# **Title and Abstracts**

# TSMIF Sanya workshop: From Approximation Theory to Real World Applications, December 11-15, 2017

1. Martin Buhmann, University of Giessen, Germany

Title: Recent Results on Multiply Monotone Radial Functions

**Abstract:** The purpose of this talk is to establish new results on interpolation to continuous functions of multiple variables. Specifically, in order to classify radial basis and other functions that are useful for scattered data interpolation from vector spaces spanned by translates of basis functions (kernels) in any (high) dimensional space, we study multiply monotonic functions. We collect special properties of such multiply monotonic functions. Furthermore, we generalize results recently established for completely monotone functions to this class of functions. (Joint work with Janin Jäger.)

**2.** Di-Rong Chen, Beijing University of Aeronautics and Astronautics, China **Title:** Estimations of Cross-covariance Operators

**Abstract:** In this talk we discuss the estimations of cross-covariance operators, which are widely used in statistics and learning theory. For Kernel cross-covariance operators, we construct a class of shrinkage estimators and propose a completely data-driven shrinkage estimator. It is root-n consistent and have a good empirical performance. In addition, the minimax rate of order  $n^{-1/2}$  is established under some conditions. For sparsely and irregularly observed functional data, the cross-covariance functions are estimated with kernel smoothing method.

## 3. Falai Chen, University of Science and Technology of China, China

Title: Low Rank Approximation for Planar Domain Parameterization

**Abstract:** Construction of spline surfaces from given boundary curves is one of the classical problems in computer aided geometric design, which regains much attention in iso-geometric analysis in recent years and is called domain parameterization. However, for most of the state-of-the-art parametrization methods, the rank of the spline parameterization is usually large for complex computational domains, which results in higher computational cost in solving numerical PDEs. In this talk, we propose a low-rank representation for the spline parameterization of planar domains using low-rank tensor approximation technique, and apply quasi-conformal mapping as the framework of the spline parameterization. Under given correspondence of boundary curves, a quasi-conformal map with low rank and low distortion between a unit square and the computational domain can be obtained by solving a non-linear optimization problem. We propose an efficient algorithm to compute the quasi-conformal map by solving two convex optimization problems alternatively. Experimental results show that our approach can produce a low-rank parametric spline representation of planar domains.

4. Daoqing Dai, Sun Yat-sen University, China

Title: Sparse Dictionary Regression with Applications

**Abstract:** Regression with dictionary in high dimensional setting is attracting attention recently. In this talk I shall report constructions of discriminant dictionary and their applications to face recognition.

#### 5. Feng Dai, University of Alberta, Canada

**Title:** Riesz Transforms and Fractional Integrations for Weighted Orthogonal Expansions on the Unit Sphere

Abstract: I will report some recent progress on weighted orthogonal polynomial expansions (WOPEs) on the unit sphere of the Euclidean space R<sup>d</sup> with weights being invariant under a general finite reflection group. For many problems in this area, one of the main difficulties comes from the fact that explicit closed formulas for reproducing kernels of the weighted orthogonal polynomial spaces are not available unless the finite reflection group is Abelian. In this talk, I will first show that highly localized estimates for certain polynomial kernels associated with the WOPEs can be established by using the Christoffel functions with doubling weights and a well-known result of M. Rosleron Dunkl's intertwining operators. After that, I will show how the resulting estimates can be applied to determine the sharp power of the fractional Dunkl-Laplace-Beltrami operator for which the weighted Hardy-Littlewood-Sobolev inequality holds. I will also deduce anew decomposition of the Dunkl-Laplace-Beltrami operator for the WOPEs using angular derivatives with respect to Euler angles and certain differential-difference operators which are easier to compute. Such a decomposition combined with the theory of fractional integration is used to introduce Riesz transforms on weighted spheres. Finally, I will discuss briefly how these results can be used to establish similar results for WOPEs on the balls and simplexes. Our results on the ball extend a classical inequality of Muckenhoupt and Stein on conjugate ultraspherical polynomial expansions. This is a joint work with Han Feng (University of Oregon).

