The Mathematical Texts in East Asia Mathematical History Workshop

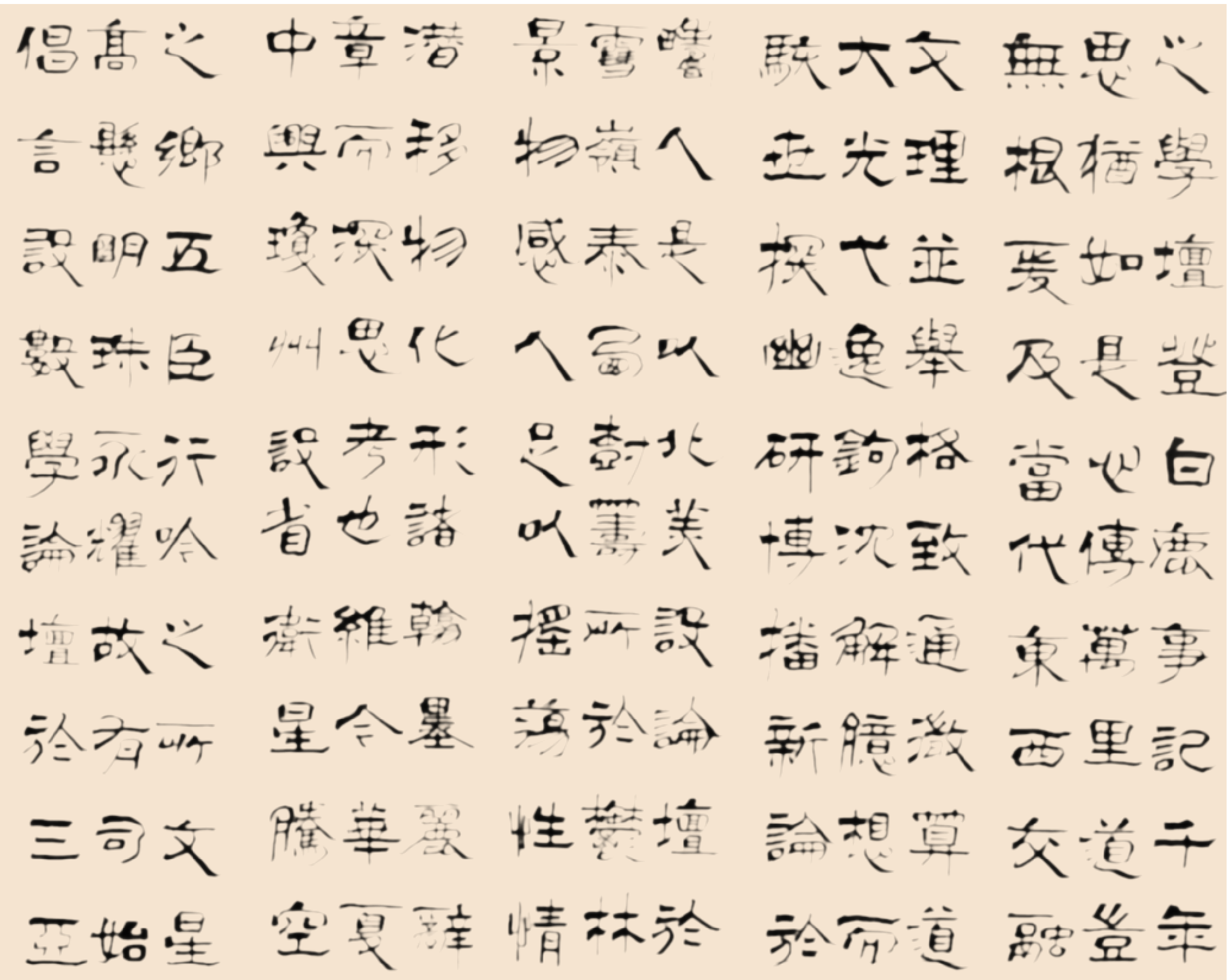
东亚数学典籍研讨会

Tsinghua Sanya International Mathematics Forum (TSIMF)

清华三亚国际数学论坛

Sanya, Hainan, P. R. China，March 11-15, 2016

2016 年 3 月 11 日至 15 日，中国・海南・三亚



心學壇登白鹿事記千年
思栖如是也傳萬里道豈
無根髮及當代東西交融
文理並舉格致通徹算道
大光弋逸鉤沉解臆想而
馱丕探幽研博播新論於
曙人長以北美設論壇於
雪嶺泰岛封籌所於藝林
景物感人足以搖蕩性情
潛移物化形諸翰墨羅辭
章而深思考也維令華夏
中興境州設省衛星騰空
之鄉五臣行吟之所文星
島懸明珠承耀故有司始
倡言設數學論壇於三亞

The Mathematical Texts in East Asia
Mathematical History Workshop

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Description 描述

Mathematics books and literature are the carriers of mathematics knowledge. They play important roles in the transmission, exchange, and advancement of mathematics knowledge. In East Asia, China, Japan, Korea and Vietnam all belonged to the Sinosphere, the region that used Chinese characters as recent as 100 years ago. Hence, they have much in common in terms of heritage in mathematics culture. For a long time, Chinese mathematics books were a linkage of the mathematics culture of different countries within the Sinosphere. Because of the widespread use of Chinese characters as medium of communication and the circulation of Chinese mathematics books, a mathematics cultural system distinct from that in the West was developed in East Asia. There were not much academic activities in this area until recent years. In order to advance the research of Chinese mathematics ancient books and the history of mathematics, to share the research information, to exchange academic views, the "Mathematical Texts in East Asia Mathematical History" workshop will hold at Tsinghua Sanya International Mathematics Forum (TSIMF) from March 11-15, 2016. The aim of this workshop is to bring together Eastern Asia mathematics history researchers and scholars to report new results and exchange ideas in the following topics:

(i) Contents and editions of East Asian mathematics books

(ii) The spread and the influence of Chinese mathematics books in Chinese character cultural region

(iii) Translation of mathematics books from the West and its influence

(iv) Mathematics books in East Asian mathematics history.

Registration and Accommodation 注册与食宿：

TSIMF will cover all expenses in room and board during the workshop for delegates, other expense by delegates themselves.

Organizers 组织机构：

Yau Mathematical Sciences Center, Tsinghua University

Institute for History of Science and Technology & Ancient Texts, Tsinghua University

The History of Mathematics Branch of Chinese Mathematical Society

Organizing Committee 组织委员会：

Chiefs 主席

Professor Feng Lisheng 冯立昇，Institute for History of Science and Technology & Ancient Texts,Tsinghua University, P. R. China，清华大学科学技术史暨古文献研究所

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Schedule 日程安排

March 10-11, 3 月 10-11 日，Registration 注册

Time 时间	Venue 地点
All day 全天	Check in the reception desk of Room 200, Building B (1st Floor), Tsinghua Sanya International Mathematics Forum.清华三亚国际数学论坛，接待室 B 栋一层 200 室前台办理入住

Morning of March 12, 3 月 12 日上午—Workshop 会议

Opening ceremony 开幕式，Host 主持人：JI Zhigang 纪志刚		
8:30—9:10	FENG Lisheng 冯立昇致辞	
9:10—9:50	Cheng-Yih Chen 程貞一	An Analysis of Thought Processes in Early Mathematics in Chinese Civilization 早期中華數學思路的分析
9:50—10:30	Jens Høyrup 简斯•休儒	Seleucid, Demotic and Mediterranean mathematics versus Chapters 8 and 9 of the Nine Chapters: accidental or significant similarities?
10:30—10:35	Photo time 合影	
10:30—10:50	Coffee break 茶歇	
Workshop 大会报告，Host 主持人：FENG Lisheng 冯立昇		
Time 时间	Speaker 报告人	Title 题目
10:50—11:30	LUO Jianjin 罗见今	An Analysis on the Achievement of Counting Theory in the Book “Mengxi Bitan” by Shen Kuo 沈括《梦溪笔谈》中计数成就探析
11:30—12:10	Tatsuhiko Kobayashi 小林龍彦	On the Acceptance of Surveying of the Suanfa tongzong ≪算法統宗≫ by Wasan-mathematician
12:10—14:00	Lunch 午餐	

Afternoon of March 12, 3 月 12 日下午—Workshop 会议

Workshop 大会报告，Host 主持人：XU Zelin 徐泽林

Time 时间	Speaker 报告人	Title 题目
14:00—14:40	Tsukane Ogawa 小川束	English Translations of Mathematical Books in Pre-modern Japan
14:40—15:20	Mitsuo Morimoto 森本光生	The Full Expansion Formula of Determinants Given in the Taisei Sankei
15:20—16:00	Sung Sa HONG 洪性士	Mathematical Structures of Gugo Wonlyu (勾股源流) by Jeong Yag-yong (丁若鏞)
16:00—16:20	Coffee break 茶歇	
Workshop 大会报告，Host 主持人：Seisho Yoshiyama 吉山青翔		
16:20—17:00	HAN Qi 韩琦	1713: A Year of Significance
17:00—17:40	Young Wook KIM 金英郁	Gougushu in Joseon and Yu-ssi Gugosulyo Dohae
18:00—	Banquet 晚宴	

Morning of March 13, 3 月 13 日上午—Workshop 会议

Workshop 大会报告，Host 主持人：Mitsuo Morimoto 森本光生		
Time 时间	Speaker 报告人	Title 题目
8:30—9:10	XIAO Can 肖灿	秦人对于数学知识的重视与运用
9:10—9:50	ZOU Dahai 邹大海	A Study on a Type of Wedge-shaped Solid in Early Chinese Mathematical Documents 中国早期数学文献中一类楔形体的研究
9:50—10:30	TIAN Miao 田淼	Study on the solution of indeterminate equations of first degree in traditional mathematics in China 中国传统数学中的一次不定方程问题研究
10:30—10:50	Coffee break 茶歇	
Workshop 大会报告，Host 主持人：Tatsuhiko Kobayashi 小林龍彦		
10:50—11:30	DENG Kehui 邓可卉	Discuss again Jiudaoshu 再议九道术
11:30—12:10	SIU Man Keung 蕭文強	Figures and history on either side, rendered dynamically — GeoGebra to go hand in hand with ancient or medieval Chinese mathematical texts in the mathematics classroom 左圖右史，別有「動」天——在數學課堂上結合 GeoGebra 與中國古代數學史素材
12:10—14:00	Lunch 午餐	

Afternoon of March 13, 3 月 13 日下午—Workshop 会议

Workshop 大会报告，Host 主持人：ZOU Dahai 邹大海		
Time 时间	Speaker 报告人	Title 题目
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14:40—15:20	JI Zhigang 纪志刚	The Translation of Tongwen suanzhi and its Mathematics Knowledge Sources
15:20—16:00	CHENG Chun Chor, Litwin 鄭振初	Equations of Inscribed Circle in Ceyuan Haijing
16:00—16:20	Coffee break 茶歇	
Workshop 大会报告，Host 主持人：TIAN Miao 田淼		
16:20—17:00	DONG Jie 董杰	A Study of Xue Fengzuo's Method for Building the Trigonometric Tables
17:00—17:40	GAO Hongcheng 高红成	A Textual Research on the Original English Text of Yuanzhui Quxianshuo Translated by Li Shanlan and Joseph Edkins 李善蘭、艾約瑟合譯《圓錐曲線說》英文底本考
17:40—18:20	DENG Liang 邓亮	A Primary Study on Lin Chuanjia's *Weiji Jizheng* 林传甲《微积集证》初探
18:20—	Dinner 晚餐	

Morning of March 14, 3 月 14 日上午—Workshop 会议

Workshop 大会报告，Host 主持人：Young Wook Kim 金英郁		
Time 时间	Speaker 报告人	Title 题目
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9:10—9:50	Seisho Yoshiyama 吉山青翔	A Study of Zhang Shenfu's Letters to Yoshio Mikami:A Scene in the Exchange History for Study of the East Asia Mathematical History between Japan and China 张申府致三上义夫书简研究：日中东亚数学史研究交流史的一个断面
9:50—10:30	ZHANG Hong 张红	Mathematics exchanges between China and Japan and mathematics education modernization process in 3ichuan 中日数学交流与数学教育在四川的现代化过程

