



**Tsinghua Sanya International Mathematics Forum**

(清华三亚国际数学论坛)

**Recent Advances in Extremal  
Combinatorics Workshop**

**May 22-26, 2017**

No.100 Tsinghua Rd., Tianya District, Sanya, Hainan.

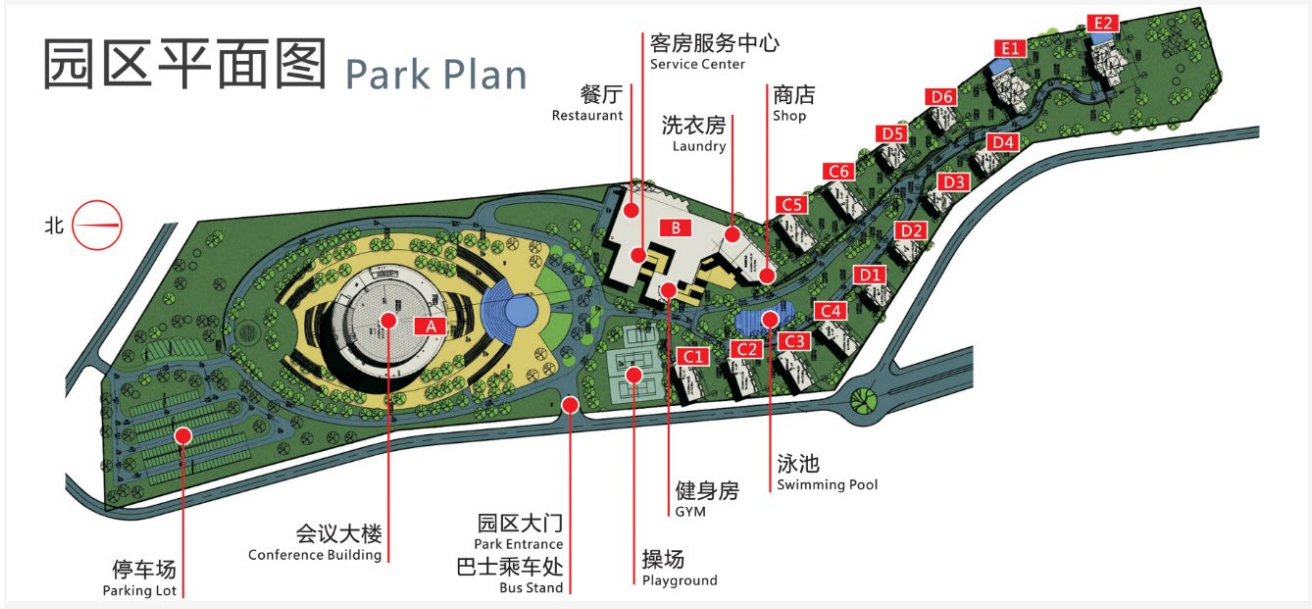
(海南省三亚市天涯区清华路100号)

## **Welcome to TSIMF**

The facilities of TSIMF are built on a 23-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 29,000 square meter that includes state-of-the-art conference facilities (over 10,000 square meter) to hold many international workshops simultaneously, two libraries, a guest house (over 10,000 square meter) and the associated catering facilities, a large swimming pool, gym and sports court and other recreational facilities.

Mathematical Sciences Center (MSC) of Tsinghua University, assisted by TSIMF's International Advisory Committee and Scientific Committee, will take charge of the academic and administrative operation of TSIMF. The mission of TSIMF is to become a base for scientific innovations, and for nurturing of innovative human resource; through the interaction between leading mathematicians and core research groups in pure mathematics, applied mathematics, statistics, theoretical physics, applied physics, theoretical biology and other relating disciplines, TSIMF will provide a platform for exploring new directions, developing new methods, nurturing mathematical talents, and working to raise the level of mathematical research in China.

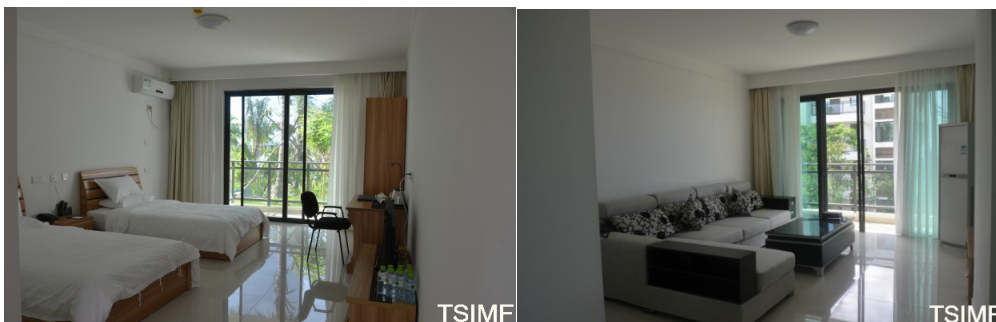
## About Facilities



## Registration

Conference booklets, room keys and name badges for all participants will be distributed at the Registry. Please take good care of your name badge. It is also your meal card and entrance ticket for all events.

## Guest Room



Conference Center can receive about 378 people having both single and double rooms, and 42 family rooms.

All the rooms are equipped with: free Wi-Fi, TV, air conditioning and other utilities.

Family rooms are also equipped with kitchen and refrigerator.