#### 6. Say Song Goh, National University of Singapore, Singapore

Title: Functions with Piecewise Constant Spectra

**Abstract:** Univariate functions whose Fourier transforms are piecewise constant functions extend the celebrated sinc function. Our goal is to provide detailed analysis of them when the breakpoints of their spectra are periodic. In that case, we obtain various desirable properties of these functions including stability, orthonormality, interpolation, sampling, refinability, biorthogonality and tight frames. The success of our program relies heavily upon materials on periodic wavelet analysis which we readily make use of. The approach provides a new perspective to the interplay between functions with piecewise constant spectra and periodic functions. It produces, among others, new refinable and wavelet functions with piecewise constant spectra that generate biorthogonal wavelet bases and tight wavelet frames. One of the highlights is that by taking appropriate step function approximations of the spectra of well-known stationary refinable and wavelet functions, we can obtain a rich family of nonstationary refinable and wavelet functions with piecewise constant spectra. This is joint work with Charles A. Micchelli.

#### 7. Bin Han, University of Alberta, Canada

**Title:** Academic Career and Scientific Contributions of Professor Rong-Qing Jia **Abstract:** As one of the major leading experts in approximation theory and wavelet analysis, Professor Jia has been making significant contributions with great and wide impacts in many areas of mathematics. This Sanya workshop is to celebrate Professor Rong-Qing Jia's retirement. As one of his academic descendants, in this talk I will first briefly outline Prof. Jia's academic career and provide a list of graduate students under his supervision. Then I will present some of Prof. Jia's selected major scientific contributions and achievements on splines and box splines, shift-invariant spaces, and properties of refinable functions and wavelets. In order for everyone to appreciate the beautiful mathematical results developed by Prof. Jia, necessary background and details will be provided for better understanding.

#### 8. Don Hong, Middle Tennessee State University, USA

#### Title: Computational Learning Methods for Neuroimaging Data Analysis

**Abstract:** The studies on brain science including many neuropsychiatric disorders using fMRI technology usually incorporate data spatial information and investigate functional connectivity of the brains at the network level. However, the identification of region of interests for disease and differential expressions in connectivity networks remains challenging on theory, computation, and statistical inferences. In this talk, we'd like to report some recent progress working with recent Ph.D. students on fMRI data analysis using computational learning schemes including multi-task learning, spatial regularization neural network, and non-Gaussian penalized PARAFAC analysis for fMRI data processing with applications in autism and Alzheimer disease (AD) studies, as well as a project on sleeping disorder studies.

#### 9. Janin Jäger, Institute fur Numerische Mathematik, Germany

#### Title: Multiply Monotone Functions for Spherical Interpolation

**Abstract**: In this talk we show how univariate functions with certain monotonicity properties can be used to perform scattered data interpolation on the sphere  $S^{d-1}$ . We use spherical basis functions, dependent on the geodetic distance, which can be derived in different ways form a univariate function  $\phi$ , and show that their positive definiteness can be determined from the monotonicity of the underlying function  $\phi$ . We use results for Euclidean interpolation to study the interrelations of the monotonicity properties of spherical and Euclidean basis functions and show how the new result we found, together with existing results, can be used to construct new spherical basis functions. (Joint work with Martin Buhmann.)

#### **10.** Hui Ji, National University of Singapore, Singapore

#### Title: Spatially-variant Image Deblurring: Models and Algorithms

**Abstract**: Blind Image de-convolution is one very challenging non-linear inverse problem with many applications in practice. Blind deconvolution aims at recovering the clear image from its blurred observation without knowing how it is blurred. Even worse, in many cases, the blurring process is non-stationary in the sense that different image regions are blurred by different kernels, which makes it even more difficult. In this talk, I will present several mathematical models and techniques toward solving blind image deblurring problems arising from digital photography, including both motion blurring and out-of-focus blurring.