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10:50—11:30	GUO Shirong 郭世荣	Japanese Scholar Riken Fukuda's Understanding and Adaption of the Structure of the Astronomical Treatise Tantian (談天)
11:30—12:10	XU Zelin 徐泽林	The view on mathematics in Chinese-character cultural circle from Sūdo Shōdan 从《数度霄谈》看汉字文化圈的数学论
12:10—13:30	Lunch 午餐	

Afternoon of March 14, 3 月 14 日下午—Vist 考察参观

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Time 时间	Speaker 报告人	Title 题目
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9:10—9:50	Guo Shuchun 郭书春	关于《中华大典·数学典》
9:50—10:10	Han Yihua 韩义华	关于《中华大典·数学典》
10:10—10:30	Closing ceremony 闭幕式，Tsukane Ogawa 小川束 & JI Zhigang 纪志刚致辞	
10:30—	Coffee break 茶歇	

Abstract 论文摘要

Cheng-Yih Chen 程貞一 An Analysis of Thought Processes in Early Mathematics in Chinese Civilization 早期中華數學思路的分析

An Analysis of Thought Processes in Early Mathematics in Chinese Civilization

Joseph C. Y. Chen 程貞一

(Department of Physics, University of California at San Diego)

Abstract: By analyzing early works on the *Gōu-Gŭ* 勾股 principle in Chinese civilization, one realizes that thought process in obtaining the *Gōu-Gŭ* principle is based on 'derivative establishment' (i. e. *tuī dăo dé chéng* '推導得成'). This thought process of 'derivative establishment' is different from the thought process of 'axiomatic proof' of the early Greek civilization. In the west, the *Gōu-Gŭ* 勾股 principle is called the Pythagorean principle. But there is no existing records of Pythagoras's proof of the principle. The earliest available proof of the Pythagorean principle in the Greek civilization is that from the Euclid's *Elements* dated to c. 300BC. In the Chinese civilization, the derivation of the *Gōu-Gŭ* 勾股 principle first appeared in a dialog between Shāng Gāo 商高 and Zhōu Gōng 周公 of the West Zhōu 周. The earliest existing record of this derivation is found in the *Zhōu-Bì Suàn-Jīng*《周髀算經》(*The Mathematics Classic of Zhōu Gnomons*). The present paper presents an analyses on the effect of the different thought processes on the development of mathematics in the early Greek and early Chinese civilizations.

The derivation of the *Gōu-Gŭ* 勾股 principle first appeared in a dialog between Shāng Gāo 商高 and Zhōu Gōng 周公 of the West Zhōu 周. The earliest existing record of this derivation is found in the *Zhōu Bì Suàn Jīng*《周髀算經》(*The Mathematics Classic of Zhōu Gnomons*).

Between the thought processes of 'derivative establishment' of the Chinese civilization and the 'axiomatic proof' of the Greek civilization.

The derivation of the *Gōu-Gŭ* 勾股 principle first appeared in a dialog between Shāng Gāo 商高 and Zhōu Gōng 周公 of the West Zhōu 周. The earliest existing record of this derivation is found in the *Zhōu Bì Suàn Jīng*《周髀算經》(*The Mathematics Classic of Zhōu Gnomons*).

早期中華數學思路的分析

程貞一
(聖迭戈加州大學物理系)

摘要：分析勾股定律在中華文明的各種早期記載，可體會到古代中華數學的思路偏重於定律的“推導得成”。這與古代希臘“公理証明”的思路有顯著的差異。西方現存最早勾股定律的証明記錄出現於古希臘歐几里德 (Euclid) 的几何代表著 Elements (約公元前 300 年) 一書中，由此記錄可見歐几里德給勾股定律的公理証明是非常複雜的。在中華文明，勾股定律出現於西周商高與周公的對話中，現存最早記錄出現於《周髀筭經》中的“商高篇”。本文分析 “演繹証明”與“推導得成”兩思路的異同，并評估此兩思路對古代中西數學發展的影響。

Jens Høyrup 简斯 • 休儒 Seleucid, Demotic and Mediterranean mathematics versus Chapters 8 and 9 of the Nine Chapters: accidental or significant similarities?

Seleucid, Demotic and Mediterranean mathematics versus Chapters 8 and 9 of the Nine Chapters: accidental or significant similarities?

Jens Høyrup
(Roskilde University, Denmark)

Abstract: Similarities of geometrical diagrams or arithmetical structures of problems have often been taken as evidence of transmission of mathematical knowledge or techniques between China and "the West". Confronting on one hand some problems from Chapter VIII of the Nine Chapters with similar problems known from Ancient Greek sources, on the other a Seleucid collection of problems about rectangles with a subset of the triangle problems from Chapter IX, it is concluded,

(1) That transmission of some arithmetical riddles without method – not "from Greece" but from a transnational community of traders – is almost certain, and that these inspired the Chinese creation of the fangcheng method, for which Chapter VIII is a coherent presentation;

(2) That transmission of the geometrical problems is to the contrary unlikely, with one possible exception, and that the coherent presentation in Chapter IX is based on local geometrical practice.

Seleucid, Demotic and Mediterranean mathematics versus Chapters 8 and 9 of the *Nine Chapters:* accidental or significant similarities?

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Contribution to the workshop
Mathematical Texts in East Asia Mathematical History
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preliminary version
(if quoting, indicate this status!)
21 February 2016

Abstract

Similarities of geometrical diagrams or arithmetical structures of problems have often been taken as evidence of transmission of mathematical knowledge or techniques between China and "the West". Confronting on one hand some problems from Chapter VIII of the *Nine Chapters* with similar problems known from Ancient Greek sources, on the other a Seleucid collection of problems about rectangles with a subset of the triangle problems from Chapter IX, it is concluded,

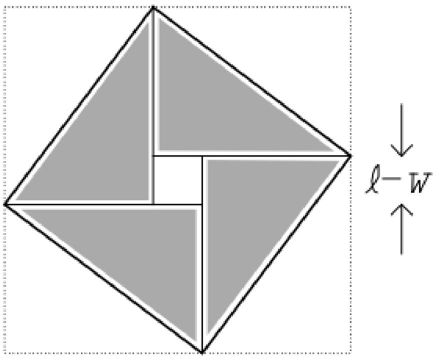
(1) that transmission of some arithmetical riddles without method – not "from Greece" but from a transnational community of traders – is almost certain, and that these inspired the Chinese creation of the *fangcheng* method, for which Chapter VIII is a coherent presentation;

(2) that transmission of the geometrical problems is to the contrary unlikely, with one possible exception, and that the coherent presentation in Chapter IX is based on local geometrical practice.

Contents

Two pictures and a winged transmission

First of all, as introduction, two pictures.



To the left, the decoration of the "Mathematics and System Science" tower in the campus of the Chinese Academy of Sciences in Beijing, borrowed from Zhao Shuang's third-century CE commentary to the *Gnomon of the Zhou* – cf. [Cullen 1996: 206; Chemla & Guo 2004: 695–701]; to the right, my own diagram reconstructed from the description of the procedure of the Old Babylonian problem Db_2-146 (from c. 1775 BCE), in which the sides of a rectangle are to be determined from its area and the diagonal [Høyrup 2002a: 257–259]. When I made it I did not know about the Chinese diagram.

A first reaction may be that the two must be connected; indeed, when Joseph Needham [1959: 96 n.a, 147] finds the same diagram in Bhaskara II (he gives no reference) and believes that it is found nowhere else, he finds it "extremely probable that Bhaskara's treatment derives from" Zhao Shuan's commentary.

A cnnection between the Mesopotamian and the Chinese diagram can certainly not be excluded *a priori;* there can be no doubt that the shared problem of the "hundred fowls" is really shared. The earliest known occurrence is in the fifth-century *Zhang Quijuan Suanjing* [van Hee 1913], but soon it turns up not only with the same mathematical structure but also with shared parameters and dress (100 units of different prices and a total price of 100) in Carolingian Western

Europe, in India and in the Islamic World – see, e.g. [Libbrecht 1974]; this cannot be imagined to be an accident.

We may add an observation, to my knowledge not made before. The early Islamic, Indian and Chinese occurrences speak of fowls, the Carolingian *Propositiones ad acuendos iuvenes* [ed. Folkerts 1978] has various dresses but none with fowls. This supports Jean Christianidis' suggestion [1991: 7] that the problem has developed from an early form represented in a Greek papyrus from the second century CE, where the units are already 100 but the price 2500.[1] Once the more striking version 100/100 was invented, that was the one that spread east and west – but the fowls only eastward, for which reason this latter invention must be presumed to be a secondary accretion.

Other *possible* interactions

The hundred fowls is an arithmetical riddle. Another riddle that *may* have travelled far is the one known in the cultures connected to the Mediterranean as the "purchase of a horse". Like the "hundred fowls" it circulates with varied numerical parameters, but a typical example states that three men go to the market in order to buy a horse. The first says that he has enough to pay the price if he can have half of the possession of the other two; the second only needs one third, the last only one fourth of what the other two have. The possession of each and the price of the horse is asked for. Sometimes the prie of the horse is given.

The problem seems to be hinted at in book I of Plato's *Republic* (333B–C).[2] In any case there is no doubt that it turns up, undressed as pure-number problems, in Diophantos' *Arithmetica* I:24–25 [ed. Tannery 1893: I, 56–69]; propositions 22–23, moreover, ask a question which, if dealing with a purchase, would make each participant ask for the fractions $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ of the possession not of all the others but of the one that precedes in a circle.

The latter type has an interesting parallel in Chapter 8, problem 13 of the *Nine Chapters* (my English from the French of [Chemla & Guo 2004: 643]):

[1] Text in [Winter 1936: 39].

[2] Trans. [Shorey 1930: I, 332*f*]. Socrates asks when one needs an expert; and as usually he answers himself: for example when you go to the market to buy or sell a horse in common. Since horses did not serve in agriculture but only for military purposes, they would never be bought in common in real life. (I owe the discovery of the Platonic passage to the late Benno Artmann).

> Let us assume that with five families sharing a well, that what is missing for the two ropes of Jia [in order to reach [the bottom of the well], that is as one rope of Yi, that what is missing for the three ropes of Yi, that is as one rope of Bing, that what is missing for the four ropes of Bing, that is as one rope of Wu, that what is missing for the six ropes of Wu, that is as one rope of Jia; and that, if each gets the rope that is missing for him. all will reach the bottom of the well. One asks for the depth of the well and for the length of the ropes.

The mathematical structure of the problem is the nearly the same, including the characteristic attractive sequence of fractions. However, in order to avoid cutting the ropes, one of *n* ropes is spoken of instead of the fraction $^1/_n$ of the totality of each; moreover, the request is made to the following participant, not to the predecessor in the circle.

Problems 3 and 12 (*ibid.* p. 625, 641) are determinate but otherwise similar in structure. No. 3 speaks of 2 bundles of millet of high quality, 3 of medium quality and 4 of low quality, and in similar combinations they are to produce 1 *dou*; no. 12 deals with the hauling capacity of horses of different strength. Problems 14 and 15 (*ibid.* pp. 645, 647) are sophisticated variants – we shall return to them.

The coincidences may seem striking – but are they evidence *of connection* or *of parallel experiences* of fascination? *If* the far from obvious dress had also been shared, as in the case of the hundred fowls, then connection would seem next to certain. Since it is not, we cannot decide on the basis of these problems alone.

However, problem 10 (*ibid.* p. 639) supports the connection hypothesis. It presents us with something like a two-participant version of the problems we have just examined. Two persons own money; if Jia gets half of what Yi possesses, he will have 50 coins; and if Yi gets $^2/_3$ of what Jia possesses, he will have as much.

With only two participants, there is of course no difference between Diophantos's two types. We notice, firstly, that here fractions and not "one out of *n*" are spoken about; secondly, that the dress is the familiar "give-and-take" type.

This dress is used for a slightly different mathematical structure in problems from late Mediterranean Antiquity (and later). In Book XIV of the *Greek Anthology* [ed. trans. Paton 1916: V, 105], no. 145 runs

> *A*. Give me ten minas, and I become three times as much as you. *B*. And if I get the same from you I am four times as much a you.

No. 146 uses different numerical parameters (two minas, twice, four times) but is otherwise identical. Prop. XV of Diophantos's *Arithmetica* I [ed. trans. Tannery

1893: I, 36f] is an undressed version of the same problem type.