## Library

**Opening Hours: 08:00am-22:00pm**

TSIMF library is available during the conference and can be accessed by using your room card. There is no need to sign out books but we ask that you kindly return any borrowed books to the book cart in library before your departure.

## Restaurant



All the meals are provided in the Chinese Restaurant (Building B1) according to the time schedule.

Breakfast	07:30-08:30
Lunch	12:00-13:30
Dinner	17:30-19:00

## Laundry

**Opening Hours: 24 hours**

The self-service laundry room is located in the Building 1 (B1), next to the shop.

## Gym

The gym is located in the Building 1 (B1), opposite to the reception hall. The gym provides various fitness equipment, as well as pool tables, tennis tables and etc.

## Playground

Playground is located on the east of the central gate. There you can play basketball, tennis and badminton. Meanwhile, you can borrow table tennis, basketball, tennis balls and badminton at the reception desk.

## Swimming Pool



Please note that there are no lifeguards. We will not be responsible for any accidents or injuries. In case of any injury or any other emergency, please call the reception hall at +86-898-38882828.

## Outside Shuttle Service:

We have shuttle bus to take participants to the airport for your departure service. Also, we would provide transportation at the Haipo Square (海坡广场) of Howard Johnson for the participants who will stay outside TSIMF. If you have any questions about transportation arrangement, please feel free to contact Ms. Li Ye (叶莉), her cell phone number is (0086)139-7679-8300.

## Free Shuttle Bus Service at TSIMF:

We provide free shuttle bus for participants and you are always welcome to take our shuttle bus, all you need to do is wave your hands to stop the bus.

Destinations: Conference Building, Reception Room, Restaurant, Swimming Pool, Hotel etc.



## Contact Information of Administration Staffs

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## Schedule for the Workshop on Extremal Combinatorics, May 22-26, 2017

Time & Date	Monday (May 22)	Tuesday (May 23)	Wednesday (May 24)	Thursday (May 25)	Friday (May 26)		
7:30-8:30	<i>Breakfast</i>						
<i>Chair</i>	<i>Xingxing Yu</i>	<i>Prasad Tetali</i>	<i>Tibor Szabo</i>	<i>Andrzej Rucinsk</i>	<i>Hao Huang</i>		
8:45-9:20	Balazs Patkos	Asaf Shapira	Shagnik Das	Louis DeBiasio	Fan Wei		
9:25-10:00	Andrey Kupavskii	Michael Krivelevich	Allan Lo	Andrzej Dudek	Linyuan Lu		
10:00-10:30	<i>Tea Break</i>						
10:30-11:05	Tokushige Norihide	Mathias Schacht	Po-Shen Loh	Andrew Treglown	Xing Peng		
11:10-11:45	Hao Huang	Caroline Terry	Richard Mycroft	Douglas West	Discussion		
12:00-13:00	<i>Lunch</i>						
<i>Chair</i>	<i>Penny Haxell</i>	<i>Xuding Zhu</i>	—	<i>Guantao Chen</i>	—		
13:00-14:25	Discussion	Group Photo 14:10-14:20	Free Discussion 13:30-17:00				
14:25-15:00	David Conlon	Alexandr Kostochka				Discussion	Penny Haxell
15:00-15:30	<i>Tea Break</i>					<i>Tea break</i>	
15:30-16:05	Zoltán Füredi	Andrzej Rucinski				Tibor Szabó	Free Discussion 13:00-18:00
16:10-16:45	Jozsef Solymosi	Yuejian Peng	Banquet 18:00-20:00				
16:45-18:00	Discussion	Problem Session 16:55-17:55				Liana Yepremyan	Discussion
18:00-19:30	<i>Dinner</i>		<i>Dinner</i>				

**Ps: As for the "Problem Session" on the afternoon of Tuesday, please kindly prepare some problems for this session.**

## Titles and Abstracts

1. David Conlon, University of Oxford, United Kingdom

**Title:** Unavoidable patterns in words

**Abstract:** A word  $w$  is said to contain the pattern  $P$  if there is a way to substitute a nonempty word for each letter in  $P$  so that the resulting word is a subword of  $w$ . Bean, Ehrenfeucht and McNulty and, independently, Zimin characterised the patterns  $P$  which are unavoidable, in the sense that any sufficiently long word over a fixed alphabet contains  $P$ . Zimin's characterisation says that a pattern is unavoidable if and only if it is contained in a Zimin word, where the Zimin words are defined by  $Z_1 = x_1$  and  $Z_n = Z_{n-1}x_nZ_{n-1}$ . We study the quantitative aspects of this theorem, obtaining essentially tight tower-type bounds for the function  $f(n, q)$ , the least integer such that any word of length  $f(n, q)$  over an alphabet of size  $q$  contains  $Z_n$ .

Joint work with Jacob Fox and Benny Sudakov.

2. Shagnik Das, Freie Universität Berlin, Germany

**Title:** A Semi-Random Construction of Small Covering Arrays

**Abstract:** Given a set  $S$  of  $v \geq 2$  symbols, and integers  $k \geq t \geq 2$  and  $N \geq 1$ , an  $N \times k$  array  $A \in S^{N \times k}$  is an  $(N; t, k, v)$ -covering array if all sequences in  $S^t$  appear as rows in every  $N \times t$  subarray of  $A$ . These arrays have a wide variety of applications, driving the search for small covering arrays. The covering array number,  $\text{CAN}(t, k, v)$ , is the smallest  $N$  for which an  $(N; t, k, v)$ -covering array exists.