### 11. Rong-Qing Jia, University of Alberta, Canada

#### Title: Applications of Quasi-interpolation in Analysis

**Abstract:** Quasi-interpolation is a powerful tool in approximation theory. In this talk we first review some fundamental results on approximation power of quasi-interpolation schemes. Next we will discuss several unexpected applications of quasi-interpolation to various areas of analysis such as function spaces, harmonic analysis, wavelet analysis, numerical analysis, and partial differential equations.

#### 12. Hong Jiang, Nokia Bell Labs, USA

#### Title: Minimizing Diffraction Effects in Computational Imaging

**Abstract**: In computational imaging acquisitions, a coded mask is often used to provide to spatial diversity in acquired image data. As the size of acquisition devices becomes small, the coded patterns of the mask introduce a significant amount of diffraction in the light field near sensors where image data is acquired. The diffraction causes reconstructed images to be blurred and places a limit on how high a resolution of the image can be. In this paper, we analyze the diffraction effect of a coded mask on image quality. The analysis provides a mechanism to alleviate diffraction effects during reconstruction of images. Furthermore, we will show that the diffraction effect will be much reduced when measurements are taken by pairs, for each pattern and its complimentary pattern. This result is a generalization of Babinet's principle to any light propagation field.

#### 13. Qingtang Jiang, University of Missouri-St. Louis, USA

**Title:** Instantaneous Frequency Estimation Based on Synchrosqueezing Transform **Abstract:** In this talk we will discuss some techniques for signal instantaneous frequency estimation based on wavelet synchrosqueezing transform (SST). We will consider the instantaneous frequency-embedded continuous wavelet transform and its corresponding SST. We will also discuss the choice of time-varying parameters with the Morelet type wavelets for sharp representation of nonstationary signals. (Joint work with Bruce Suter, Lin Li, Haiyan Cai and Hongbing Ji)

#### 14. Seng Luan Lee, National University of Singapore, Singapore

## Title: From Orthogonal Polynomials to Biorthogonal Spline Functions

**Abstract:** A natural extension of orthogonal polynomials to spline functions is in the construction sequences of polynomials and spline functions that form a biorthogonal pair. We construct a class of polynomials that are biorthogonal to the linear combinations of shifts of a continuous L\_1-function  $\phi$ , which includes refinable functions and the uniform B-splines of any order. These linear combinations turn out to be the finite differences of  $\phi$ , and the corresponding biorthogonal polynomials inherit some properties of the Appell sequences. These results are extended to variation diminishing spline functions, which are linear combinations of nonuniform B-splines. In particular, we also consider B-splines on a finite interval with multiple knots at the end points, which reduce to Bernstein polynomial basis functions when there are no interior knots, in which case the corresponding variation diminishing spline functions together with the biorthogonal polynomials reduce to Jacobi polynomials with integer indices. In this sense the variation diminishing spline functions can be considered as a spline extension of the Jacobi polynomials, and they inherit some behaviors of the Jacobi polynomials and related hypergeometric functions. Results on Hermite and Laguerre polynomials can also be obtained from the biorthogonal splines as limiting cases.

#### 15. Junjiang Lei, Mentor Graphics Corp, USA

Title: Practical Aspects to Applications of B-splines and Quasi-interpolation.

**Abstract:** The cubic B-splines are widely used in many engineering applications, e.g., data smoothing, low pass filtering, interpolation, upsampling, downsmapling, and function representation. In practice, a performance benchmark may be carried out in order to make a decision of whether to use B-splines and quasi-interpolation or to use other possible options, for each of these applications. This talk will present examples of how B-splines and quasi-interpolation are compared with their alternative solutions, for applications involving band-limited functions. Only univariate functions are discussed, as the tensor product is often used for multivariate functions.