On the other hand, Fibonacci's first example of a "purchase of a horse" [ed. Boncompagni 1857: 228] has the same mathematical structure as the Chinese give-and-take problem, apart from being indeterminate. In Fibonacci's problem, the first man asks for $\frac{1}{3}$ of the possession of the second, while the second asks for $\frac{1}{4}$ of what the first has. In both cases, each will have enough to buy the horse (whence the same).

Again, this coincidence on its own suggests but does not prove a connection. However, the alternative explanation here cannot be fascination with interesting numbers but only accident. If we take together all the problems we have looked at, independent invention in the two areas becomes unlikely – not least because the case of the "hundred fowls" provides us with firm evidence that transmission could and sometimes did take place.

But what exactly can have been transmitted? All the problems from the *Nine Chapters* come from Chapter 8, and all are used to train the *fangcheng* method. Diophantos's methods are quite different – and no closer are variants of the "Bloom of Thymarides" [Heath 1921: I. 94–96], which may have been used already around or before Plato's times to solve similar problems.

In connection with the "hundred fowls", Ulrich Libbrecht [1974: 313] points out that

> This implies that several mathematical problems were transmitted only as questions, without any method, as we can clearly state in Alcuin's work [the *Propositiones ad acuendos iuvenes*/JH]; in different places methods were developed - wrong or right - to solve these problems. Perhaps they were considered more as games than as serious problems, as we can prove from several Chinese and European works.

The same is clearly the case here. In terms I have used in [Høyrup 1990] (not knowing by then about Libbrecht's observation), the problems have circulated as subscientific mathematics, professional riddles of mathematical practitioners; once taken up by groups which in some way can be characterized as scholarly mathematicians,[3] these developed their own ways to deal with them, and in some cases they expanded the range of questions these methods could be applied to. Since the riddles functioned precisely as *riddles* in the community of practitioners (in anthropological parlance as *neck riddles*) it is not even certain that the practitioners always had a *mathematical* method for solving them – a

[3] That is, people who are engaged in or linked to a school-based (as opposed to an apprenticeship-based) educational system, and who in that connection shape and transmit mathematical knowledge.

riddle asks for an answer, not for a calculation or a logical derivation, and a guess followed by a verification may have been enough.[4]

That is where problems no. 14 and 15 of Chapter 8 of the *Nine Chapters* come in. No 14, similar to no. 3, deals with groups of unit fields with millet with different yields – say $2A$, $3B$, $4C$ and $5D$. But this time $2A+B+C = 3B+C+D = 4C+D+A = 5D+A+B = 1$ *dou*. This is too complex to present a nice recreational riddle – to keep track of it without material support would be difficult. No 15 deals with three groups of bundles of millet of different weights – in symbols, $2A–B = 3B–C = 4D–A = 1$ *dan*.

Such extensions of the range of "recreational" riddles by variation and systematization are a common occurrence in history, from Old Babylonian times to Pedro Nuñez and beyond. Diophantos, in *Arithmetica* I, replaces variation by generalization – but his choice of examples betray the recreational starting point.

At times, however, "scholarly mathematicians" have made a further step, and used the recreational material as the starting point or inspiration for the creation of a whole mathematical discipline. That is the way Old Babylonian second- and third-degree "algebra" was generated.[5] In the whole corpus, there is not a single problem derived from a genuine practical problem that might present itself to a Babylonian scribal calculator, even though the entities occurring as "unknowns" would be familiar to him – dimensions of fields and excavations, prices, manpower, etc. Only recreational riddles stating, for instance, the area and the sum of the sides of a rectangle, would do.

If we now consider Chapter VIII of the *Nine Chapters* as a whole, the parallel becomes obvious. Although the *Nine Chapters* as a whole teach administrators' mathematics, Chapter VIII does not present us with a single instance of this. True, *the entities* that occur would (mostly) be of the kind dealt with by calculating bureaucrats (the combined ropes hardly); but *the problems* would never turn up in their offices. Moreover, the book as a whole is a theoretical unity. Since the topic is absent from the *Suàn shù shū* [Cullen 2004: 6; *id.* 2007: 29; Dauben 2008: 97, 131], Chapter VIII can be assumed to be the outcome of recent systematic establishment of a well-defined mathematical field – inspired in all

[4] This is precisely what Abū Kāmil reproaches those who enjoyed the "hundred fowls" in his times and surroundings – it was "a particular type of calculation, circulating among high-ranking and lowly people, among scholars and among the uneducated, at which they rejoice, and which they find new and beautiful; one asks the other, and he is then given an approximate and only assumed answer, they know neither principle nor rule in the matter" – my translation from [Suter 1910: 100].

[5] See, for instance, [Høyrup 2001].

likelihood by select recreational problems, since administrative mathematics in itself would not lend itself adequately to that role.

All in all, Chapter VIII and its Mediterranean kin thus appears to present us with all the facets involved in questions about transmission:

1. transmission of *problems* as riddles from an unidentified *somewhere* to both the classical Mediterranean area and Han China (and other locations).[6]
2. Local creation of adequate *methods*.
3. A creation of a mathematical *discipline* on this foundation in China, in a process that is parallel to what can be seen in Old Babylonian mathematics. This parallel was based on shared sociological conditions and certainly did not involve any kind of transmission of metamathematical ideals.[7]

Problems about "combined works" present themselves easily in all cultures of scribal mathematical administration, and there are basically only two reasonable ways to solve them (obviously algebraically equivalent); neither the occurrence of such problems in different places nor a shared way to proceed can thus be taken as evidence of transmission. An unlikely dress can, however (as in the case of the "hundred fowls").

Such a case is present in Chapter VI, problem 6 of the *Nine Chapters* [ed. trans. Chemla & Guo 2004: 541]. Here, a pool is filled from five streams. As it is, filling is a preferred dress for such problems in the *Greek Anthology* XIV – thus no. 7, 130–133, 135 [ed. trans. Paton 1916: V, 31, 97, 99]. Shared transmission from somewhere is thus likely – but since this is no favourite dress of the problem type in the *Nine Chapters* (other instances – no. 22, 23 and 25 – really concern working rates), an isolated borrowed recreational problem may simply have been inserted in an adequate place of the *Nine Chapters*, the writer having recognized an already familiar type.[8]

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[6] The *somewhere* must be emphasized. In questions of this kind it is misleading to take for granted that the ultimate source must be one of the literate, "nationally" defined high cultures we know about – "the Chinese", "the Greeks", "the Indians", etc. "Proletarians have no fatherland", it was claimed – until the experience of the First World War proved the opposite. Merchants and technicians (even highly qualified technicians à la Werner von Braun) still have none.

[7] Those of the Old Babylonian school had died with the school itself around 1600 $_{\text{bce}}$, more or less at the time of the earliest oracle bones. Even if that had not been the case, however, transmission could be safely excluded – institutional ideals cannot be exported with understanding without export of the institution itself.

[8] No. 21 and 22 are in the dress of travel times, which also shows up elsewhere in later times. No. 27 and 28, of structure "box problems", are in the dress of repeated taxation,

All of these cases of credible transmission, from the "hundred fowl" onward, are number problems; they are of the kind that would allow an accountant and a travelling merchant to show their mathematical proficiency. Since accountants are likely to stay more or less in their place, travelling merchants constitute the plausible carrying community for these riddles; at an earlier occasion [Høyrup 1990: 74] I have spoken of them as the "Silk Route group".

Seleucid and Demotic Geometry

We started with a suggestive geometric diagram, and then shifted focus to the possible transmission of arithmetical riddles. Let us return to geometry.

Our initial diagram is too isolated to be worth pursuing. More intriguing is the geometry of Chapter IX of the *Nine Chapters* in relation to certain geometric problems from Seleucid Mesopotamia (third to second century BCE) and Hellenistic-Demotic Egypt.

The Seleucid problems in question (mainly) deal with rectangles with a diagonal. They have a family relationship with the Old Babylonian so-called "algebra" – apparently not by direct descent but via shared borrowing from the riddles of practical surveyors.[9]

Most of the problems in question are known from the tablet BM 34568,[10] undated but probably from the later third or earlier second century bce. Its problems can be described as follows:[11]

(1) $l = 4$, $w = 3$; d is found as $\frac{1}{2}l+w$ – first formulated as a general rule, next done on the actual example.

also familiar from India and elsewhere in the later first and early second millennium. Chapter VI thus serves, it seems, as a receptacle for several widely circulating recreational problems (much as Diophantos's *Arithmetica* I), for which it presents the earliest written evidence.

[9] The evidence for this is in part linguistic, in part it has to do with strong reduction (followed by expansion) at the level of mathematical substance. See [Høyrup 2002: 389–399].

The kinship does not imply that the Seleucid problems represent an "algebra". Whether the Old Babylonian technique does so is a matter how we define algebra, but no reasonable definition will cover the Seleucid rectangle problems.

[10] [Ed. trans. Neugebauer 1935: III, 14–19], partial edition, "conformal" translation and commentary in [Høyrup 2002: 392–399].

[11] l stands for the length, w for the width, d for the diagonal and A for the area of a rectangle. The sexagesimal place value numbers are transcribed into Arabic numerals. Entities that are found but not named in the text are identified in ⟨ ⟩.

With minor corrections, I draw the list from [Høyrup 2002{g}: 13*f*].

(2) $l = 4$, $d = 5$; w is found as $\sqrt{d^2-l^2}$.

(3) $d+l = 9$, $w = 3$; l is found as $\frac{½\cdot([d+l]^2-w^2)}{d+l}$, d as $(d+l)-l$.

$d+w = 8$, $l = 4$; solution corresponding to (3).

(5) $l = 60$, $w = 32$; d is found as $\sqrt{l^2+w^2} = 68$.

(6) $l = 60$, $w = 32$; A is found as $l\cdot w$.

(7) $l = 60$, $w = 25$; d is found as $\sqrt{l^2+w^2} = 65$.

(8) $l = 60$, $w = 25$; A is found as $l\cdot w$.

(9) $l+w = 14$, $A = 48$; $\langle l-w\rangle$ is found as $\sqrt{(l+w)^2-4A} = 2$, w as $½\cdot([l+w]-\langle l-w\rangle)$ and l finally as $w+\langle l-w\rangle$.

(10) $l+w = 23$, $d = 17$; $\langle 2A\rangle$ is found as $([l+w]^2-d^2) = 240$, $\langle l-w\rangle$ next as $\sqrt{(l+w)^2-4A} = 7$ – whence l and w follow as in (9).

(11) $d+l = 50$, $w = 20$; solved as (3), $l = 21$, $d = 29$.

(12) deals with a reed leaned against a wall, cf. imminently; a corresponding rectangle problem is $d-l = 3$, $w = 9$; d is found as $\frac{½\cdot(w^2+[d-l]^2)}{d-l} = 15$, l as $\sqrt{d^2-w^2} = 12$.

(13) $d+l = 9$, $d+w = 8$; $\langle l+w+d\rangle$ is found as $\sqrt{(d+l)^2+(d+w)^2-1} = 12$, where 1 obviously stands for $(l-w)^2 = ([d+l]-[d+w])^2$; next, w is found as $\langle l+w+d\rangle-(d+l) = 3$, d as $(d+w)-w$, and l as $(d+l)-d$.

(14) $l+w+d = 70$, $A = 420$; d is found as $\frac{½\cdot([l+w+d]^2-2A)}{l+w+d} = 29$.

(15) $l-w = 7$, $A = 120$; $\langle l+w\rangle$ is found as $\sqrt{(l-w)^2+4A} = 23$, w as $½\cdot(\langle l+w\rangle-[l-w]) = 8$, l as $w+(l-w)$.

(16) A cup weighing 1 mina is composed of gold and copper in ratio 1:9.[12]

(17) $l+w+d = 12$, $A = 12$; solved as (14), $d = 5$.

(18) $l+w+d = 60$, $A = 300$; not followed by a solution but by a rule formulated

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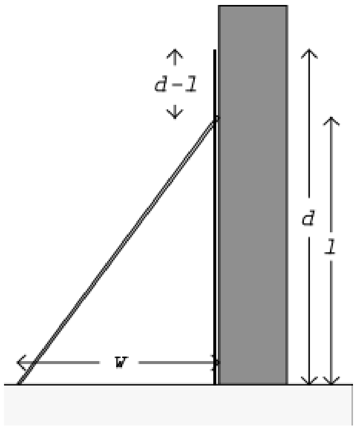
[12] Obviously an intruder, which however shows that problems from the "Silk Road group" were already known in Mesopotamia at the time. In the present context there is no reason to discuss this connection in depth.