In this talk we will combine probabilistic and linear algebraic constructions to improve the upper bounds on  $\text{CAN}(t, k, v)$  by a factor of  $\ln v$ , showing that for prime powers  $v$ ,  $\text{CAN}(t, k, v) \leq (1 + o(1)) ((t-1)v^t / (2 \lg v - \lg(v+1))) \lg k$ , which also offers improvements for large  $v$  that are not prime powers. A key step, which may be of independent interest, will be the construction of an array with  $v^t$  rows that covers the maximum possible number of subsets of size  $t$ .

This is joint work with Tamás Mészáros.

3. Louis DeBiasio, Miami University, United States

**Title:** Infinite graph-Ramsey theory

**Abstract:** Ramsey's theorem guarantees a monochromatic copy of any countably infinite graph  $G$  in any  $r$ -coloring of the edges of the complete graph  $K_{\mathbb{N}}$ . It is natural to wonder how "large" of a monochromatic copy of  $G$  we can find with respect to some measure – for instance, the (upper) density of the vertex set of  $G$  in  $\mathbb{N}$ . Unlike finite graph-Ramsey theory, where this question has been studied extensively, the infinite version has been mostly overlooked.

Erdős and Galvin proved that in every 2-coloring of  $K_{\mathbb{N}}$ , there exists a monochromatic path whose vertex set has upper density at least  $2/3$ , but it is not possible to do better than  $8/9$ . They also showed that there exists a monochromatic path  $P$  such that for infinitely many  $n$ , the set  $\{1, 2, \dots, n\}$  contains the first  $\frac{n}{3+\sqrt{3}}$  vertices



of  $P$ , but it is not possible to do better than  $2n/3$ . We improve both results, in the former case achieving an upper density at least  $3/4$  and in the latter case obtaining a tight bound of  $2/3$ . Inspired by this, we consider infinite analogs of well-known finite results on directed paths, trees (connected subgraphs), and graphs of bounded maximum degree/chromatic number.

Joint work with Paul McKenney.

4. Andrzej Dudek, Western Michigan University, United States

**Title:** Multicolor Ramsey Properties of Random Graphs and Hypergraphs

**Abstract:** First we focus on the size-Ramsey number of a path  $P_n$  on  $n$  vertices. In particular, we show that  $5n/2 - 15/2 \leq \hat{r}(P_n) \leq 74n$  for  $n$  sufficiently large. This improves the previous lower bound due to Bollobás, and the upper bound due to Letzter. Next we study long monochromatic paths in edge-colored random graph  $G(n, p)$  with  $pn \rightarrow \infty$ . Recently, Letzter showed that a.a.s. any 2-edge coloring of  $G(n, p)$  yields a monochromatic path of length  $(2/3 - o(1))n$ , which is optimal. Extending this result, we show that a.a.s. any 3-edge coloring of  $G(n, p)$  yields a monochromatic path of length  $(1/2 - o(1))n$ , which is also optimal. We will also discuss this problem for an arbitrary number of colors. We also consider a related problem and show that for any  $r \geq 2$ , a.a.s. any  $r$ -edge coloring of  $G(n, p)$  yields a monochromatic connected subgraph on  $(1/(r-1) - o(1))n$  vertices, which is also tight. Finally, we discuss some extensions of the above results for random hypergraphs.

Joint work with Paweł Prałat and also with Patrick Bennett, Louis DeBiasio, and Sean English.

5. Zoltán Füredi, MTA Rényi Institute of Mathematics, Hungary and University of Illinois at Urbana-Champaign, United States

**Title:** Almost similar configurations

**Abstract:** This is a talk about how to use extremal (hyper)graph theory, i.e., Turán type results, to solve combinatorial geometry problems. We illustrate the method with the question of almost similar configurations.

Let  $A \subset \mathbb{R}^2$  be a fixed  $k$ -set. Two points sets  $A = \{a_1, \dots, a_k\}$  and  $B = \{b_1, \dots, b_k\}$  are  $\varepsilon$ -similar if  $1 - \varepsilon < \frac{|a_i a_j|}{|b_i b_j|} < 1 + \varepsilon$  for all  $i \neq j$ . Let  $h(n, A, \varepsilon)$  denote the maximum number of  $\varepsilon$ -similar copies of  $A$  in an  $n$ -element planar set. An obvious construction shows that  $h(n, A, \varepsilon) \geq (n/k)^k$  for any  $\varepsilon > 0$ . Conway, Croft, Erdős, and Guy (1979) studied the case when  $A$  is the regular triangle. Improving their result we show that for all triangles  $T$  that are close enough to a regular one there exist  $\delta = \delta(T) > 0$  such that  $h(n, T, \varepsilon) = (1 + o(1))n^3/24$  as  $n \rightarrow \infty$  and  $\varepsilon \in (0, \delta(T))$  is fixed. However, there are triangles with  $h(n, T, \varepsilon) > n^3/15$ .

It is a joint work with Imre Bárány (MTA Rényi Institute, Budapest, Hungary, and Department of Mathematics, University College London, U.K.)

