Titles and Abstracts

1. Gregory Beylkin, University of Colorado at Boulder, USA

   Title: On Long Distance Wave Propagation In a Variable Medium

   Abstract: Numerical methods for wave propagation in variable medium are essential in both global seismology and seismic exploration. In global Earth models and large scale models in seismic exploration the distance of propagation (measured in units of the dominant wavelength) approaches $10^3 - 10^5$ wavelengths. This presents a number of challenges for numerical methods addressing these problems.

   To reduce the computational burden in such problems, we discuss the advantages of using a basis for bandlimited functions instead of orthogonal polynomials. We also emphasize two additional tools: using separated representations of operators and operator exponentials for time stepping. It has been observed that, over distances of approximately one wavelength, the operator exponentials maintain a certain useful form that can be exploited to accelerate the computation.

   A basis for bandlimited functions, the so-called Prolate Spheroidal Wave Functions (PSWFs), were introduced in 1960s for signal processing and found new numerical applications in the early 2000s. Separated representations for multivariate functions are a generalization of the usual sepa-ration of variables and is an active area of research.

2. Jiubing Chen, Tongji University, China

   Title: Pseudo-spectral methods for elastic wave modeling and mode decomposition in anisotropic media

   Abstract: Seismic modeling, structure imaging, source and medium parameter inversion in areas with complex geology all require high-accuracy numerical algorithms for time extrapolation of waves. The finite-difference (FD) method still remains one of the most popular numerical methods, mainly due to its computational efficiency and due to the need for high-performance computing for many popular seismic modeling applications, e.g., pre-stack reverse-time migration (RTM), time-reversal (earthquake or micro-seismic) source imaging, and full-waveform inversion (FWI). However, apart from numerical dispersion which hampers the FD modeling, these applications remains exposed to a range of complexities induced by multicomponent data, such as multi-mode (e.g., P and S) coupling and conversions of seismic waves. Increasing studies have found that, elastic wave mode decomposition can not only help seismic migration to produce physically interpretable images, but also provide more opportunity to mitigate the parameter trade-offs in waveform inversion.

   For isotropic media, far-field P and S waves can be separated by taking divergence and curl in the extrapolated elastic wavefields. Alternatively, one can extrapolate vector P and S modes “separately” in an elastic medium by decomposing the wave equation into P and S-wave components. In anisotropic media, one cannot, so simply, derive explicit single-mode time-space-domain differential wave equations. Generally, elastic wavefield is extrapolated using the full-wave modeling engine (e.g., using FD solution) and then decomposed into P and S-wave fields using the polarization orthogonality. In heterogeneous anisotropic media, mode decomposition involves space-wavenumber-domain Fourier integral operators (FIOs) of the general form, of which the computation complexity is extremely high.

In this study, we have proposed an efficient pseudo-spectral method to propagate and decompose the elastic waves using the displacement or velocity-stress wave equations for general anisotropic media. The primary strategy is to merge the numerical solutions for time extrapolation and mode decomposition into a unified Fourier integral framework and speed up the solutions using the low-rank approximation. Moreover, we have developed a new pseudo-spectral method to solve the velocity-stress wave equations for anisotropic media exhibiting high-contrasts or with symmetry lower than orthorhombic. 2-D and 3-D synthetic examples will be shown to demonstrate the performance of the proposed approach.

3. Yalchin Efendiev, Texas A&M University, USA
   Title: Multiscale model reduction for wave propagation in heterogeneous media
   Abstract: Common applications in modeling and inversion aim to apply numerical simulations of elastic and acoustic wave propagation to increasingly large and complex models, which can provide more realistic and useful results. The computational cost of these simulations increases rapidly, however, and this may make it impossible to address some problems. I will discuss the application of a newly developed multiscale finite element algorithm, the Generalized Multiscale Finite Element Method (GMsFEM), to address this challenge in simulating acoustic wave propagation in heterogeneous media. The wave equation is solved on a coarse grid using multiscale basis functions that are chosen from the most dominant modes among those computed by solving relevant problems on a fine-grid representation of the model. These multiscale basis functions are computed one time in an offline stage prior to the simulation of wave propagation. Because these calculations are localized to individual coarse cells, one can improve the accuracy of multiscale methods by revising and updating these basis functions during the simulation. These updated bases are therefore known as online basis functions. I will present our new algorithm, some important ingredients of the method, and numerical results.