Similar connections *may* be in play in the seemingly aberrant problem 1 (extensively discussed, but with a different aim, in [Gonçalves 2008]).

in general terms and corresponding to (14) and (17).

(19) $l+d$ = 45, $w+d$ = 40; again, a general rule is given which follows (13).

No. 2 is obviously an application of what I prefer to call the "Pythagorean rule", no theorem being involved; it corresponds to knowledge that was amply around in Old Babylonian times. No. 10 is not identical with the Old Babylonian problem which I referred to initially (Db_2-146); but it is closely related and solved by means of the same diagram.



The reed leaned against the wall.

All the others (disregarding here and in what follows no. 1 and no. 16) represent innovations within the surveyor's riddle tradition. No. 12 deserves particular discussion. It deals with a reed of length *d* first standing vertically against a wall, next in a slanted position, in which the top descends to height *l* (descending thus d–l); at the same time, the foot moves a distance *w* away from the wall.

In the Old Babylonian text BM 85196 we find a similar dress, but there *d* and *w* is given. To find *l* thus requires nothing but direct application of the Pythagorean rule (similarly to no. 2 here). In the present case, instead, as stated, the descent d–l is given together with *w*.

It is possible to find plausible geometric explanations of the procedures used to solve all the "new" problems – see [Høyrup 2002b: 13–18]; that, however, is of no interest in the present connection.

BM 34568 is not our only source for this kind of rectangle problems. Firstly, the Seleucid text AO 6484 ([ed. Neugebauer 1935: I, 96–99]; early second century BCE) contains a rectangle problem of the same type as nos. 14, 17 and 18 of BM 34568. Secondly, the Demotic papyrus P. Cairo J.E. 89127–30, 89137–43 from the third century BCE[13] contains eight problems about the reed leaned against the wall – three of the easy Old Babylonian type where *d* and *w* are given, three of the equally simple type where *d* and d–l are given; and two, finally, where d–l and *w* are given, as in BM 34568 no. 12. Apart from the numerical parameters, moreover, two of its problems are identical with that of the Old Babylonian Db_2-146, and thus closely related to BM 34568 no. 10.

[13] [Ed. trans. Parker 1972: 13–53], with summary *ibid.* pp. 3*f.*

There can be no doubt that the ultimate source for this whole cluster of geometric problems is Mesopotamia – Pharaonic mathematics contains nothing similar. It is also easy to pinpoint a professional community that could transmit it: For half a millennium, Assyrian, Persian and Macedonian military surveyors and tax collectors (the latter no doubt trained in the Near Eastern tradition) had walked up and down Egypt.[14]

Travelling geometry – travelling how far?

It is less easy to identify the channels through which these problems came to be adopted into Jaina mathematics, as they certainly did [Høyrup 2004]. Our evidence is constituted by Mahāvīra's *Gaṇita-sāra-saṅgraha* from the ninth century CE, but it is obvious that by then these problems were considered old, native and venerable by the Jainas. It is also highly plausible that what reached them had already been digested and somewhat transformed by a broader Mediterranean community

If they reached India, could they also have inspired China's mathematical bureaucrats, or at least the author of Chapter IX of the *Nine Chapters* (which has no more to do with real bureaucratic tasks than Chapter VIII)?

At a first glance, problems 6 to 12 and 24 might suggest so. In that case, however, the inspiration has certainly been digested – the topic of Chapter IX is the right triangle, and with exception of the reed problem the Seleucid-Demotic problems deal with rectangles. Moreover, the Chinese variant of the reed against the wall (no. 8 – the only one where the dress is suggestive) compares the slanted and the *horizontal* position.

If we express the Chinese problems in the same symbolic form as used for BM 34568, we get the following:[15]

(6) $d–l = 1$, $w = 5$ (mathematically an analogue of the Seleucid reed problem BM 34568 no. 12, but dealing with a reed in a pond, in vertical and slanted position). l is found as $\frac{w^2-[d-l]^2}{2\cdot(d-l)}$, whence d, where BM 34568 finds d as $\frac{½\cdot(w^2+[d-l]^2)}{d-l}$ and next l. We observe that the Chinese procedure does not halve before dividing by $(d–l)$, which suggests that a geometric justification, *if* once present, had been forgotten.

[14] Macedonians excepted, this explains why Greek authors, from Herodotos onward, could believe the Egyptians to have invented geometric techniques which we now recognize as Mesopotamian.

[15] Once more I borrow (this time more freely) from [Høyrup 2002b].

(7) $d-l = 3$, $w = 8$. The mathematical structure of the problem is the same, but the concrete dress wholly other. The solution proceeds differently than everything Seleucid-Demotic: $\langle d+l\rangle$ is found as $\frac{w^2}{d-l} = \frac{(d+l)\cdot(d-l)}{d-l}$, and d as $\frac{1}{2}\cdot(\langle d+l\rangle+[d-l])$.

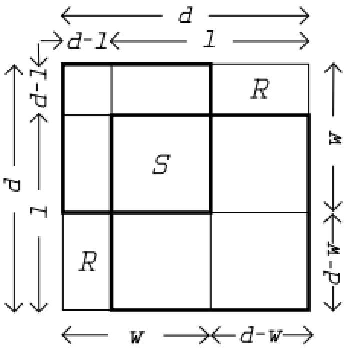
(8) $d-l = 1$, $w = 10$. Same mathematical structure and same procedure as no. 7 – but the dress is now a pole first leaning against a wall and then sliding down to horizontal position.

(9) Another variation of no. 7.

(10) Yet another variation of no. 7.

(11) $d = 100$, $l-w = 68$. $\langle\frac{l+w}{2}\rangle$ is found as $\sqrt{\frac{d^2-2\cdot(\frac{l-w}{2})^2}{2}}$, and w then as $\frac{1}{2}\cdot(\langle\frac{l+w}{2}\rangle+\langle\frac{l-w}{2}\rangle])$. Not Seleucid-Demotic in style with its use of average and deviation – nor however similar in detail to anything from the older Mesopotamian tradition, where these quantities were fundamental.

(12) $d+l = 10$, $w = 3$. $\langle d-l\rangle$ is found as $w^2/(d+l)$, and l as $\frac{1}{2}\cdot([d+l]+[d-l])$, once more different from the Seleucid calculation.



The possible geometric basis for problem IX.24 of the *Nine Chapters*.

(24) $d-l = 2$, $d-w = 4$. The solution builds on the observation that $\square(d-[d-l]-[d-w]) = 2(d-l)\cdot(d-w)$. The problem type is not found in BM 34568, but the solutions can be argued from a diagram that can also be used to solve problem 13 of that text, $d+l = \alpha$, $d+w = \beta$. In the present case, the full square $\square(d)$ must equal the sum of the squares $\square(l)$ and $\square(w)$; therefore, the overlap $S = \square(d-[d-l]-[d-w])$ must equal the area which they do not cover, that is, $2R = 2$⊏⊐$(d-l,d-w)$.

All in all, the similarities boil down to the mathematical structures of the questions. The dresses are generally quite different, and the procedures used to obtain the solutions are also others, often as different in character as the subject allows. All in all, no decisive internal evidence speaks in favour of transmission.

External evidence is equally unfavourable to the transmission thesis: arithmetical riddles might be carried along the Silk Road network by travelling merchants and exchanged as camp fire fun or challenges. But where can we find likely carriers of geometrical questions?

All in all, Chapter IX of the *Nine Chapters* is likely to be just as much an original Han creation as Chapter VIII – but with the difference that the underlying inspiration must be sought in local geometric practice, and not in the practice or riddles of any transnational professional community.

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Luo Jianjin 罗见今 An Analysis on the Achievement of Counting Theory in the Book "*Mengxi Bitan*" by Shen Kuo

An Analysis on the Achievement of Counting Theory
in the Book "*Mengxi Bitan*" by Shen Kuo

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Abstract: The Piling Method ("Duo ji Shu")proposed in the book "*Mengxi Bitan*" by Shen Kuo (1031-1095) is the first research on the piling method in the history of mathematics in China, which promoted the research on the series theory. Shen Kuo also proposed three methods to calculate the digits 3361 of different go game chessboards ("Qiju Dushu"). This paper discusses these two questions from the perspective of counting theory, makes an analysis on Shen Kuo's methods, and also indicates the significance of these achievements in enumeration theory.

Key words: Shen Kuo(沈括); *Mengxi Bitan* (*Dream Brook Essays*,《梦溪笔谈》); piling methods("Duo ji Shu",垛积术); the digits 3361 of different go game chessboards ("Qiju Dushu", 棋局都数), enumeration or counting theory (计数论)

沈括《梦溪笔谈》中计数成就探析

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摘要：沈括（1031-1095）在他的《梦溪笔谈》中提出了"隙积术"，开创中算垛积术之先河，推进了级数论的研究；在"棋局都数"中他提出三种方法判断围棋不同棋局总数 3361 的位数。本文从计数论的角度论述这两个名题，分析沈括的方法，指出这些成就在计数论中的意义。

关键词：沈括；《梦溪笔谈》；隙积术；棋局都数；计数论

引言

沈括是中国文化史、科学史上的巨人。《宋史·沈括传》称"括博学善文，于天文、方志、律历、音乐、医药、卜算，无所不通，皆有所论著。又纪平日与宾客言者为《笔谈》，多载朝廷故实、耆旧出处，传于世"。沈括既是一位朝廷命官，同时又是一位学者。他一生勤奋好学，博古通今，著作有 40 种之多，其中有经类 8 种、史类 11 种、子类 18 种、集类 3 种，涉及易、礼、乐、春秋、仪注、刑法、地理、儒家、农家、小说家、历算、兵书、杂艺、医书、别集、总集、文史等 17 类。

《宋史·艺文志》著录沈括的著作共有 22 种：《乐论》、《乐器图》、《三乐谱》、《乐律》、《春秋机括》、《熙宁详定诸色人厨料式》、《熙宁新修凡女道士给赐式》、《诸敕式》、《诸敕令格式》、《诸敕格式》、《天下郡县图》、《忘怀录》、《笔谈》、《清夜录》、《熙宁奉元历》、《熙宁奉元历经》、《熙宁奉元历立成》、《熙宁奉元历备草》、《比较交蚀》、《良方》、《苏沈良方》、《集贤院诗》。

《梦溪笔谈》（包括《补笔谈》、《续笔谈》，下同）具体分为故事、辩证、乐律、象数、人事、官政、权智、艺文、书画、技艺、器用、神奇、异事、谬误、讥谑、杂志、药议等节。

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科学史家胡道静（1913-2003）先生是研究《梦溪笔谈》的国际知名学者，他把该书条目分为 609 条[1]，认为属社会科学和掌故、见闻的 420 条，属自然科学的 189 条。自然科学的内容约占全书三分之一。因此，《梦溪笔谈》既是一部自然科学著作，也是一部人文科学著作。

《梦溪笔谈》对科学多有贡献，他的发现和认识成为中国科学史上的早期记录，例如：天文测量方面简化、改革浑仪、浮漏和用表测影的方法，记载在《浑仪议》、《浮漏议》和《景表议》等三文中，晚年提出用“十二气历”代替阴阳合历；地学方面发现地磁偏角；据太行山有螺蚌壳和卵形砾石的带状分布，推断出此系远古海滨；指出华北由黄河等携带泥沙形成的沉积平原；据浙东雁荡山诸峰地貌，指出此系水蚀之结果；沈括认识到石化的竹笋、松树、鱼蟹等系古生物遗迹；化学方面记录“胆水炼铜”：即用铁置换出铜的生产过程；定义“石油”并预言这种液体的性质和用途；等等，不胜枚举[2]。