6. Penny Haxell, University of Waterloo, Canada

**Title:** Algorithms for independent transversals

**Abstract:** An *independent transversal* (IT) in a vertex-partitioned graph  $G$  is an independent set in  $G$  consisting of one vertex in each partition class. This is a very basic notion that comes up in many combinatorial problems. There are various criteria that guarantee the existence of an IT in a given graph  $G$ . For example, if each partition class has size at least  $2\Delta(G)$  then  $G$  has an IT.

The known proofs of these criteria do not give efficient algorithms for actually finding an IT. Here we discuss appropriate weakenings of such results that do have effective proofs.

7. Hao Huang, Emory University, United States

**Title:** Degree versions of some classical results in Extremal Combinatorics

**Abstract:** In this talk, I will describe how to use algebraic methods to prove the following degree version of the celebrated Erdős-Ko-Rado theorem: given  $n > 2k$ , every intersecting  $k$ -uniform hypergraph  $H$  on  $n$  vertices must contain a vertex that lies on at most  $\binom{n-2}{k-2}$  edges. We have also developed a degree version of Hilton-Milner theorem for sufficiently large  $n$ .

The talk is based on joint works with Peter Frankl, Jie Han and Yi Zhao.

8. Alexandr Kostochka, University of Illinois at Urbana-Champaign, USA and Sobolev Institute of Mathematics, Novosibirsk, Russia

**Title:** Two results on cycles in graphs

**Abstract:** The goal of the talk is to discuss two recent results on cycle structure of graphs.

Let  $V_{\geq t}(G)$  (respectively,  $V_{\leq t}(G)$ ) denote the set of vertices in  $G$  of degree at least (respectively, at most)  $t$ . Dirac and Erdős in 1963 extended the Corrádi-Hajnal Theorem as follows: *If  $k \geq 3$  and  $G$  is a graph with  $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)| \geq k^2 + 2k - 4$ , then  $G$  has  $k$  disjoint cycles.* They also presented a series of examples of graphs  $G$  with  $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)| = 2k - 1$  that do not have  $k$  disjoint cycles.

We show that if  $n \geq 19k$ , then each graph  $G$  with  $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)| \geq 2k$  has  $k$  disjoint cycles and conjecture that this holds if  $n \geq 4k + 1$ . This is joint work with H. Kierstead and A. McConvey.

For  $n > 2a$ , let  $H(n, a)$  be the  $n$ -vertex graph obtained from  $K_{n-a}$ , say with vertex set  $A$ , by adding  $a$  vertices of degree  $a$  each of which is adjacent to the same  $a$  vertices in  $A$ . By construction,  $H(n, a)$  is not hamiltonian and has  $h(n, a) = \binom{n-a}{2} + a^2$  edges. Erdős proved in 1962 that *for any  $d < n/2$  each nonhamiltonian  $n$ -vertex graph with minimum degree at least  $d$  has at most  $\max\{h(n, d), h(n, \lfloor \frac{n-1}{2} \rfloor)\}$  edges.* We generalize and sharpen this result. This is joint work with Z. Füredi and R. Luo.

9. Michael Krivelevich, Tel Aviv University, Israel

**Title:** Finding and using expanders in locally sparse graphs

**Abstract:** Our main goal is to find large expanders in locally sparse graphs. For constants  $c_1 > c_2 > 1, 0 < a < 1$ , a graph  $G$  on  $n$  vertices is called a  $(c_1, c_2, a)$ -graph if it has at least  $c_1 \cdot n$  edges, but every vertex subset  $W$  of size at most  $a \cdot n$  spans less than  $c_2 \cdot |W|$  edges. (Putting it informally, the local density of  $G$  is sizably

smaller than its global density.) For example, sparse random graphs  $G$  drawn from  $G(n, C/n)$  are typically locally sparse. We show that every  $(c_1, c_2, a)$ -graph  $G$  with bounded degrees contains a spanning expander of linear order. The proof can be made algorithmic. Time permitting, we will discuss applications of this result to problems about embedding graph minors, and to positional games

10. Andrey Kupavskii, Moscow Institute of Physics and Technology, Russia and Ecole Polytechnique Fédérale de Lausanne, Switzerland

**Title:** Families with forbidden subconfigurations

**Abstract:** Put  $[n] := \{1, 2, \dots, n\}$  and let  $2^{[n]}$  denote the power set of  $[n]$ . A subset  $\mathcal{F} \subset 2^{[n]}$  is called a *family of subsets of  $[n]$* , or simply a *family*.

The maximum number of pairwise disjoint members of a family  $\mathcal{F}$  is denoted by  $\nu(\mathcal{F})$  and is called the *matching number* of  $\mathcal{F}$ . It is a classical question due to Erdős to determine, how big a family  $\mathcal{F} \subset 2^{[n]}$  could be, if  $\nu(\mathcal{F}) < s$  for some integer  $s$ . Let us denote this number  $e(n, s)$ . The value  $e(n, s)$  was found by Kleitman [4] for  $n = sm, sm - 1$ . Quinn [5] found the value of  $e(3m + 1, 3)$ .

In our recent work, we determined the values  $e(sm - 2, s)$  for all  $m$ , as well as the values of  $e(sm - l, s)$  for  $s > lm + 3l + 3$ . In this talk, I wanted to discuss the method we used to reprove Quinn's result, as well as to determine  $e(4m + 2, 4)$  (see [1]). This method seems to be rather general, and it already allowed us to resolve completely the following problem, also posed by Erdős: what is the maximum size of  $\mathcal{F} \subset 2^{[n]}$ , such that  $\mathcal{F}$  does not contain two disjoint sets and their union? The answer for  $n = 3m + 1$  was obtained by Kleitman [3], and we settled [2] the other two cases:  $n = 3m$  and  $n = 3m + 2$ .