4. Brittany Froese, New Jersey Institute of Technology, USA
   Title: Comparison of Seismic Signals Using the Wasserstein Metric
   Abstract: We propose the Wasserstein metric as a measure of misfit between seismic signals. It exhibits properties of the more traditional L^2or travel-time differences, but offers additional advantages relating to convexity and insensitivity to noise. The Wasserstein metric is computed using recently developed numerical methods for the Monge-Amp$\grave{e}$re equation and optimal transport. Gradients of the discrete Wasserstein metric are easily computed as part of the Monge-Amp$\grave{e}$re

solve, and can be used in minimisation. Simple computational examples illustrate the advantages of this approach for registration and full waveform in-version.

5. Sergey Fomel, The University of Texas at Austin, USA
   Title: Classic problems in applied computational seismology and the common task framework
   Abstract: Applied computational seismology is concerned with creating images of the subsurface and extracting subsurface properties using seismic waves. I will review the history of the field and will make connections between some of the recent research advancements and the pioneering work of Francis Muir (1926-2015), Jerry Gardner (1926-2009), and Tury Taner (1927-2010). The particular topics are time-domain imaging, anisotropic wave propagation, and seismic velocity estimation.

   The common task framework (CTF) is a technology for accelerating data analysis research by employing publicly available datasets. The recent development of shared open-source computational platforms makes it possible to apply CTF to common tasks in computation seismology.

6. Richard L. Gibson Jr., Texas A&M University, USA
   Title: Applications of the Generalized Multiscale Finite Element Method to Seismic Modeling and Imaging
   Abstract: Simulation of seismic wave propagation in models that incorporate free surface topography and large contrasts in velocities incurs significant challenges for numerical methods for solving the acoustic and elastic wave equations. An important example is modeling and imaging in arid regions where the surficial materials are very low velocity sand materials, underlain by high velocity carbonate rock with dissolution features ranging in scale from centimeters to many meters. These karst features can be filled with water or low velocity sediment, creating high contrast scatterers of wave energy. Common finite difference methods face challenges with topography, and grid discretization is controlled by the very low velocity regions of the model. On the other hand, finite element methods can face challenges in speed and mesh generation. The Generalized Multiscale Finite Element Method (GMsFEM) is designed to address these challenges. It employs a local model reduction approach where basis functions are computed on a fine-grid by solving physically-relevant equations, thereby incorporating the influence of heterogeneity. These basis functions are then utilized in a Discontinuous Galerkin finite element approach on a coarse grid that overlies the fine grid. The basis functions need only be computed once, and all simulations are conducted on the coarse grid, so that the effects of finely detailed heterogeneity are included in fast simulations on a coarse grid. We compare results of simulations of seismic wave propagation in an arid region karst model to those from the spectral element method to demonstrate the accuracy and speed gains of GMsFEM. In addition, we apply GMsFEM within a reverse time migration algorithm to accelerate the generation of subsurface images. Migration

is generally an iterative process that must be repeated as the seismic velocity model is corrected, and an important strength of the GMsFEM is that we can reduce the number of basis functions when the model is poorly known. Though this provides a less accurate approximation of the solution, tests show that the resulting images can still be used to improve the velocity model. The usage of few basis functions at early stages allows for significant reductions in computation time in the process of iteratively improving the velocity model to generate the final subsurface image.

7. Martin J. Gander, University of Geneva, Switzerland
   Title: Rigorous Convergence Results and Numerical Experiments for the Shifted Helmholtz Preconditioner
   Abstract: The shifted Helmholtz operator has received a lot of attention over the past decade as a preconditioner for the iterative solution of the Helmholtz equation. The idea is that if one uses a small complex shift, the shifted Helmholtz operator is close to the original one and thus an effective preconditioner, and if one shifts enough, the shifted Helmholtz preconditioner can be effectively inverted by standard multigrid or domain decomposition methods. We present in this talk two recent rigorous results for the shifted Helmholtz preconditioner: first, we show that if the shift is less than the wavenumber, one can guarantee that iteration numbers for GMRES preconditioned with the shifted Helmholtz operator are independent of the wavenumber [1], and second that if the shift is bigger than the wavenumber squared, multigrid is a robust solver for the shifted Helmholtz preconditioner [2]. We then show that for the regime between shifts of the size of the wavenumber and the wavenumber squared, it will be difficult to obtain any rigorous result on the performance of the shifted Helmholtz preconditioner. We thus address this region by extensive numerical experiments, which show that in this regime, using one or a few multigrid cycles to approximately invert the shifted Helmholtz operator, GMRES iteration numbers will in general grow like the wave number squared for this approximately inverted shifted Helmholtz preconditioner applied to the Helmholtz equation, like when using no preconditioner. In addition, in contrast to current practice, the best choice is in general to shift by a bit less than the wavenumber squared.