#### 16. Song Li, Zhejiang University, China

## Title: A few Problems in Compressed Sensing

**Abstract:** In my talk, I shall investigate some problems in compressed sensing. First of all, we will confirm a conjecture on RIP which was proposed by T. Cai and A. Zhang and also establish uniformly optimal RIP bounds for 1-p minimization problem with 0<p<1. Secondly, we will discuss compressed data separation problem, as a results, we answer an open problem as proposed by E. Candes, Y. Eldar, D. Needell and P. Randall. Finally, we introduce RE conditions adapted to frame D (called D-RE condition), which was really extension of the famous RE condition proposed by P. Bickel, Y. Ritovusing and A. Tsybakov. By using D-RE condition, we investigate two models called ALASSO and ADS (introduced by Junhong Lin and Song Li).

## 17. Rongrong Lin, Sun Yat-sen University, China

**Title:** An Optimal Convergence Rate for the Gaussian Regularized Shannon Sampling Series

**Abstract:** In this talk, I will discuss the reconstruction of a bandlimited function from its finite localized sample data. Truncating the classical Shannon sampling series results in an unsatisfactory convergence rate due to the slow decay of the sinc function. To overcome this drawback, a simple and highly effective method, called the Gaussian regularization of the Shannon series, was proposed in the engineering and has received remarkable attention. It works by multiplying the sinc function in the Shannon series with a regularized Gaussian function. Recently, it was proved that the upper error bound of this method can achieve the convergence rate of the order  $O(\frac{1}{\sqrt{1}})$ , where  $O(\frac{1}{\sqrt{1}})$  is the bandwidth and  $numerical verification all regularized methods for the Shannon sampling series. The main objective of my work is to present the theoretical justification and numerical verification that such convergence rate is optimal when <math>O(\frac{1}{\sqrt{1}})$  by estimating the lower error bound of the truncated Gaussian regularized Shannon sampling series.

18. Zhongxuan Luo, Dalian University of Technology, China

Title: Quadrilateral and Hexahedral Mesh Generation Based on Surface Foliation Theory

**Abstract:** For the purpose of isogeometric analysis, one of the most common ways is to construct structured hexahedral meshes, which have regular tensor product structure, and fit them by volumetric T-Splines. This theoretic work proposes a novel surface quadrilateral meshing method, colorable quad-mesh, which leads to the structured hexahedral mesh of the enclosed volume for high genus surfaces. The work proves the equivalence relations among colorable quad-meshes, finite measured foliations and Strebel differentials on surfaces. This trinity theorem lays down the theoretic foundation for quadrilateral/hexahedral mesh generation, and leads to practical, automatic algorithms. The work proposes the following algorithm: the user inputs a set of disjoint, simple loops on a high genus surface, and specifies a height parameter for each loop; a unique Strebel differential is computed with the combinatorial type and the heights prescribed by the user's input; the Strebel differential assigns a flat metric on the surface and decomposes the surface into cylinders; a colorable quad-mesh is generated by splitting each cylinder into two quadrilaterals, followed by subdivision; the surface cylindrical decomposition is extended inward to produce a solid cylindrical decomposition of the volume; the hexahedral meshing is generated for each volumetric cylinder and then glued together to form a globally consistent hex-mesh.

19. Alexander M. Powell, Vanderbilt University, USATitle: Sharp Balian-Low Theorems and Fourier MultipliersAbstract: The classical Balian-Low theorem is a strong form of the uncertainty

principle that constrains the time-frequency localization of Gabor systems that form orthonormal bases. We discuss a generalization of the Balian-Low theorem that provides a sharp scale of constraints on the time-frequency localization of Gabor systems under a weaker form of spanning structure associated with so-called exact  $C_q$  systems. Admissibility conditions on Fourier multipliers play an important role in the proofs and, as an additional application, yield sharp Balian-Low type theorems in the setting of shift-invariant spaces. This is joint work with Shahaf Nitzan and Michael Northington.