We also discuss the uniform case of the first problem, also known as the Erdős Matching Conjecture. This is joint work with Peter Frankl.

#### REFERENCES

- [1] P. Frankl, A. Kupavskii, *The largest families of sets with no matching of sizes 3 and 4*, arXiv:1701.04107
- [2] P. Frankl, A. Kupavskii, *Families of sets with no two disjoint sets and their union*, preprint
- [3] D.J. Kleitman, *On families of subsets of a finite set containing no two disjoint sets and their union*, J. Combin. Theory 5 (1968), N3, 235–237.
- [4] D.J. Kleitman, *Maximal number of subsets of a finite set no  $k$  of which are pairwise disjoint*, Journ. of Comb. Theory 5 (1968), 157–163.
- [5] F. Quinn, PhD Thesis, Massachusetts Institute of Technology (1986).

11. Allan Lo, University of Birmingham, United Kingdom

**Title:** Designs beyond quasirandomness

**Abstract:** The Existence conjecture for combinatorial designs states that every large complete  $k$ -graph can be edge-decomposed into small cliques (subject to the obvious divisibility condition). This conjecture was proved by a recent breakthrough of Keevash. In joint work with Stefan Glock, Daniela Kühn and Deryk Osthus, we gave a new proof of this result, based on the method of iterative absorption. In fact, ‘regularity boosting’ allows us to extend our main decomposition result beyond the

quasirandom setting and thus to generalise the results of Keevash. In particular, we obtain a resilience version and a minimum degree version.

12. Po-Shen Loh, Carnegie Mellon University, United States

**Title:** Induced Turán numbers

**Abstract:** The classical Kővári-Sós-Turán theorem states that if  $G$  is an  $n$ -vertex graph with no copy of  $K_{s,t}$  as a subgraph, then the number of edges in  $G$  is at most  $O(n^{2-1/s})$ . We prove that if one forbids  $K_{s,t}$  as an *induced* subgraph, and also forbids *any* fixed graph  $H$  as a (not necessarily induced) subgraph, the same asymptotic upper bound still holds, with different constant factors. This introduces a nontrivial angle from which to generalize Turán theory to induced forbidden subgraphs, which this paper explores. Along the way, we derive a nontrivial upper bound on the number of cliques of fixed order in a  $K_r$ -free graph with no induced copy of  $K_{s,t}$ . This result is an induced analog of a recent theorem of Alon and Shikhelman and is of independent interest.

13. Linyuan Lu, University of South Carolina, United States

**Title:** A Bound on the Spectral Radius of Hypergraphs with  $e$  Edges

**Abstract:** For  $r \geq 3$ , let  $f_r: [0, \infty) \rightarrow [1, \infty)$  be the unique analytic function such that  $f_r(\binom{k}{r}) = \binom{k-1}{r-1}$  for any  $k \geq r-1$ . We prove that the spectral radius of an  $r$ -uniform hypergraph  $H$  with  $e$  edges is at most  $f_r(e)$ . The equality holds if and only if  $e = \binom{k}{r}$  for some positive integer  $k$  and  $H$  is the union of a complete  $r$ -uniform hypergraph  $K_k^r$  and some possible isolated vertices. This result generalizes the classical Stanley's theorem on graphs. Joint work with Shuliang Bai.

14. Richard Mycroft, University of Birmingham, United Kingdom

**Title:** Unavoidable trees in tournaments

**Abstract:** We say that an oriented tree  $T$  is *unavoidable* if  $T$  is contained in every tournament on the same number of vertices. Bender and Wormald showed that almost all oriented trees are 'nearly-unavoidable', in the sense that they are contained in almost all tournaments on the same number of vertices, and conjectured that in fact almost all oriented trees are unavoidable. We prove this conjecture by showing that every oriented tree with a certain property is unavoidable, and that almost all oriented trees have this property.

Joint work with Tássio Naia

15. Balazs Patkos, Alfrd Rnyi Institute of Mathematics, Hungary

**Title:** Generalized forbidden subposet problems

**Abstract:** A subfamily  $\{F_1, F_2, \dots, F_{|P|}\} \subseteq \mathcal{F}$  of sets is a copy of a poset  $P$  in  $\mathcal{F}$  if there exists a bijection  $\phi: P \rightarrow \{F_1, F_2, \dots, F_{|P|}\}$  such that whenever  $x \leq_P x'$  holds, then so does  $\phi(x) \subseteq \phi(x')$ . For a family  $\mathcal{F}$  of sets, let  $c(P, \mathcal{F})$  denote the number of copies of  $P$  in  $\mathcal{F}$ , and we say that  $\mathcal{F}$  is  $P$ -free if  $c(P, \mathcal{F}) = 0$  holds. For any two posets  $P, Q$  let us denote by  $La(n, P, Q)$  the maximum number of copies

of  $Q$  over all  $P$ -free families  $\mathcal{F} \subseteq 2^{[n]}$ , i.e.  $\max\{c(Q, \mathcal{F}) : \mathcal{F} \subseteq 2^{[n]}, c(P, \mathcal{F}) = 0\}$ . This generalizes the well-studied parameter  $La(n, P) = La(n, P, P_1)$  where  $P_1$  is the one element poset. In this talk we consider the problem of determining  $La(n, P, Q)$  when  $P$  and  $Q$  are small posets, like chains, forks, the  $N$  poset, etc. Our main result determines  $La(n, P_{h(Q)}, Q)$  up to some polynomial factor where  $Q$  is any complete multi-level poset,  $P_j$  is the chain of length  $j$  and  $h(Q)$  is the height of  $Q$  (the length of the longest chain in  $Q$ ). To obtain this, we solve (up to a polynomial factor) a problem on  $r$ -wise intersections in antichains.