8. Ralf Hiptmair, ETH Z$\ddot{u}$rich, Switzerland
   Title: Plane Wave Discontinuous Galerkin Methods
   Abstract: This lecture reviews the current convergence theory for a special class of Trefftz-type discontinuous Galerkin (TDG) methods that rely on plane waves for approximating solutions of the homogeneous Helmholtz equation $-\Delta \mathrm{u} - \omega^2 u = 0$ locally. These methods have been designed as a cure for the notorious pollution effect that haunts standard low-order Galerkin schemes for the simulation of wave propagation. A prominent representative is the so-called ultra-weak variational (UWVF) invented by Cessenat and Despres.
   A powerful tool in the analysis of plane wave discontinuous Galerkin methods

(PWDG) are duality techniques borrowed from least squares methods. They paved the way for quasi-optimality results. Together with new approximation estimates for plane waves [5, 4], this allowed detailed a priori predictions of convergence. However, the early versions of the theory [1] were confined to convex domains, and, even worse, could not accommodated locally refined meshes, which is very unfortunate, because numerical experience suggests that PWDG should be used on such meshes. In short, the sophisticated hp-refinement strategy that ensures exponential convergence (in the number of degrees of freedom) for classical polynomial Galerkin finite element approximation of second-order elliptic boundary value problem should also be adopted for PWDG.

Only recently, the theory could be extended to the full hp-setting that involves families of trial spaces generated by dyadic mesh refinement towards corners combined with a global increase of the number of plane waves used in each element. The key idea is to make clever use of the freedom to choose flux parameters in the DG method, which makes it possible to establish quasi-optimality without assuming quasi-uniformity. The second ingredient is new estimates for the approximation of analytic Helmholtz solutions by plane waves. Thus, exponential convergence of hp-PWDG for the Helmholtz equation in 2D on domains with piecewise analytic boundaries could be established [2].

9. Maarten V. de Hoop, Purdue University, USA

   Title: Seismic inverse problems: Iterative reconstruction with bounded frequency boundary data

   Abstract: We discuss the seismic inverse problem with its different aspects including a dual time-frequency point of view. Central in the analysis is the formulation as an inverse boundary value problem with the Dirichlet-to-Neumann map or Neumann-to-Dirichlet map as the data. We present various conditional Lipschitz stability estimates for this problem for coefficients containing discontinuities, and with partial boundary data, which involves the introduction of an unstructured tetrahedral mesh. We elaborate on results pertaining to the stable recovery of material parameters defined on the mesh and the stable recovery of (the shape of) the mesh itself. A coarse-scale segmentation of the mesh facilitates a robust algorithm for mesh updating making use of level sets. The unstructured tetrahedral mesh, simultaneously, provides the framework for discretization. One might apply numerical homogenization to arrive at (approximate) coefficient representations leading to the well-posedness. Quantitative estimates of the stability constants play a critical role in analyzing convergence for iterative reconstruction schemes, which make use of Hausdorff warping, and lead to a multilevel approach requiring hierarchical, multi-scale compression of coefficient representations on the mesh with local refinement. We will show various computational experiments.

   In collaboration with E. Beretta, K. Datchev, E. Francini, S. Vessella and J. Xia.

10. ShingYu Leung, The Hong Kong University of Science and Technology, Hong Kong
    Title: Eulerian Approaches For Traveltime Tomography
    Abstract: The talk discusses Eulerian formulations for solving the first arrival traveltime tomography arising from important applications such as seismic imaging and medical imaging. In the first approach, we define a mismatch functional and derive the gradient of the nonlinear functional by an adjoint state method. A recent approach formulates the traveltime tomography problem as a variational problem of a certain cost functional explicitly with respect to both traveltime and sound speed. The cost functional is penalized to enforce the nonlinear equality constraint associated with the underlying eikonal equation, biharmonically regularized with respect to traveltime, and harmonically regularized with respect to sound speed. This resulting system is associated with an initial value problem which can be efficiently solved by an operator-splitting based solution method. This is a joint work with Jianliang Qian, Wenbin Li and Roland Glowinski.