我国著名科学家竺可祯（1890-1974）先生高度评价沈括的科学精神：“在括当时能独违众议，毅然倡立新说，置怪怨攻骂于不顾，其笃信真理之精神，虽较之于伽利略，亦不多让也。”钱宝琮（1892-1974）先生称沈括为“伟大的科学家”。沈括在诸多领域均有建树，被英国李约瑟称作“中国整部科学史中最卓越的人物”，并称他的《梦溪笔谈》为“中国科学史的里程碑”。美国科学史家席文（Natham Sivin，1931- ）则称沈括是“中国科学与工程史上最多才多艺的人物之一”。上世纪初，日本数学家三上义夫（1875-1950）认为：“沈括这样的人物，在全世界数学史上找不到，唯有中国出了这样一个。我把沈括称做中国数学家的模范人物或理想人物，是很恰当的。”

本文选择《梦溪笔谈》的部分内容做分析，指出沈括对“隙积术”和“棋局都数”等的研究都十分深刻，属于重要计数成果。他是中算家级数论的创始者。另外，沈括对“甲子纳音”的研究，颇具特色，在构造中使用了计数（enumeration or counting）方法，将另文讨论。

1 沈括是科学史上的伟人

1.1 沈括的生平

沈括（1031-1095）[3]，字存中，号梦溪丈人，浙江钱塘（今杭州市）人。北宋杰出的政治家、科学家。父沈周在简州（今四川简阳）、润州（今江苏镇江）和福建泉州等地做官。沈括从小接受儒学教育，幼年随父南迁，到父任所各地和京城开封，见识广博，眼界开阔。十四、五岁时住金陵（今南京市），研究医药，宋仁宗皇祐二年（1050）著《苏沈良方》二卷。沈周翌年去世。六年，以父荫任海州沭阳县主簿。

图 1 沈括画像

宋仁宗嘉祐六年（1061），任安徽宁国县令，撰《圩田五说》、《万春圩图记》。八年，考中进士，任扬州司理参军，掌刑讼审讯。宋英宗治平二年（1065）荐入京师昭文馆，校编典籍，治天文历算。

宋神宗熙宁二年（1069），协助王安石变法，任三司使，管理全国财政。参与删定三司条件。四年，任太子中允，检正中书刑房公事。五年，兼提举司天监，观测天象，推算历书。“迁提举司天监，日官皆市井庸贩法象图器，大抵漫不知。括始置浑仪、景表、五壶浮漏，后皆施用。”（《宋史·沈括传》）荐举淮南盲人卫朴编制《奉元历》。六年，任集贤院校理。多次出使，赴两浙考察农田、水利、差役。撰《浑仪议》、《浮漏议》、《景表议》、《修

城法式条约》、《营阵法》。七年，主管军器监，详定《九军阵法》。八年，以翰林侍读学士出使辽国，交涉划界，获成而还。绘辽国地图《使虏图抄》。九年，任权三司使，整顿陕西盐政。力主减免下户役钱，变法失败受牵连，罢官。十年，任宣州知州（今安徽宣城）。

宋神宗元丰二年（1079），御史李定、何正臣等上表弹劾苏轼，即乌台诗案。沈括曾指责苏轼。三年，知延州（今陕西延安），兼鄜延路经略安抚使。率军与西夏军作战，收复失地。五年，升龙图阁直学士。因首议筑永乐城（今陕西米脂县西），宋军战败，损兵二万，连带担责，被贬随州[①]（今湖北随县）团练副使。八年，改置秀州（今浙江嘉兴）。

宋哲宗元祐二年（1087），绘成《守令图》（天下郡县图），获准到汴京进呈。三年，《良方》定稿。四年，叙以光禄少卿分司南京衔，获允自由迁居。自五年始，移家润州（今江苏镇江市东），隐居“梦溪园”，《梦溪笔谈》定稿，撰成《补笔谈》、《续笔谈》、《梦溪忘怀录》及《长兴集》等。宋哲宗绍圣二年（1095）去世，享年六十四岁。

1.2 从“梦溪笔谈序”看作者的著作思想

“梦溪笔谈序　　沈括存中述

予退处林下，深居绝过从，思平日与客言者，时纪一事于笔，则若有所晤言，萧然移日。所与谈者，唯笔砚而已，谓之《笔谈》。圣谟国政及事近宫省，皆不敢私纪；至于系当日士大夫毁誉者，虽善亦不欲书，非止不言人恶而已。所录唯山间木荫，率意谈噱，不系人之利害者，下至闾巷之言，靡所不有。亦有得于传闻者，其间不能无缺谬。以之为言则甚卑，以予为无意于言可也。”

图 2 沈括塑像

《梦溪笔谈》序言字数并不多，阐明作者隐居梦溪的写作原则和态度。这需要了解当时变法的时代背景。熙宁初年，宋神宗重用王安石任宰相，推行机构、赋税、军队、科举等制度改革，兴修水利。各项新法触犯了皇室、官员、豪强等的利益，受到阻挠和反对。后来宋神宗态度动摇，熙宁七年（1074）、九年王安石两次去职。神宗去世，年仅 10 岁的哲宗继位，高太后垂帘听政，以司马光任宰相，元祐年间（1086-1093）废除新法，史称“元祐更化”。

从沈括履历来看，在政治上和军事上至少受到过两次重大挫折：首先，王安石变法受到保守势力的强烈反对，作为被重用的沈括，虽然业绩卓然，但由于反对力量强大，熙宁末年他被贬宣州。乌台诗案是双方交恶的结果，形成文字狱，双方都心怀警戒；在互相攻讦之中都受到严重伤害。其次，元丰五年（1082）永乐城败绩，此役沈括虽非首责，但他参与督军，加以救援不力，战败形势严重不利。这次以待罪之身被贬，形同流放，不得自由迁徙。由此可知，沈括在序言中说：“圣谟国政及事近宫省，皆不敢私纪；至于系当日士大夫毁誉者，虽善亦不欲书，非止不言人恶而已。”为什么采取这样的态度，就完全可以理解了，这是当时的政治环境所造成的，也是文字狱的恶果。为政者是非、善恶、毁誉皆无所萦怀，或不值一提，也与他本人的地位和心理状况相符。在这种背景下，把注意力转向自然、社会，“所录唯山间木荫，率意谈噱，不系人之利害者，下至闾巷之言，靡所不有”，撰成十一世纪科学与社会学的杰作。所以从某种意义上可以说，《梦溪笔谈》是在“元祐更化”背景下产生的。沈括在序言中所表明的莫谈国是、一心钻研自然科学的政治态度为后世隐者效法，

[①]一说,被贬均州(今湖北省均县).见百度沈括传.

不介入政事，只希望在揭示自然现象方面贡献个人的才智和观察力。

1.3 纪念沈括对科学的杰出贡献

沈括晚年居住在今镇江市区梦溪园，约 10 亩，一条溪水流经园内，原有多种建筑。1985 年部分恢复，占地 2 亩，前幢建筑门上方嵌有茅以升题写的“梦溪园”大理石横额；后幢厅房内有沈括雕像和文字图片、模型、实物，展现出沈括在天文、地理、数学、化学、物理、生物、地质、医学等方面的科研成就。沈括墓在杭州市良渚镇安溪下溪湾自然村北的太平坞，已得到很好的保护。1979 年 7 月 1 日为了纪念沈括，中国科学院紫金山天文台将该台在 1964 年 11 月 9 日发现的一颗小行星 2027 号命名为“沈括星”。

2.沈括“隙积术”首开中算垛积术

2.1 《梦溪笔谈》卷十八技艺“隙积术”原文[4]

“算术求积尺之法，如刍萌、刍童、方池、冥谷、堑堵、鳖臑、圆锥、阳马之类，物形备矣，独未有隙积一术。古法，凡算方积之物，有立方，谓六幕①（幂）皆方者，其法再自乘则得之。……有刍童，谓如复斗者，四面皆杀，其法：倍上长加入下长，以上广乘之，倍下长加入上长，以下广乘之，并二位法，以高乘之，六而二②（一）。

隙积者，谓积之有隙者，如累棋、层坛，及酒家积罂之类，虽以（似）复斗，四面皆杀，缘有刻缺及虚隙之处，用刍童法求之，常失于数少。予思而得之，用刍童法为上行，下行别列：下广以上广减之，余者以高乘之，六而一，并入上行。假令积罂最上行纵横各二罂，最下行各十二罂，行行相次，先以上二行相次，率至十二，当十一行也。以‘刍童法’求之：倍上行长得四，并入下长得十六，以上广乘之得三十二，又倍下长得十六③（二十四），并入上长得四十六④（二十六），以下广乘之，得三百一十二，并二倍⑤（位）得三百四十四，以高乘之，得二（三）千七百八十四。重列下广十二，以上广减之，余十，以高乘之，得一百一十，并入上行，得三千八百九十四，六而一，得六百四十九，此为罂数也。刍童求见实方之积，隙积求见合角不尽，益出羡积也。”

2.2 解说和证明

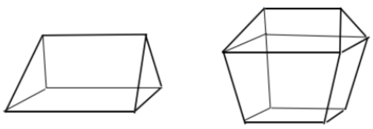

图 3 刍甍、刍草垛形状

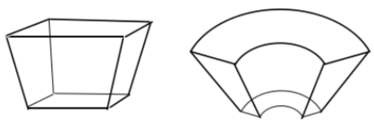

图 4 刍童、曲池形状

“求积尺”即求体积。沈括所举 8 种体积，均《九章算术》提出的名词，前几种的几何形状见图 3 和图 4。按照他的定义，设“上长”为 a，“下长”为 c，“上广”（宽）为 b，“下广”为 d，高为 n，据术文“刍童公式”则为：

$$V_{刍童}=\frac{1}{6}[(2a+c)b+(2c+a)d]n$$

此式所有字母都未必是正整数，上下长宽之间、长宽与高之间也无连带关系。但是，如果遇到按照刍童的形状堆垛起来的棋子、酒坛等，虽然也像扣着的斗一样下宽上窄，四面皆有斜度，需要计算的却是一个个物体，a，b，c，d，n 都是正整数。因为每个物体间都有空

①校点本未指出“幕”字为“幂”（平面面积）之误.
②校点本未指出“六而二”为“六而一”(用六去除)之误.
③“十六”为印刷错误.据该例,设定下长为十二,“倍下长”应为二十四.
④“四十六”为印刷错误.据该例,设定上长为二,“并入上长”应为二十六.
⑤“并二倍”当为“并二位”之误.据上文,有“并二位法”.

隙，如果利用刍童公式计数，其结果就“失于数少”。这是为什么呢？

设刍童垛积下层长宽都比上层多 1，则下长、下宽都与层数相关：$c=a+n-1$，$d=b+n-1$，

将这两式带入上面刍童公式，a，b 为已知，减少了两个变量，可得该垛积为：

$$V_{刍童垛}=nab+\frac{1}{2}n(n-1)(a+b)+\frac{1}{3}n(n-1)^2$$

沈括说这个结果不对。本文给出证明。记刍童状长方垛积为 $V_{刍长垛}$（图 5 上），则

$$V_{刍长垛}=\sum_{k=0}^{n-1}(a+k)(b+k)=nab+(a+b)\sum_{k=1}^{n-1}k+\sum_{k=1}^{n-1}k^2$$

$$=nab+\frac{1}{2}n(n-1)(a+b)+\frac{1}{6}n(n-1)(2n-1).$$

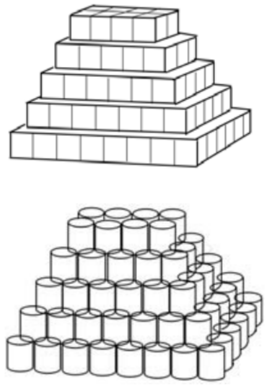

图 5 累棋层坛（上）及酒家积罂示意图

显然垛的构成下层长宽都比上层多 1，属于刍童状的垛积，其结果应是正确的。于是

$$V_{刍长垛}-V_{刍童垛}=\frac{1}{6}n(n-1)(2n-1)-\frac{1}{3}n(n-1)^2=\frac{1}{6}n(n-1).$$

这就是说，按照刍童公式算出的垛积要比实际的少 $n(n-1)/6$，依沈括的说法，“下广以上广减之，余者以高乘之，六而一，并入上行”，即是在 $V_{刍童垛}$的结果后添加一个“羡积”：$\frac{1}{6}(d-b)n$，上面已知 $d=b+n-1$，因此$\frac{1}{6}n(n-1)=\frac{1}{6}(d-b)n$，于是问题得证。