16. Xing Peng, Tianjin University, China

**Title:** Turan problems for infinite graphs

**Abstract:** There is a large volume of literature studying Turan type problems of finite graphs. However, the case of infinite graphs is much less known. Motivated by a result of Erdos on the infinite Turan number of increasing paths, we will introduce the infinite Turan number for general graphs and discuss recent progress on this topic. Joint work with Craig Timmons.

17. Yuejian Peng, Hunan University, China

**Title:** Lagrangian densities of hypergraph paths and Turán numbers of their extensions

**Abstract:** Given a positive integer  $n$  and an  $r$ -uniform hypergraph  $F$ , the *Turán number*  $ex(n, F)$  is the maximum number of edges in an  $F$ -free  $r$ -uniform hypergraph on  $n$  vertices. The *Turán density* of  $F$  is defined as  $\pi(F) = \lim_{n \rightarrow \infty} \frac{ex(n, F)}{\binom{n}{r}}$ . The *Lagrangian density* of  $F$  is  $\pi_\lambda(F) = \sup\{r!\lambda(G) : G \text{ is } F\text{-free}\}$ . It was observed by Sidorenko and Pikhurko that  $\pi(F) \leq \pi_\lambda(F)$ , and  $\pi(F) = \pi_\lambda(F)$  if every pair of vertices in  $F$  is contained in an edge of  $F$ . Recently, Lagrangian densities of hypergraphs and Turán numbers of their extensions have been studied actively. We obtain the Lagrangian density of the 3-uniform linear paths. Applying it, we get the Turán numbers of their extensions, and show the uniqueness of the extremal hypergraphs.

18. Andrzej Rucinski, Adam Mickiewicz University, Poland

**Title:**  $\frac{5}{9}$  or Minimum Vertex Degree Condition for Tight Hamiltonian Cycles in 3-Uniform Hypergraphs

**Abstract:** The study of Dirac type questions for hypergraphs was initiated in the seminal paper by Gyula Y. Katona and Hal Kierstead ([2]). Improving upon earlier results obtained in [1], [3], and [4], we show that every 3-uniform hypergraph with  $n$  vertices and minimum degree at least  $(\frac{5}{9} + o(1))\binom{n}{2}$  contains a tight Hamiltonian cycle. Owing to known lower bound constructions, this degree condition is asymptotically optimal. This is joint work with Christian Reiher, Vojtěch Rödl, Mathias Schacht, and Endre Szemerédi.

## REFERENCES

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- [2] Gy. Y. Katona, H. Kierstead, *Hamiltonian chains in hypergraphs*, J. Graph Theory, 30(3) (1999) 205–212.
- [3] V. Rödl, A. Rucinski, *Families of triples with high minimum degree are Hamiltonian*, Discuss. Math. Graph Theory, 34(2) (2014) 361–381.
- [4] V. Rödl, A. Rucinski, M. Schacht, E. Szemerédi *On the Hamiltonicity of triple systems with high minimum degree* Annals of Combin., 21(1) (2017) 95–117.

19. Mathias Schacht, Universität Hamburg, Hamburg, Germany

**Title:** Forcing Quasirandomness with Triangles

**Abstract:** We study *forcing pairs* for *quasirandom graphs*. Chung, Graham, and Wilson initiated the study of families  $\mathcal{F}$  of graphs with the property that if a large graph  $G$  has approximately homomorphism density  $p^{e(F)}$  for some fixed  $p \in (0, 1]$  for every  $F \in \mathcal{F}$ , then  $G$  is quasirandom with density  $p$ . Such families  $\mathcal{F}$  are said to be *forcing*. Several forcing families were found over the last three decades and characterising all bipartite graphs  $F$  such that  $(K_2, F)$  is a forcing pair is a well known open problem in the area of quasirandom graphs, which is closely related to Sidorenko’s conjecture. In fact, most of the known forcing families involve bipartite graphs only.

We consider forcing pairs containing the triangle  $K_3$ . In particular, we show that if  $(K_2, F)$  is a forcing pair, then so is  $(K_3, F^\Delta)$ , where  $F^\Delta$  is obtained from  $F$  by replacing every edge of  $F$  by a triangle (each of which introducing a new vertex). For the proof we first show that  $(K_3, C_4^\triangleright)$  is a forcing pair, which strengthens related results of Simonovits and Sós and of Conlon et al. This is joint work with Chr. Reiher.