11. Jianwei Ma, Harbin Institute of Technology, China
    Title: Sparse representation of seismic data and some applications
    Abstract: Most of the signal decomposition methods are based on the same idea: the measured data consists out of a small number of important components plus noise. Thus, decomposition methods are often applied in order to reconstruct the individual components. If the number of significant components is small, we name the method as a sparse decomposition. Sparse representation is very important for seismic denoising, interpolation, modeling and inverse. In this talk, I will report some related work in my group from data-driven tight frame (DDTF) transform, Monte Carlo DDTF, tensor DDTF, to asymmetric Chirplet transform, and their applications in seismic denoising and interpolation. Finally, I will report our recent work on the splitting algorithms (can be seen as a sparse decomposition) for high-order compact finite difference scheme in wave equation modeling.

12. Jianliang Qian, Michigan State University, USA
    Title: Babich's Expansion and the Fast Huygens Sweeping Method for the Helmholtz Wave Equation at High Frequencies
    Abstract: In some applications, it is reasonable to assume that geodesics (rays) have a consistent orientation so that the Helmholtz equation can be viewed as an evolution equation in one of the spatial directions. With such applications in mind, starting from Babich's expansion, we develop a new high-order asymptotic method, which we dub the fast Huygens sweeping method, for solving point-source Helmholtz equations in inhomogeneous media in the high-frequency regime and in the presence of caustics. The first novelty of this method is that we develop a new Eulerian approach to compute the asymptotics, i.e. the traveltime function and amplitude coefficients that arise in Babich's expansion, yielding a locally valid solution, which is accurate close enough to the source. The second

novelty is that we utilize the Huygens-Kirchhoff integral to integrate many locally valid wavefields to construct globally valid wavefields. This automatically treats caustics and yields uniformly accurate solutions both near the source and remote from it. The third novelty is that the buttery algorithm is adapted to accelerate the Huygens-Kirchhoff summation, achieving nearly optimal complexity O(NlogN), where N is the number of mesh points; the complexity prefactor depends on the desired accuracy and is independent of the frequency. To reduce the storage of the resulting tables of asymptotics in Babich's expansion, we use the multivariable Chebyshev series expansion to compress each table by encoding the information into a small number of coefficients.

The new method enjoys the following desired features. First, it precomputes the asymptotics in Babich's expansion, such as traveltime and amplitudes. Second, it takes care of caustics automatically. Third, it can compute the point-source Helmholtz solution for many different sources at many frequencies simultaneously. Fourth, for a specified number of points per wavelength, it can construct the wavefield in nearly optimal complexity in terms of the total number of mesh points, where the prefactor of the complexity only depends on the specified accuracy and is independent of frequency. Both two-dimensional and three-dimensional numerical experiments have been carried out to illustrate the performance, efficiency, and accuracy of the method.

This is a joint work with Wangtao Lu and Robert Burridge.

13. Huihui Weng, Chinese University of Hong Kong, Hong Kong

Title: Effects of fault heterogeneities on earthquake rupture propagation

Abstract: Crustal faults have been suggested to be heterogeneous, such as material contrast and stress heterogeneities etc. How these fault heterogeneities affect earthquake rupture propagation is important to advance our understanding of earthquake physics, and yet remains poorly understood. Recently developed numerical methods combining fault constitutive equations and elastic equations allow us to resolve dynamic rupture problem considering different material and stress heterogeneities, such as finite element method and finite difference method. For instant, a finite element code, PyLith (Aagaard et al., 2013), could simulate the fault relative motion by constructing cohesive cells with zero volume from the vertices on both sides of the fault interface and Lagrange multiplier constraints.

Using these numerical methods, we investigate the effects of fault heterogeneities on rupture propagation through numerical modeling on a strike-slip planar fault governed by a linear slip-weakening friction law. We first investigate the effects of a patch with elevated effective normal stress (barrier) on the rupture propagation. Except for the distance d between the barrier and the nucleation zone, its width w, and the additional effective normal stress $\Delta\sigma_n$, all other parameters are kept constant for all the simulated models. We find that the barrier may slow down or stop coseismic ruptures, but may also induce supershear ruptures. Moreover, there is a sharp boundary between stopping the rupture and making very strong supershear ruptures. Furthermore, we demonstrate that the supershear rupture may emerge in a region that is delineated by two approximate linear boundaries for parameters d and w. The duration of supershear ruptures increases as the barrier sizes grow from the lower to the upper boundary, which are proportional to the reduction in rupture speeds caused by the barrier. Because supershear ruptures may produce more destructive near-field ground shaking, thus our findings have important implications for earthquake hazard preparation.