沈括具体举出一例来说明：设酒坛垛积上层长宽各 2，即 $a=2$，$b=2$，最下层长宽各 12，即 $c=12$，$d=12$，层数 $n=11$。按照 $V_{刍童}$公式＋羡积$(d-b)n/6$，可以逐步求出 $V_{刍童垛}=649$。因有计数公式，原文排印舛错易于纠正。也可按 $V_{刍长垛}$公式，直接求出酒坛总数为 649。此外，沈括所举之例较特别，是从 2 至 12 的自然数平方和，亦为幂和公式的基本一例：

$$V_{酒罂垛}=2^2+3^2+\cdots+12^2=\sum_{k=2}^{12}k^2-1=\frac{1}{6}12(12+1)(2\cdot12+1)-1=649.$$

最后，沈括总结道：“刍童求见实方之积，隙积求见合角不尽，益出羡积也。”就是说仅用求“实方”（无空隙）的刍童公式，未将边角之处“不尽”的体积计入，隙积术就是要把多出的“羡积”再加上。

附记一 所添加的羡积也可以是：

$$\frac{1}{6}(c-a)n\text{，因 } c=a+n-1\text{，}\frac{1}{6}(c-a)n=\frac{1}{6}n(n-1)=\frac{1}{6}(d-b)\text{。}$$

因此，沈括也可以说“下长以上长减之，余者以高乘之，六而一。”即“下行别列”有两种等值的表示方法。但两者只能取其一。

附记二 羡积是否正整数与层数 n 相关。对于 $n(n-1)/6$，$n=3,4,6,7,9,10,12,\cdots$时羡积为正整数。沈括所举例子 $n=11$ 时，羡积不为整数，但这并不妨碍求得酒罂垛之积，他的方法是：将余项分子与刍童垛分子相加后再除以六。

这样，沈括应用他创造的“隙积术”，将一个连续的几何体积（即“实方”）公式有目的地改造成为一个离散的（即“合角不尽”）计数公式，成为中算史上垛积术的发轫之作。

对连续和离散体积的这种细微区别，不知道沈括是怎样发现的；对垛积公式的证明，也

不知道他是怎样进行的。人们猜测：他建起两者的模型，对比计数方法，做出大量演算，才能深刻指出两者的不同。证完之余，我们惊异于沈括的洞察力和简洁的表达方式。上面应用的虽是数列求和的基本公式，所表示的关系并非轻易能被发现，其推导过程亦非可一蹴而就。

沈括的这一计数公式见证了从计算连续的几何体积转化为计算离散的堆垛体的过程，成为嗣后 800 年中算家朱世杰、杨辉、李善兰等发展垛积术之嚆矢。

3.《梦溪笔谈》棋局都数

围棋是中国文化的重要标志之一，在国际上特别是东亚形成了围棋爱好者的众多群体。

3.1 从围棋的发展看问题的提出

相传尧发明围棋，晋人张华在《博物志》中说：“尧造围棋以教子丹朱”；《路史后记》记载尧为调教儿子丹朱下围棋“以闲其情”。围棋用以开发智慧、陶冶性情。后来，舜觉得儿子商均不甚聪慧，也曾用围棋教子。一说，“夏人乌曹作赌博围棋”，用于赌博。

陕西西安半坡出土的新石器时代陶罐上，绘有 10~13 道纵横条纹，格子齐整，很象围棋盘，被称为棋盘纹图案。唐人皮日休在《原弈》中认为，围棋始于战国，为纵横家所创，“有害诈争伪之道”，此说不足为信。打开网上围棋的历史，有关出土信息俯仰皆是。

1977 年内蒙古敖汉旗发掘辽代古墓，出土围棋方桌，高 10cm，围棋盘长宽各 30cm，纵横各 13 道，布有黑子 71 枚，白子 73 枚。辽代陈国公主墓里也发现了木质围棋子。

咸阳西汉中晚期甲 M6 墓葬出土石棋盘一件，长 66.4cm，厚 32cm，以黑线画出棋格 15×15=225 格。湖南湘阴县一唐代古墓出土随葬品中有一正方围棋盘，纵横各 15 道。

1954 年河北望都东汉古墓出土一石棋局，高 14cm，边长 69cm，上刻有纵横 17 道线。三国时魏邯郸淳《艺经》说“棋局纵横各十七道，合二百八十九道，白黑棋子各一百五十枚”。1975 年山东邹县发掘西晋刘宝墓，有一副装在灰色陶盒里的围棋，黑白共 289 子。

敦煌莫高窟石室所藏南北朝《棋经》中载明当时棋局是“三百六十一道，仿周天之度数”。

迄今围棋盘皆 19×19=361 格，黑 181 子，白 180 子。棋局千变万化，举不胜举。19^2 条线的任一交点上只能出现白子、黑子和无子的 3 种情况之一。对于任意一局，如果不考虑具体走法，就会产生一个问题：从理论上来说，从 361 子中无论取多少、无论怎样摆放，能够形成多少种不同的棋局？初看起来，只要反复计算就可以解决；但事实上连计数单位都不够用，将出现一个常人无法理解的天文数字。沈括就认真思考这个大数难题，并给出了自己的答案。

3.2 《梦溪笔谈》卷十八技艺中**棋局都数原文**[5]

“小说：唐僧一行曾算棋局都数凡若干局尽之。余尝思之，此固易耳，但数多非世间名数可能言之。今略举大数。凡方二路，用四子，可变八[千]①十一局；方三路，用九子，可变一万九千六百八十三局；方四路，用十六子，可变四千三百四万六千七百二十一局；方五路，用二十五子，可变八千四百七十二亿八千八百六十万九千四百四十三局；古法十万为亿，十亿为兆，万兆为秭。②合（算）家以万万为亿，万万亿为兆，万万兆为垓。今且以算家数计之。方六路，用三十六子，可变十五兆九十四万六千三百五十二亿八千二百三万一千九百二十六局；方七路以上，数多无名可纪。尽三百六十一路，大约连书万字五十二，即是局之大数。万字五十二，最下万字是万局，第二是万万局，第三是万亿局，第四是亿兆局，第五是万兆局，第六是万万兆，谓之一垓，第七是垓局，第八是万万垓，第九是万倍万万垓，此外无名可纪。但五十二次万倍乘之

① “千”为衍字。凡当删之字标在括号[]内。

②秭，音 zǐ，不作[禾弟]。按《数术记遗》下等进位法“秭”为 10^9；沈括认为古法“万兆为秭”即 10^{10}。

即是都大数，零中数不与。其法初一路可变三局一黑一白一空，自后不以横直，但增一子，即三因之，凡三百六十一增，皆三因之，即是都局数。又法：先计循边一行为法，凡十九路得一十（一）[③]亿六千二百二十六万一千四百六十七局。凡加一行，即以法累乘之，乘终十九行，亦得上数。又法：以自法相乘。得一百三十五兆八百五十一万七千一百七十四亿四千八百二十八万七千三百三十四局，此是两行凡三十八路变得此数也。下位副置之，以下乘上，又以下乘下，置为上位，又副置之，以下乘上，以下乘下，加一法，亦得上数。有数法可求，唯此法最径捷。只五次乘便尽三百六十一路。千变万化，不出此数，棋之局尽矣。

3.3 解说和计算

《易经》里，任选阴、阳爻可重复的 2 个符号置于 3 个位置，可得八卦：$2^3=8$；任选八卦可重复的 8 个符号置于下上两个位置，可得六十四卦：$(2^3)^2=8$。这称为有重排列（rearrangement）。在求棋局都数时，也遇到要把 3 个元素置于 19^2 个位置的问题。

⑴计数单位：沈括上文首先明确计数单位。“世间名数”无法表示特别大的数，古代计数单位不足为凭，需要以算家使用的单位“以万万 $10^4\cdot10^4$ 为亿 10^8，万万亿 $10^8\cdot10^8$ 为兆 10^{16}，万万兆 $10^8\cdot10^{16}$ 为垓 10^{32}”来计数。从第 2 章 4 节可知，这是《数术记遗》所述“中数”记法。目标不是算出精确的数字，而是“略举大数”，从他的下文看，“大数”就是位数。

⑵沈括求棋局都数**第一法：**设黑子个数为 a，白子为 b，沈括视空子是与黑白子相同的一类，记作 c。文中的“子数”也就是棋盘上的交点数，为三者之和 $a+b+c$。他由浅入深列出计数步骤，递次枚举出指数 n 为自然数平方（$n=2^2$，3^2，…，19^2）的幂级数数列$\{3^n\}$：

①棋盘方 2 路，用 4 子，可变 $3^4=81$ 局；②方 3 路，9 子，可变 $3^9=19\ 683$ 局；③方 4 路，16 子，可变 $3^{16}=43\ 046\ 721$ 局；④方 5 路，25 子，可变 $3^{25}=847\ 288\ 609\ 443$ 局；⑤方 6 路，36 子，可变 $3^{36}=15\ 009\ 463\ 528\ 231\ 926$ 局。接着他说“方七路以上，数多无名可纪”。棋局都数最后的结果，“大约连书万字五十二 $10^{4\times52}=10^{208}$”。他解释说：从第 1 个万字 10^4 开始，到第 7 个万字，相当于垓 10^{24}；第 9 个万字是“万倍万万垓 10^{32}”，“此外无名可纪”。总的算法：“但增一子，即三因之，凡三百六十一增，皆三因之，即是都局数”，即 3^{361}。

⑶沈括求棋局都数**第二法：**“先计循边一行为法”，一边上有 19 个交点，3 元素共可形成 $3^{19}=1162261467$ 局；然后逐行计出后累乘。如将第 n 项($n=1,2,\cdots,19$)记作$(3^{19})_n$，“乘终十九行”，于是获得：$(3^{19})_1(3^{19})_2\cdots(3^{19})_{19}=(3^{19})^{19}=3^{361}$。

⑷沈括求棋局都数**第三法：**“以自法相乘”：$(3^{19})(3^{19})=3^{38}=135\ 851\ 717\ 448\ 287\ 334$，作为“上位”，记作$(3^{38})_{上}$；另列为“下位”，记作$(3^{38})_{下}$，相乘$(3^{38})_{上}(3^{38})_{下}=3^{76}$，再乘$(3^{38})_{下}(3^{38})_{下}=3^{76}$，作为上位$(3^{76})_{上}$；另列$(3^{76})_{下}$，相乘$(3^{76})_{上}(3^{76})_{下}=3^{152}$，再乘$(3^{76})_{下}(3^{76})_{下}=(3^{152})_{上}$，最后“加一法”，即再乘以 3^{19}，于是经过 5 次计算，得到$(3^{38})(3^{76})(3^{76})(3^{152})(3^{19})=3^{361}$。
以上三种算法都具有一定复杂性，沈括力求计算量达于最小。

⑸ 现今的分析：为判定棋局都数的位数，可对 3^{361} 取对数，知：

$$\log3^{361}=361\times0.4771212547=172.24077295,$$

对数首数加 1 得位数：172＋1=173，表明 3^{361} 是一个 173 位的数：$10^{174}>3^{361}>10^{173}$。网上有的说该数为 10^{164}，偏小；有的说为 10^{768}，偏大。“大约连书万字四十三”才是正确的。

在上述计算中，任将 3 个可重复的元素置于 19^2 个位置，由于是有重排列，就会出现包括 $181<a\leqslant361$，$180<b\leqslant361$，$c=361$ 在内的情况，即出现围棋盘上 361 交点全黑、或全白，或全部无子，等等，与围棋规定相悖（$c=361$ 表示尚未下棋，不能算作一局）。另外，还没

[③]此处原文缺“一”字，添入括号( )中。因 $3^{19}=1162261467$，而非 1062261467。

有考虑“打劫”、“提子”等情况。这些都给精通围棋的数学爱好者提出了更深刻的课题。

实际上，沈括提出“棋局都数”的意义，主要在于构造指数 n 为自然数平方的幂级数数列$\{3^n\}$，使人们认识到指数以平方增长，数列便迅猛扩张，立即达到无法掌控的境地。沈括的成就，向人们展示出：数学家的任务，就是须要提供最优算法，对巨大的数作出估计。

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[5] 同上,129-130.