20. Asaf Shapira, Tel Aviv University, Israel

**Title:** Removal Lemmas with Polynomial Bounds

**Abstract:** Addressing a problem of Alon and Fox, we prove new sufficient and necessary criteria, guaranteeing that a graph property admits a removal lemma with a polynomial bound. Although both are simple combinatorial criteria, they imply almost all prior positive and negative results of this type, as well as many new ones. In particular, we show that every semi-algebraic graph property admits a polynomially bounded removal lemma. This confirms a conjecture of Alon.

Joint work with L. Gishboliner

21. Jozsef Solymosi, University of British Columbia, Canada

**Title:** Extremal Combinatorics in Discrete Analysis

**Abstract:** In this talk I will present examples how to translate arithmetic problems into questions in extremal combinatorics. Most of the problems I will consider have a hypergraph model where we ask for the maximum number of edges under certain conditions. A typical example is a problem of Katz and Tao: If the sumset of an

$n$ -element set of an abelian group is at most  $n$  along a graph,  $G_n$ , then the difference set along the edges of  $G_n$  should be much less than quadratic in  $n$ . Katz and Tao gave the  $n^{11/6}$  upper bound. We will see how is this problem related to the question of extremal  $C_5$ -free graphs of 3-uniform hypergraphs.

Joint work with Dhruv Mubayi

22. Tibor Szabó, Freie Universität Berlin, Germany

**Title:** On the complexity of Ryser's Conjecture

**Abstract:** Ryser's Conjecture states that for every  $r$ -partite  $r$ -uniform hypergraph it is possible to find a vertex cover of size only  $(r - 1)$ -times the matching number. For  $r = 2$ , the conjecture reduces to König's Theorem, for  $r = 3$  it was proved by Aharoni by topological methods, while for larger  $r$  it is wide open. In the talk we will be focusing on those hypergraphs that are extremal for Ryser's Conjecture, that is their vertex cover number is exactly  $(r - 1)$ -times their matching number. An intricate extension of Aharoni's topological method (obtained with Penny Haxell and Lothar Narins) provides a characterization of the 3-uniform Ryser-extremal hypergraphs. In the talk we describe a recent joint work with Ahmad Abu-Khazneh, Jnos Bart, and Alexey Pokrovskiy, where we construct exponentially many non-isomorphic Ryser-extremal hypergraphs, for a new infinite sequence of uniformities  $r$ . We speculate that these constructions provide further evidence that Ryser's Conjecture, if true in general, will not be solved by purely combinatorial methods.

23. Caroline Terry, University of Maryland, United States

**Title:** The jump to the fastest speed of a hereditary property of  $r$ -uniform hypergraphs

**Abstract:** A hereditary graph property is a class of finite graphs closed under isomorphism and induced subgraphs. Given a hereditary graph property  $\mathcal{H}$ , the *speed* of  $\mathcal{H}$  is the function which sends  $n$  to the number of distinct elements in  $\mathcal{H}$  with underlying set  $\{1, \dots, n\}$ . There are many wonderful results concerning what functions can occur as speeds of hereditary graph properties. More specifically, these results show there are discrete "jumps" in the possible speeds of hereditary graph properties. In this talk we extend one of these results to the setting of  $r$ -uniform hypergraphs, for  $r \geq 2$ . This result uses a generalization of VC-dimension which was first developed in model theory.

24. Norihide Tokushige, Ryukyu University, Japan

**Title:** The maximum product of measures of cross  $t$ -intersecting families

**Abstract:** For a positive integer  $n$  let  $[n] := \{1, 2, \dots, n\}$  and let  $\Omega := 2^{[n]}$  denote the power set of  $[n]$ . A family of subsets  $\mathcal{A} \subset \Omega$  is called  $t$ -intersecting if  $|A \cap A'| \geq t$  for all  $A, A' \in \mathcal{A}$ . Let  $p \in (0, 1)$  be a fixed real. We define the product measure  $\mu : 2^\Omega \rightarrow [0, 1]$  by  $\mu(\mathcal{A}) := \sum_{A \in \mathcal{A}} p^{|A|} (1 - p)^{n - |A|}$  for  $\mathcal{A} \subset 2^\Omega$ . Ahlswede and Khachatryan proved that if

$$\frac{r}{t + 2r - 1} \leq p \leq \frac{r + 1}{t + 2r + 1}$$

and  $\mathcal{A} \subset \Omega$  is  $t$ -intersecting, then  $\mu(\mathcal{A}) \leq \mu(\mathcal{F}_r^t)$ , where  $\mathcal{F}_r^t$  is a  $t$ -intersecting family defined by  $\mathcal{F}_r^t := \{F \subset [n] : |F \cap [t+2r]| \geq t+r\}$ .

We extend this result to two families. We say that two families  $\mathcal{A}, \mathcal{B} \subset \Omega$  are cross  $t$ -intersecting if  $|A \cap B| \geq t$  for all  $A \in \mathcal{A}, B \in \mathcal{B}$ . In this case it is conjectured that  $\mu(\mathcal{A})\mu(\mathcal{B}) \leq \mu(\mathcal{F}_r^t)^2$  under the assumption of (). In my talk I will report that this conjecture is true if  $t \gg r$ . I will also discuss a related stability result.

This is joint work with Sang June Lee and Mark Siggers.