#### 20. Hao Shan, Xinjiang astronomical observatory, China

Title: The idea of compressed sensing in radio interferometric deconvolution

**Abstract:** The sampling rate of the traditional Shannon/Nyquist sampling theorem must be at least twice the maximum frequency of signals. A new idea named compressive sensing(CS) says that a compressible unknown signal can be recovered from far fewer number of measurements than those required by traditional methods.. The CS measurement only obeys sub-Shannon rate without the limitation of Fourier bandwidth. In this talk, we introduce a CS-based deconvolution method for radio interferometric imaging. It can reconstruct both point sources and extended sources by using the wavelet and the shearlet transforms as the basis functions in the reconstruction step. In the experiments, We compare the introduced CS-based deconvolution methods. Both visual and numerical results of the comparison are provided. The proposed method has competitive and practical prospects.

#### 21. Yi Shen, Zhejiang Sci-Tech University, China

Title: Iterative Hard Thresholding Method and its variation

**Abstract:** This talk treats the problem of minimizing a general function with sparsity constrains. The problem has been considered in previous literature and will serve as prototype models throughout the talk are described in several examples, such as compressed sensing,  $\ell$ 2-regularized logistic regression and multiple kernel learning. The projected gradient/subgradient method are proposed to solve the non-convex problem. The theoretical convergence of these methods are studied. The algorithm and results are illustrated by numerical examples.

#### 22. Zuowei Shen, National University of Singapore, Singapore

#### Title: Image Restoration and Beyond

**Abstract:** We are living in the era of big data. The discovery, interpretation and usage of the information, knowledge and resources hidden in all sorts of data to benefit human beings and to improve everyone<sup>1</sup>s day to day life is a challenge to all of us. The huge amount of data we collect nowadays is so complicated, and yet what we expect from it is so much. This provides many challenges and opportunities to many fields. As images are one of the most useful and commonly used types of data, in this talk, we start from reviewing the development of the wavelet frame (or more general redundant system) based approach for image restoration. We will observe that a good system for

any data, including images, should be capable of effectively capturing both global patterns and local features. One of the examples of such system is the wavelet frame. We will then show how models and algorithms of wavelet frame based image restoration are developed via the generic knowledge of images. Then, the specific information of a given image can be used to further improve the models and algorithms. Through this process, we shall reveal some insights and understandings of the wavelet frame based approach for image restoration and its connections to other approaches, e.g. the partial differential equation based methods. Finally, we will also show, by many examples, that ideas given here can go beyond image restoration and can be used to many other applications in data science.

**23. Jiachang Sun,** Institute of Software, Chinese Academy of Sciences, China **Title:** Multivariable Approximation on Iso-Spetral Problems. **Abstract:** 

#### 24. Qiyu Sun, University of Central Florida, USA

**Title:** Phaseless Sampling and Reconstruction of Signals in Shift-invariant Space **Abstract:** A function is defined by its evaluation on its domain. In this talk, we discuss the problem whether a function can be determined by its magnitudes on its domain or a subset of its domain. The above problem is related to phase retrievable and it is called a phaseless sampling and reconstruction. In this talk, we will discuss mathematical foundation and numerical implementation for phaseless sampling and reconstruction for signals to reside in a shift-invariant space.

#### 25. Wenchang Sun, Nankai University, China

#### Title: Local and global phaseless sampling in spline spaces

**Abstract:** We study the recovery of functions in real spline spaces from unsigned sampled values. We consider two types of recovery. The one is to recover functions locally from finitely many unsigned samples. And the other is to recover functions on the whole line from infinitely many unsigned samples. In both cases, we give characterizations for a sequence of distinct points to be a local or global phaseless sampling sequence.

#### 26. Keith Taylor, Dalhousie University, Canada

Title: Continuous Wavelet Transforms in Three Dimensions

**Abstract:** For each dimension *n*, there are closed subgroups of  $GL_n(R)$  which have open and free orbits in R^n. For any such *H*, one can develop an analog of the continuous wavelet transform. Besides the one dimensional case, the shearlet transform arises in this way and has found applications in image processing. Novel three dimensional versions are presented in this talk. All closed subgroups of  $GL_3(R)$ which admit open free orbits have recently been classified by Currey, Fuhr, and Oussa. This provides a wide variety of different ways of moving a suitable analyzing wavelet around to represent functions of three variables. We will present and discuss the resulting transforms and reconstruction formulas.