Tatsuhiko Kobayashi 小林龍彦 On the Acceptance of Surveying of the *Suanfa tongzong* ≪算法統宗≫

On the Acceptance of Surveying of the *Suanfa tongzong* ≪算法統宗≫
by Wasan-mathematician

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Abstract: Mathematics book of ancient Chinese, the *Suanxue qimeng* ≪算学啓蒙≫ in 1299 and the *Suanfa tongzong* ≪算法統宗≫ in 1592 had greatly influenced on the development of the pre-modern Japanese mathematics, Wasan. Wasan-mathematician learned particularly how to use a celestial element, Tianyuan-yi (天元一) which means an unknown number form the former, and learned using of Chinese abacus and practical mathematics from the latter.

It is well-known that Wasan-mathematician Yoshida Mitsuyoshi learned the *Suanfa tongzong* and published a mathematics book, entitled the *Jinko-ki* ≪塵劫記≫ in 1627. After this, the *Suanfa tongzong* was studied by a lot of Wasan-mathematicians and has had no little influence on the development of Wasan. Muramatsu Shigekiyo, Yuasa Tokuyuki, Isomura Yoshinori, Seki Takakazu etc. were Wasan-mathematicians who acquired mathematical idea of the *Suanfa tongzong* in the second half of the 17th century. An ancient surveying was also included as one of such mathematical idea.

In this symposium, we will introduce that what kind of surveying in the *Suanfa tongzong* was accepted by Wasan-mathematician.

Tsukane Ogawa 小川東 English Translations of Mathematical Books in Pre-modern Japan

English Translations of Mathematical Books in Pre-modern Japan

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Abstract: International Symposiums on the history of pre-modern Japanese mathematics have increased in recent years. Some proceedings of them were published in English. At this time, standardizing the English titles or mathematical terms in Pre-modern Japanese mathematics is required to avoid confusions.

First of all, I will summarize present issues about English translations of mathematical books in pre-modern Japan. Then I will give some translations of book titles. I will also argue Romanizing titles of books and names of authors.

Mitsuo Morimoto 森本光生 The Full Expansion Formula of Determinants Given in the *Taisei Sankei*

The Full Expansion Formula of Determinants Given in the *Taisei Sankei*

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Abstract: The *Taisei Sankei* is an encyclopedic work of mathematics written by three mathematicians Seki Takakazu (1640s-1708), Takebe Kata'akira (1661-1716) and Takebe Katahiro (1664-1739) during 1683 – 1711. The work is composed of twenty volumes preceded by an introductory volume. In Volume 17, the full expansion formula of determinant of orders 2, 3, 4 and 5 are given in connection with the elimination theory of the system of algebraic equations with several unknowns. We examine the exactitude of the description of determinants.

Sung Sa Hong 洪性士 Mathematical Structures of *Gugo Wonlyu* (勾股源流) by Jeong Yag-yong (丁若鏞)

Mathematical Structures of *Gugo Wonlyu* (勾股源流) by Jeong Yag-yong (丁若鏞)
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Abstract: Since *Jiuzhang Suanshu*, the main tools in the theory of right triangles, i.e., gougushu were algebraic identities about their three sides derived from the Pythagorean theorem. Using tianyuanshu up to siyuanshu, Song--Yuan mathematicians could skip over those identities in the theory. Chinese Mathematics in the 17--18th centuries were concerned with western geometrical proofs of the identities. Jeong Yag-yong (1762--1836), a well-known Joseon scholar and writer of the school of Silhak (實學), noticed that those identities can be derived through algebraic approach and then wrote *Gugo Wonlyu* (勾股源流) in the early 19th century. We show that Jeong reveals the algebraic structure of polynomials with the three indeterminates in the book along with their order structure, namely inequalities of polynomials. Although the title refers to right triangles, it is the first pure algebra book in Joseon mathematics.

HAN Qi 韩琦 1713: A Year of Significance

1713: A Year of Significance

1713 年：中国数学史上重要的一年

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摘要：康熙时代的数学传播是明清科学史乃至中国数学史上最为重要的篇章。受杨光先反教案的影响，康熙开始学习西学。1688 年，法国"国王数学家"来华之后，很快成为康熙的御用数学教师。但在 1713 年之前，数学活动都集中在宫廷，为皇帝所掌控。1713 年，康熙谕令建立蒙养斋算学馆，招募擅长算学的文人，编纂算学著作。本文将依据中西文献，重构 1713 年所从事的算学活动，这将帮助我们理解《数理精蕴》成书的复杂历程。

Young Wook Kim 金英郁 *Gougushu* in Joseon and *Yu-ssi Gugosulyo Dohae*

Gougushu in Joseon and *Yu-ssi Gugosulyo Dohae*

KIM, Young Wook
(Korea University)

Abstract: *Gougushu* 勾股术 is one of the most important subject in the oriental mathematics since the beginning of history. It has been studied by most of the authors in China, Japan and Korea. The theory has complex history of ups and downs and shows many interesting aspects of mathematics in theeast.

The *gougushu* advanced remarkably in Tang and Song dynasties in China which was succeeded by Korean mathematicians probably in Goryeo and definitely in Joseon. *Gougushu* is studied by most of the Joseon mathematicians and is treated in all of their writings. One of the most interesting of them is *Yu-ssi Gugosulyo Dohae* 劉氏勾股術要圖解 by Nam Byeong-gil 南秉吉 which is the explanations with proofs by diagrams of *Yu's Gugosulyo* (presumably an 18th century text of Korea). We review some of the Korean *gougushu* and see some special aspect of *Yu-ssi Gugosulyo Dohae*.

XIAO Can 秦人对于数学知识的重视与运用

秦人对于数学知识的重视与运用

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摘要：本文基于出土文献中的数学史料分析了秦人对数学的运用情况，论证了秦人普及的是“实用算法式数学”，这种注重实效的数学被广泛运用于管理田地租税、仓储物资、劳役工程、军战事务等，取得了高效，从而使秦人在兼并他国中占有优势。

关键词：秦，数学，实用算法

Key Words：Qin Period, mathematics, practical algorithm

秦统一的原因是多方面的，王子今先生在《秦统一原因的技术层面考察》一文中认为：“秦在技术层次的优越，使得秦人在兼并战争中取得优势”，文中提到了“天文历算数术之学也为秦人所重视。里耶秦简中九九乘法表的发现，为当时数学知识的普及提供了例证。”[1] 这种看法非常有道理。本文拟依据出土的秦代数学文献，对此作一探寻。

首先，数学确实为秦人所重视。

在多批出土的秦代文献里都发现有数学内容。除了广为人知的里耶秦简乘法表，还有两部较完整的数学书，一是北京大学藏秦简《算书》，一是岳麓书院藏秦简《数》。《算书》正在整理、部分已公布，它出自 2010 年北京大学获赠的一批秦简牍，数量达到 400 余枚，整理者命名为《算书》，根据简的形制、简文内容、书写特征将其分为甲种、乙种、丙种，主要内容除了田亩、赋税、粮食兑换等实际问题的算法外，还有一段长达 800 多字的“数论”，命名为《鲁久次问数于陈起》（以下简称《陈起》），论述了数学的起源及社会功能。《数》出自 2007 年底入藏岳麓书院的一批秦简，简上有自题篇名“数”（写在一枚简的背面），它的图版、释文、注释已于 2011 年底出版，即《岳麓书院藏秦简（贰）》，有两百多枚简，六千多字，主要是些具体算题，也有描述算法的“术”，还记载有多种谷物之间兑换的计算比率、衡制、乘法口诀等。

出土的秦代文献里明确提出数学的重要性。《陈起》篇简文：“鲁久次问数于陈起曰：‘久次读语、计数弗能立彻，欲彻一物，可（何）物为急？’陈起对之曰：‘子为弗能立彻，舍语而彻数，（数）可语殹（也），语不可数殹（也）。’久次曰：‘天下之物，孰不用数？’陈起对之曰：‘天下之物，无不用数者……’”[2]大意是“数”比“语”更重要，天下事物诸如宇宙构成、身体疾病、社会管理和生产活动无不用到数学，这倒有点像古希腊毕达哥拉斯学派的思想了，数是万物的本源。

其次，秦人重视的数学是一种“实用算法式数学”。

秦人重视数学，但只重视一种适于应用在实际生产生活的数学，邹大海先生称之为“实用算法式数学”，这种提法最早见于他对张家山汉简《算数书》的研究论文 3，秦简《数》公布后，对比研究证明秦汉数学均属于这种“实用算法式数学”，汉承秦制，数学也沿袭。之所以说秦汉数学是“实用算法式”的，是因为我们在秦简《数》、《算书》的算题里见到的

1 王子今：《秦统一原因的技术层面考察》，载《社会科学战线》2009 年第 9 期, 第 222-231 页。

2 韩巍：《北大藏秦简<鲁久次问数于陈起>初读》，载《北京大学学报(哲学社会科学版)》2015 年第 2 期(第 52 卷)，第 29-35 页。

3 邹大海：《出土算数书初探》，载《自然科学史研究》2001 年第 3 期(第 20 卷)，第 193-205 页。

都是对具体算法的描述，例如：

“〔仓广〕二丈五尺，问袤几可（何）容禾万石？曰：袤卌丈。术曰：以广乘高法，即曰，禾石居十二尺，万石，十二万（0498）尺为貣（实），（实）如法得袤一尺，其以求高及广皆如此。（0645）”[4]

题设条件是已知粮仓的“广”是 25 尺，求“袤”为多少时能容纳 1000 石禾。又补充已知条件：1 石禾堆积高度 12 尺，或说体积是 12 立方尺，因底面积默认是 1 平方尺。[5]这就好比是 1 石禾堆成个 1 尺×1 尺×12 尺的四棱柱体，那么 10000 石禾就是把 10000 个四棱柱体置放一起，每行 25 个，共有几行呢？古代数学书里指称计算方法的专用词是“术”，依照这道算题简文的“术”计算如下：

12×25=300 平方尺为法（分母），120000 立方尺为实（分子），分子除以分母 120000÷300=400 尺，即 40 丈。

我们看算题术文的叙述，它只说怎么做而不说为什么，更无公式定理的推演，这样的方式，不需要系统教学、不需要长时间练习、不需要学习者过多思考，只需要按例题指示一步步照做，在实际生产生活中套用，就可以完成计算工作。即使遇到较为复杂的问题，“实用算法式数学”也能让那些没什么数学基础的人解决难题。

那么，是否先秦时期的数学就已经是这种“实用算法式”呢？显然不是的。举几个反例，比如说《庄子·天下篇》里记有数学中的“极限”概念：“一尺之棰，日取其半，万世不竭。”《墨子·经上》里记有多种几何学概念的定义：“平，同高也。”释义：“一个东西是平的，它的各处都有相同的高度。这经过一定的几何抽象，不考虑该对象的厚薄，并假设有一个平的东西（可能是地平面）作为它的参照。”“圜，一中同长也。”释义：“圆，有一个中心，它到圆的边缘每一处都具有相同的长度。”[6]与今天的几何学无二致。不妨对比一下秦简《数》里关于圆的算题：“周田述（术）曰：周乘周，十二成一；其一述（术）曰，半周半径，田即定，径乘周，四成一；半径乘周，二成一。（J07）”

这四种算法相当于圆周率取π=3 时的近似算法，[7]未见述及圆的抽象定义，直接应用于田地面积的计算，算法也是说的具体如何操作，还是近似简化计算。从《庄子》、《墨子》里的记载不难看出，先秦数学也曾往抽象思维发展，并非肇始就趋向实用算法，而是秦人的选择和强化，传至汉，影响后世，以致中国古代数学陷入实用算法格局。

附带再提一下里耶秦简的“九九乘法口诀表”，与清华大学藏战国竹简的《算表》相比，也是简化了的。《算表》形成于公元前 305 年左右的战国时期，比里耶秦简乘法表早近百年，利用《算表》不仅能快速进行 100 以内两个任意整数的乘法，还能做包含分数 1/2 的乘法运算，乘数被乘数取值最大可达 495.5，因为数值较大，不是“九九乘法”的范围，因此推测在使用《算表》计算时也需用到乘法交换律、乘法分配律，以及分数等数学原理和概念。[8]可想而知，复杂的《算表》不易推广普及，而里耶秦简的“九九乘法口诀表”则易学易用，已满足日常的生产生活的需求。秦简《数》里也有部分乘法口诀。