25. Andrew Treglown, University of Birmingham, United Kingdom

**Title:** An improved lower bound for Folkman's Theorem

**Abstract:** Folkman's Theorem asserts that for each  $k \in \mathbb{N}$ , there exists a natural number  $n = F(k)$  such that whenever the elements of  $[n] := \{1, \dots, n\}$  are two-coloured, there exists a set  $A \subset [n]$  of size  $k$  with the property that all the sums of the form  $\sum_{x \in B} x$ , where  $B$  is a nonempty subset of  $A$ , are contained in  $[n]$  and have the same colour. In 1989, Erdős and Spencer showed that  $F(k) \geq 2^{ck^2/\log k}$ , where  $c > 0$  is an absolute constant. In this talk we describe how one can improve this bound significantly by showing that  $F(k) \geq 2^{2k-1/k}$  for all  $k \in \mathbb{N}$ .

Joint work with József Balogh, Sean Eberhard, Bhargav Narayanan and Adam Zsolt Wagner

26. Fan Wei, Stanford University, United States

**Title:** Fast Permutation Property Testing and Metrics of Permutations

**Abstract:** The goal of property testing is to quickly distinguish between objects which satisfy a property and objects that are  $\epsilon$ -far from satisfying the property. There are now several general results in this area which show that natural properties of combinatorial objects can be tested with "constant" query complexity, depending only on  $\epsilon$  and the property, and not on the size of the object being tested. The upper bound on the query complexity coming from the proof techniques are often enormous and impractical. It remains a major open problem if better bounds hold.

Hoppen, Kohayakawa, Moreira, and Sampaio conjectured and Klimošová and Král' proved that hereditary permutation properties are strongly testable, i.e., can be tested with respect to Kendall's tau distance. The query complexity bound coming from this proof is huge, even for testing a single forbidden subpermutation. We give a new proof which gives a polynomial bound in  $1/\epsilon$  in this case.

Maybe surprisingly, for testing with respect to the cut metric, we prove there is a universal (not depending on the property), polynomial in  $1/\epsilon$  query complexity bound for two-sided testing hereditary properties of sufficiently large permutations. We further give a nearly linear bound with respect to a closely related metric which also depends on the smallest forbidden subpermutation for the property. Finally, we show that several different permutation metrics of interest are related to the cut metric, yielding similar results for testing with respect to these metrics.

This is a joint work with Jacob Fox.



27. Douglas West, Zhejiang Normal University, China and University of Illinois, United States

**Title:** On-line ordered size Ramsey numbers for tight paths

**Abstract:** An *ordered hypergraph* is a hypergraph  $H$  with a specified linear ordering of the vertices, and the appearance of an ordered hypergraph  $G$  in  $H$  must respect the specified order on  $V(G)$ . In on-line Ramsey theory, Builder iteratively presents edges that Painter must immediately color. The  *$t$ -color on-line ordered size Ramsey number*  $\mathring{\text{OR}}'_t(G)$  of an ordered hypergraph  $G$  is the minimum number of edges Builder needs to play (on a large ordered set of vertices) to force a Painter using  $t$  colors to produce a monochromatic copy of  $G$ . The *tight path*  $P_r^{(k)}$  is the ordered hypergraph with  $r$  vertices whose edges are the sets of  $k$  consecutive vertices.

We obtain good bounds on  $\mathring{\text{OR}}'_t(P_r^{(k)})$ . In particular, letting  $m = r - k + 1$  (the number of edges in  $P_r^{(k)}$ ), we prove  $m^{t-1}/\sqrt{2\pi t/3} \leq \mathring{\text{OR}}'_t(P_r^{(2)}) \leq tm^{t+1}$ , with the lower bound valid when  $m \in o(e^t/\sqrt{t})$ . A trivial general upper bound is  $\binom{R}{k}$ , where  $R$  is the number of vertices in the smallest complete  $k$ -uniform (ordered) hypergraph whose  $t$ -colorings all contain  $P_r^{(k)}$ . We show that  $\mathring{\text{OR}}'_t(P_r^{(k)})$  is actually much closer to  $R$ .

This is joint work with Xavier Pérez-Giménez, Paweł Prałat, and Xuding Zhu.

28. Liana Yepremyan, University of Oxford, United Kingdom

**Title:** Supersaturation of even cycles in linear hypergraphs

**Abstract:** A classic result of Bondy and Simonovits and independently, of Erdős from 1970s says that the maximum number of edges in an  $n$ -vertex graph not containing  $C_{2k}$ , the cycle of length  $2k$ , is  $O(n^{1+1/k})$ . In an unpublished work, Simonovits obtained the corresponding supersaturation result, that is, there is a constant  $c$  (depending on  $k$ ) such that for all large  $n$ , every  $n$ -vertex graph  $G$  with more than  $cn^{1+1/k}$  edges contains  $\Omega((e/n)^{2k})$  many  $C_{2k}$ 's, this bound being achieved by a random graph with the same edge density. In a recent work on the number of  $C_{2k}$ -free graphs, Morris and Saxton also obtained a 'balanced' version of the supersaturation of  $C_{2k}$ 's.

In this talk, we extend Simonovits' result to linear cycles in linear hypergraphs. Our result is self-contained and includes the  $r = 2$  case and hence is a direct extension of Simonovits' result. In comparison to Morris' and Saxton's proof for  $r = 2$ , this proof is simpler; it relies on some ideas from earlier work of Faudree and Simonovits as well as new reduction theorems that we develop. We believe that the reduction theorems are of independent interest and can be useful for other supersaturation problems. This is joint work with Tao Jiang.