## 27. Jianzhong Wang, Sam Houston State University, USA

#### Title: Analysis on Approximation of Out-of-Sample Extension

Abstract: Let Z=XUY be a data set in R^D, where X is the train set and Y is the test one. Assume that a kernel method produces a dimensionality reduction function  $\Phi: X \rightarrow R^d(d \ll D)$  that maps the high-dimensional data X to its row-dimensional representation  $\Phi(X)$ . The out-of-sample extension problem is to find the dimensionality reduction of Y using the extension of  $\Phi$  instead of re-computing the dimensionality reduction for the whole set Z. Among various out-of-sample extension methods, those based on Nystrom approximation are very attractive. Many papers have developed the algorithms for the Nystrom-type extension and shown their validity by numerical experiments. However, there is still lack of a systematical theoretical analysis for the extension. This paper develops a mathematical analysis on the Nystrom-type out-of-sample extension, particularly emphasizing on the Estimates of the approximatively errors.

## 28. Yang Wang, Hong Kong University of Science and Technology, China

Title: Data Recovery on Manifolds: A Theoretical Framework

**Abstract:** Recovering data from compressed number of measurements is ubiquitous in applications today. Among the best know examples are compressed sensing and low rank matrix recovery. To some extend phase retrieval is another example. The general setup is that we would like to recover a data point lying on some manifold having a much lower dimension than the ambient dimension, and we are given a set of linear measurements. The number of measurements is typically much smaller than the ambient dimension. So the questions become: Under what conditions can we recover the data point from these linear measurements? If so, how? The problem has links to classic algebraic geometry as well as some classical problems on the embedding of projective spaces into Euclidean spaces and nonsingular bilinear forms. In this talk I'll give a brief overview and discuss some of the recent progresses.

## 29. Zongmin Wu, Fudan University, China

## Title: Learning Physics Based on Data.

**Abstract:** Based on the data, the physical relations for a ball is falling to a non-newtonian fluid is learned. Take the Jerk in to account, a body moved in a non-newtonian media behavior as *the velocity proportional to the Haschke transform* (A Moebius transform multiply an identity transform) of the Jerk, where the velocity has been normalized and the jerk equal to be the second derivative of the normalized velocity.  $v(t)=T\{1-dv''(t) \mid v''-d\}v''(t)$ , where v is normalized by its terminal velocity or the limits of the average velocity.

**30.** Yuan Xu, University of Oregon, USA

Title: Approximation and Orthogonality in Sobolev Spaces

Abstract: The best approximation polynomials in a L^2 space are the partial sums of

the Fourier orthogonal expansions in the same space. This can be extended to a Sobolev space, for which the orthogonality is defined with respect to an inner product that contains derivatives and approximation holds for functions and their derivatives simultaneously. We explain recent results in this talk, starting with approximation via Sobolev orthogonal polynomials on an interval with the Jacobi weight and continuing to results on the unit ball and on a triangle.

**31.** YueshengXu, Sun Yat-sen University, China, and Old Dominion University, USA **Title:** Functional Reproducing Kernel Hilbert Spaces

