Finite Rank Operators

Abstract: In this talk, I will discuss when a product of two Toeplitz operators with bounded harmonic symbols is a finite rank perturbation of a Toeplitz operator on the Bergman space. We establish a Brown-Halmos type theorem for rank one perturbation. It is surprising that we construct examples that a product of two Toeplitz operators with bounded harmonic symbols is a rank greater than one perturbation of a Toeplitz operator with bounded harmonic symbol. This is a joint work with Xuanhao Ding and Yueshi Qin.