In addition to the stress heterogeneities, we investigate the effects of material heterogeneities based on field observations, which have shown the velocities of P and S waves of the damage fault zones can be reduced up to 50% compared to intact rocks. We find that the damage fault zones could promote the rupture extents and thus increase the earthquake sizes. In addition, the promotion effects increase as the width and the depth extent of the damage fault zones increase. Moreover, along-strike segmented fault zones as suggested from observations could also promote the ruptures and may lead to preferred rupture directions. The effects of the damage fault zones on rupture propagation may hold important implications on assessing earthquake risk.

References:

Aagaard, B., Knepley, M., Williams, C., 2013. A domain decomposition approach to implementing fault slip in finite-element models of quasi-static and dynamic crustal deformation. J. Geophys. Res. 118, 3059-3079.

14. Yanfei Wang, Chinese Academy of Sciences, China

    Title: Seismic diffraction extraction with first and second-order regularization

    Abstract: Seismic diffractions play an important role in characterizing and identifying discontinuous geological structures, such as tiny faults and cavities in Ordovician carbonate reservoirs. These faults and cavities are important because of their close relationship to the reservoir properties of oil and gas. The seismic responses of these objects in the sense of seismic wavelength are encoded in diffractions. Since diffractors are usually sparse and non-differentiable, we study diffraction extraction based on first and second-order of regularization model specially for detecting diffraction points, and numerical algorithms are addressed. Numerical examples based on synthetic data and field data are given.

15. Yunan Yang, The University of Texas at Austin, USA

    Title: Properties of the Wasserstein distance for application in seismology

    Abstract: Optimal transportation is today a mature area of mathematical analysis. We propose several advantages of the Wasserstein metric as a measure of misfit between seismic signals. In particular, this distance is convex with respect to several common transformations and is less sensitive to noise than the L^2 distance. Both analysis and numerical results on convexity and robustness to noise will be discussed.

16. Wensheng Zhang, Chinese Academy of Sciences, China

    Title: Effective numerical methods for modeling seismic wave propagation

Abstract: Modeling the propagation of seismic waves is a useful step in the interpretation of wave phenomena in complex media. It is also an essential step for inverse problem in seismic exploration. In this talk, I will introduce some effective numerical methods for solving wave equations. First, the finite element method for solving the acoustic wave equation is presented. In order to simulate wave propagation in an unbounded domain, the absorbing boundary conditions and the corresponding stability conditions are investigated. Then the finite volume method is developed for solving for two type model equations. One is the elastic wave equations which are solved on the unstructured meshes. With the unstructured meshes, the finite volume method has good adaptability to complex geometry. The other is the poroelastic equations which are solved in the spherical coordinate system and the singularity is eliminated. Numerical computations are completed and the results show that the presented numerical methods can simulate wave propagation effectively and accurately.

17. Jun Zou, The Chinese University of Hong Kong, Hong Kong
Title: Direct and indirect sampling methods for severely ill-posed inverse problems
Abstract: In this talk we will first review some recent important advances in mathematical understanding of inverse acoustic and obstacle scattering problems, then present several newly developed direct and indirect sampling methods for inverse scattering problems and other severely ill-posed inverse problems. Numerical experiments will be presented to demonstrate the robustness and effectiveness of the new methods. This is a joint work with Yat Tin Chow (UCLA), Kazufumi Ito (NCSU) and Bangti Jin (University College London). The work was supported by Hong Kong RGC grants (projects 14306814 and 405513).