Abstract: Motivated by the need of processing non-point-evaluation functional data, we introduce the notion of the functional reproducing kernel Hilbert space (FRKHS). This space admits a unique functional reproducing kernel which reproduces a family of continuous linear functionals on the space. The theory of FRKHSs and the associated functional reproducing kernels are established. A special class of FRKHSs, which we call the perfect FRKHSs, are studied, which reproduce the family of the standard point-evaluation functionals and at the same time another different family of continuous linear functionals. The perfect FRKHSs are characterized in terms of features, especially for those with respect to integral functionals. In particular, several specific examples of the perfect FRKHSs are presented. We apply the theory of FRKHSs to sampling and regularized learning, where non-point-evaluation functional data are used. Specifically, a general complete reconstruction formula from linear functional values is established in the framework of FRKHSs. The average sampling and the reconstruction of vector-valued functions are considered in specific FRKHSs. We also investigate in the FRKHS setting the regularized learning schemes, which learn a target element from non-point-evaluation functional data. The desired representation theorems of the learning problems are established to demonstrate the key roles played by the FRKHSs and the functional reproducing kernels in machine learning from non-point-evaluation functional data. We point out that the continuity of linear functionals, used to obtain the non-point-evaluation functional data, on an FRKHS is necessary for the stability of the numerical reconstruction algorithm using the resulting data.

32. Zhiqiang Xu, Academy of Mathematics and Systems Science, China

## **Title:** Generalized Phase Retrieval

Abstract: Phase retrieval is an active topic recently. In this talk, we will introduce the generalized phase retrieval which includes as special cases the standard phase retrieval as well as the phase retrieval by orthogonal projections. We first explore the connections among generalized phase retrieval, low-rank matrix recovery and nonsingular bilinear form. Motivated by the connections, we present results on the minimal measurement number needed for generalized phase retrieval. Our work unifies and enhances results from the standard phase retrieval, phase retrieval by projections and low-rank matrix recovery and also explore the connection among phase retrieval, nongsingular bilinear and topology. This is a joint work with Yang Wang.

#### 33. Zhiwen Zhang, University of Hong Kong, China

**Title:** Proper Orthogonal Decomposition Method to Two Dimensional forward Kolmogorov Equation and its Application to Nonlinear Filtering Problems.

**Abstract:** We investigate the proper orthogonal decomposition (POD) method to numerically solve the forward Kolmogorov equation (FKE). We test some adaptive strategies for the numerical stabilization of the proper orthogonal decomposition reduced order models for FKE. The convergence analysis of the proposed method is also presented and has been verified by the numerical simulation. As an important application and our primary motivation to study the POD method to FKE, we solve the nonlinear filtering (NLF) problems with a real-time algorithm. Our method is served as the off-line computation in this algorithm. Several numerical experiments of the NLF problems are carried out to illustrate the feasibility of our algorithm.

## 34. Ding-Xuan Zhou, City University of Hong Kong, China

## Title: Theory of Distributed Learning

**Abstract:** Analyzing and processing big data has been an important and challenging task in various fields of science and technology. Learning theory has wide applications in data science. It aims at learning function relations or data structures from samples. Distributed learning is an important topic in learning theory. It is a powerful method to handle big data. Distributed learning is based on a divide-and-conquer approach and consists of three steps: first we divide oversized data into subsets and each data subset is distributed to one individual machine, then each machine processes the distributed data subset to produce one output, finally the outputs from individual machines are combined to generate an output of the distributed learning algorithm. It is expected that the distributed learning algorithm can perform as efficiently as one big machine which could process the whole oversized data, in addition to the advantages of reducing storage and computing costs. This talk describes some analysis viewpoints of distributed learning, and demonstrates error bounds with regularization schemes in reproducing kernel Hilbert spaces.

#### 35. Xiaosheng Zhuang, City University of Hong Kong, China

#### Title: Multiscale Data Analysis: Framelets, Manifolds and Graphs

**Abstract:** While Big Data are high-volume, high-dimensional, and high complexity, they are typically concentrated on low-dimensional manifolds or can be represented by graphs, digraphs, etc. Sparsity is the key to the successful analysis of data in various forms. Multiscale representation systems provide efficient and sparse representation of various data sets. In this talk, we will discuss the characterizations, construction, and applications of framelets on manifolds and graphs. We shall demonstrate that tight framelets can be constructed on compact Riemannian manifolds or graphs, and fast algorithmic realizations exist for framelet transforms on manifolds and graphs as well as numerical examples will be shown.