18. Hongfeng Yang, Chinese University of Hong Kong, Hong Kong
Title: Potential earthquake rupture scenarios inferred from interseismic locking distributions
Abstract: In the past decade, interseismic locking distributions in subduction zones derived from geodetic studies (e.g. GPS and leveling data) are often used to qualitatively evaluate the potential for future megathrust earthquakes. Due to the long recurrence intervals between large earthquakes, it is inherently difficult to perform direct quantita-tive field evaluations of locking and eventual coseismic slip. Despite the uncertainties in such locking models, quantitative estimates of the future coseismic slip from the interseismic locking distributions are rare. Understanding the details and causes of interseismic locking and how these relate to coseismic slip in future megathrust earth-quakes is not only important for understanding fundamental earthquake physics, but also societally relevant for more-informed seismic hazard assessment.
Here we report our quantitative estimates of coseismic slip in future megathrust earthquakes from the interseismic locking distributions using numerical simulations of dynamic ruptures under the framework of a linear slip-weakening

law. For instance, we have applied such approach to infer future rupture scenarios along the Cascadia subduction zone, which stretches from Northern California, US, to British Columbia, Canada, where the Juan de Fuca plate subducts beneath the North America plate at 30-47 mm/yr. We estimate stress condition and frictional parameters on the megathrust from the locking distributions, assuming the total slip deficit will be released in next great earthquake. The simulation results show that the coseismic slip distributions are largely associated with the interseismic locking models, e.g. the down-dip rupture limit. In addition, along-strike rupture extents are dependent on the epicenter location and the size of the nucleation patch. For example, a rupture initiates from the north stops quickly near the nucleation zone, while ruptures nucleate from another two epicenters propagate much further, producing larger earthquakes. These results build a framework to derive future coseismic slip from interseismic locking distributions. Due to the long recurrence intervals between large earthquakes in Cascadia and most other subduction zones, the numerical simulation results have not yet been able to compare with field observations. On September 5th, 2012, an Mw 7.6 earthquake occurs in the Nicoya Peninsula of northwestern Costa Rica, where the oceanic Cocos plate descends beneath the Caribbean plate at 82 mm/yr. Since the Nicoya earthquake has ruptured directly over the previously determined locked patch from GPS measurements, it provides an unprecedented opportunity to develop meaningful comparisons between actual coseismic slips determined from observations and those predicted by numerical simulations based on interseismic locking distributions. We have computed stress conditions using the geodetically determined locking distributions. Updated results of rupture scenarios, as well as comparisons with coseismic observations, will be reported in the workshop.

19. Hongkai Zhao, Department of Mathematics, UC Irvine, USA

Title: Ray-based Finite Element Method for the Helmholtz Equation in High Frequency Regime

Abstract: We present a ray-based finite element method (Ray-FEM) for the Helmholtz equation in high frequency regime. In our method the ray field is introduced as an auxiliary unknown and both the ray field and wave field are solved simultaneously in an iterative process. The key idea is to probe the medium first with the same source by solving a relative low frequency Helmholtz equation and extract/learn the ray field by applying the Numerical Micro-Local Analysis (NMLA) to the computed wave field. Then the ray information is incorporated into the Galerkin space to solve the high frequency Helmholtz equation. The process can be continued to further improve the approximation for both the ray field and the solution to the high frequency Helmholtz iteratively. Our method only requires the minimum degrees of freedom, i.e., a fixed number of grid points per wave length, to achieve both stability and expected accuracy to solve the high frequency Helmholtz equation without the usual pollution effect. Numerical tests in both 2D and 3D are presented to corroborate our method.

20. Hai Zhang, Department of Mathematics, HKUST, Hong Kong

    Title: Mathematics of super-resolution in resonant media

    Abstract: We first introduce some background of super-resolution. We then focus on the particular super-resolution technique where resonant media are used. Three cases are analyzed: one is Helmholtz resonators and the others are high contrast material and plasmonic particles. In all cases, we developed rigorous mathematical theory to explain the mechanism of super-resolution.

21. Lei Zhang, Shanghai Jiao Tong University, China

    Title: Numerical Homogenization with Localized Basis

    Abstract: Numerical homogenization concerns the finite dimensional approximation of the solution space of, for example, divergence form elliptic equation with L^∞ coefficients which allows for nonseparable scales. Standard methods such as finite-element method with piecewise polynomial elements can perform arbitrarily badly for such problems. In this talk, I will introduce an approach for numerical homogenization which precomputes H^{-d} localized bases on patches of size $H \log(1/H)$. The localization is due to the exponential decay of the corresponding fine scale solutions with Lagrange type constraints. Interestingly, this approach can be reformulated as a Bayesian inference or decision theory problem. Furthermore, the numerical homogenization method can be used to construct efficient and robust fine scale multigrid solver or domain decomposition preconditioner, and generalized to time dependent problems such as wave propagation in heterogeneous media. This is a joint work with Houman Owhadi.