再次，“实用算法式数学”在秦人生产生活的各方面得到广泛运用，如：

对农田、租税的管理。重视农业，各国皆是，《汉书·食货志》记载：“李悝为魏文侯作尽地力之教，以为地方百里，提封九万顷，除山泽邑居三分去一，为田六百万亩，治田勤谨

[4] 肖灿：《岳麓书院藏秦简<数>研究》，湖南大学博士学位论文，2010 年，

[5] 一般来说，古算书中说体积 a 尺时，是从一个底为 1 平方尺的四棱柱其高为 a 尺这个角度来衡量的。例如秦简《数》里同样的表述还有：“〔稻粟〕三尺二寸五分寸二一石。麦二尺四寸一石。”（0760）“刍新积廿八尺一石。稾卅一尺一石。茅卅六尺一石。”（0834）“粟一石居二尺七寸（0801）”

[6] 邹大海：《从<墨子>看先秦时期的几何知识》，载《自然科学史研究》2010 年第 3 期，第 293-312 页

[7] 肖灿：《岳麓书院藏秦简<数>研究》。

[8] 李均明，冯立昇：《清华简<算表>概述》，载《文物》2013 年第 8 期，第 73-75 页。

则亩益三升，不勤则损亦如之。地方百里之增减，辄为粟百八十万石矣。”就是要尽地力，增产量，盈租税。而秦胜在运用数学优化管理，史料从两个方面证明这点：一是秦人把测量田地、计算产量和租税中会遇到的各种计算问题写入数学书的例题，且一定是示范的实用算法；一是相关法律条文基于数学的量化规范。在秦简《数》里有方田（矩形田）、箕田（梯形田）、周田（圆形田）面积的测算法；有大枲、中枲、细枲、干禾、生禾的产量与租税计算；税制又与田地性质有关，分为舆田、税田，如枲舆田的税率是十五分之一；针对有可能出现的误算及“匿租”情况，又有“租误券”的算题。从《数》的租税类算题来看，秦征收实物田租，按田地面积课税，且很可能采取直接划定一片田地上的出产为租税的方式，见于《数》的算题：“禾舆田十一亩，税二百六十四步，五步半步一斗，租四石八斗，其术曰：倍二〔百六十四步为〕(1654)”。至于相关法律条文的量化规范，例见于青川秦墓木牍《为田律》：“田广一步，袤八则，为畛，亩二畛，一百（陌）道。百亩为顷，一千（阡）道，道广三步。封高四尺，大称其高。捋（埒）高尺，下厚二尺。以秋八月修封捋（埒），正 彊（疆）畔，及登千（阡）百（陌）之大草。”[9]睡虎地秦墓竹简《田律》，如：“入顷刍稾，以其受田之数，无豤（垦）不豤（垦），顷入刍三石、稾二石。刍自黄𪎊及䕭束以上皆受之。入刍稾，相（8）输度，可殹（也）。田律（9）”[10]

对仓储物资的管理。秦对仓储物资管理十分严谨细致，因为责任重大，要收入农户缴纳的田租，要支付廪给。睡虎地秦简有《仓律》的各种规定，北大秦简《陈起》篇说：“和均五官，米粟（髹）桼（漆）、升料〈料〉斗甬（桶），非数无以命之”，岳麓秦简《数》里除了有算题是关于仓储事务的，还记录着各种谷物兑换的比率，其数据与后来的《九章算术》所载大多相同。而我想在本文中再提一下岳麓秦简《数》整理出版时几枚释义存疑的简：“券朱（铢）升，券两斗，券斤石，券钧般（卷），券十朱（铢）者（0836）”“百也，券千万者，百中千，券万（万）者，重百中。(0988)”“籥反十，券菽荅麦十斗者反十。(0975)”[11]推测这几枚简所记与仓储物资出入有关，似是说的简侧刻齿。在里耶秦简里见到有“刻齿简”，即在券写钱粮物资数额的简侧有一排刻槽，如锯齿，不同形状的刻槽对应不同的单位量，例如，9 万就刻几个表示“万”的凹槽，7 千就刻 7 个表示“千”的凹槽，那么 0836 简的含义就是：铢和升符号相同，两和斗符号相同，斤石、钧般（卷），亦如是。0988 简说的是：“千万”是在“百”的符号里加刻“千”的符号。0975 简“反十”的意思是正方向刻齿符号“十”（据里耶秦简，是凹刻一边长一边短的三角形）的反向符号。[12]券书数额的简侧刻齿是为了防止仓储记录中篡改数额以及收支方对契，可见管理的严谨。

对劳役和工程的管理。秦官府对手工业和土木工程的管理，律有明文，见于睡虎地秦简《工律》、《工人程》、《均工》，执行中需运用数学。北大秦简《陈起》篇说到：“具为甲兵筋革，折筋、靡（磨）矢、䦆（栝）㚏，非数无以成之。段（锻）铁鐶（铸）金，和赤白，为楘（柔）刚，磬钟竽瑟，六律五音，非数无以和之。锦绣文章，卒（萃）为七等，蓝茎叶英，别为五采（彩），非数无以别之。外之城攻（工），斩离（篱）凿豪（壕），材之方员（圆）细大、溥（薄）厚曼夹（狭），色（绝）契羡杼，斵凿梋（斧）锯、水绳规梠（矩）之所折断，非数无以折之。高阁台谢（榭），戈（弋）邋（猎）置堃（放）御（御），度池旱（岸）曲，非数无以置之。和攻（功）度事，见（视）土刚楘（柔），黑白赤黄，蓁厉（莱）津如（洳），立（粒）石之地，各有所宜，非数无以智（知）之。”[13]手工业生产各种用品，土木工程修

[9] 四川省博物馆等：《青川县出土秦更修田律木牍》，载《文物》1982 年第 1 期，第 1-13 页。
[10] 睡虎地秦墓竹简整理小组编：《睡虎地秦墓竹简》，文物出版社 1990 年版，(注释)第 21 页。
[11] 朱汉民，陈松长：《岳麓书院藏秦简(贰)》，上海辞书出版社 2011 年版，第 17、93 页。
[12] 秦简《数》这几枚简的释义与里耶秦简刻齿简的关系详见：肖灿《对里耶秦简刻齿简调研简报的理解和补充》，简帛网，武汉大学简帛研究中心，2012-10-13，http://www.bsm.org.cn/show_article.php?id=1743
[13] 韩巍：《北大藏秦简<鲁久次问数于陈起>初读》。

建城墙城楼、城外园林、农田规划都离不开数学，例如冶炼锻造需要靠计算控制铁、铜的硬度、成色、用炭量、损耗等；织染需要计算染料的调配比率；工程中，木材加工、开挖土方、筑修城墙都需测量和计算；城外园林建设及农田土质检测也都用到数学知识。[14]岳麓秦简《数》里，手工业方面有关于纺织、锻铁、制玉等算题，土木工程方面有徭役征发和计算仓、城、堤、亭、积堆、除（羡除，可看做是楔形体体积计算）的土方量的算题。

《陈起》篇说："天下之物，无不用数者"，除了上面所述几大类事务，还有其他种种，例如《数》里的"营军之术"算题记述了军营的建制。[15]秦人重视数学，普及实用算法，广泛运用这种"实用算法式数学"管理田地租税、仓储物资、劳役工程、军战事务等，取得了高效。高效率的管理，可能是秦人能够统一中原的一个重要因素。

14 肖灿：《读<陈起>篇札记》，《自然科学史研究》2015 年第 2 期(第 34 卷)，第 257-258 页。

15 对"营军之术"的解析，参见孙思旺：《岳麓书院藏秦简"营军之术"史证图解》，载《军事历史》 2012 年第 3 期，第 62-68 页。

ZOU Dahai 邹大海 A Study on a Type of Wedge-shaped Solid in Early Chinese Mathematical Documents

A Study on a Type of Wedge-shaped Solid in Early Chinese Mathematical Documents
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Abstract：This paper compares methods for calculating the volume of a particular type of wedge-shaped solid described in the Qin dynasty bamboo-strip manuscript *Shu* (数，Numbers), the Western Han dynasty bamboo-strip manuscript *Suanshu Shu* (筭數書，Book on Calculations), and the *Jiuzhang Suanshu* (九章筭術, *Nine Chapters on Mathematical Procedures*), and identifies the relation between them. The text of *Shu* provides new evidence to support the opinion that the four basic solids, cuboid, *qiandu* (塹堵, right triangular prism), *yangma* (陽馬, right rectangular-based pyramid) and *bienao* (鱉腝, tetrahedron with four right triangle faces) were used to determine solid volumes in the Qin and Pre-Qin periods. On the basis of the above opinion, this paper sets out a design that supplements the text of the method for finding the volume of a wedge in the *Shu*. It reasons that the method for calculating the volume of the solid described on bamboo strip no.0456 was found by mathematical derivation, and attempt to reconstruct how the ancients obtained this method. This paper argues that the mathematical methods recorded in the three sources have common characteristics and thus share the same origin. The common origin might have been transmitted and incorporated into an ancestor of the *Jiuzhang Suanshu* that predates the other two documents. This paper also proposes a new understanding of how the methods for finding solid volumes were worked out and then transmitted in early China.

Keywords: *Shu* 数 on Qin dynasty bamboo slips, *Suanshu Shu* 筭數書 on Han dynasty bamboo slips, *Jiuzhang Suanshu* 九章筭術, Liu Hui 劉徽, wedge solids, evolution of mathematical methods

中国早期数学文献中一类楔形体的研究

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摘要：本文对出土的秦代竹简算书《数》、西汉初年竹简算书《筭数书》和传世数学经典《九章筭术》中一类在古代较难处理的楔形体进行比较研究，挖掘它们之间的联系。利用《数》中的材料为秦及先秦处理体积问题时已采用基本立体——长方体、堑堵、阳马和鳖腝——提供了新的证据，在此基础上为《数》0456 简所述楔形体题目的术文提出了校补方案，推论这种立体的求积方法产生于推导，并提出了复原方案。本文论述三项文献中楔形体的算法具有共同的特点和渊源，而这种渊源流传到《九章》的先秦祖本比另两项文献很可能要早，同时也就上古时代体积算法的产生与流传问题提出了新的认识。

关键词：秦简《数》、汉简《筭数书》、《九章筭术》、刘徽、楔形体、算法演化

TIAN Miao 田淼 Study on the solution of indeterminate equations of first degree in traditional mathematics in China

Study on the solution of indeterminate equations of first degree in traditional mathematics in China

中国传统数学中的一次不定方程问题研究

HAN JIE, TIAN MIAO
(Institute for the History of Natural Sciences, Chinese Academy of Sciences)

Abstract: Solution of indeterminate problems, relevant to the indeterminate equations of the first degree in modern mathematics, is an important subject of mathematics in pre-modern China. The problem of“hundred fowls” in the *Zhang qiujian suanjing* (*Zhang qiujian’s mathematical canon*)is the common origin of indeterminate problems in China. This subject developed rapidly after the 13^{th} century. In this article, we point out that it is in the Ming Dynasty that Chinese mathematicians began to solve the problem of hundred fowls in the procedure of Fangcheng. Based on a detailed analysis and interpretation of the problems concerning the indeterminate equations contained in sources materials and the methods used in solving these problems, this paper tries to make clear the process of the development of the method for solving the indeterminate equations of the first degree in traditional mathematics in China.

Key Words: the method of hundred fowls, indeterminate equations of the first degree, the Procedure of Fangcheng

DENG Kehui 邓可卉 Discuss again Jiudaoshu

Discuss again Jiudaoshu

再议九道术

DENG Kehui

（College of Humanities and Social Sciences, Donghua University, Shanghai, 201620)

Abstract: The paper analyses the meaning of Jiudaoshu and thinks the ancient's understanding to draconic month was unclear. On the contrary, the Greeks had distinctive knowledge to anomalistic month and draconic month. The paper put forward a new explanation to Jiudaoshu based on *Discuss on Dayan calendar* （《大衍历议》）and Zhu Wenxin's viewpoint.

Key words: Jiudaoshu, Liu Hong, anomalistic revolution in Greece, *Discuss on Dayan calendar* 《大衍历议》，Zhu Wenxin

SIU Man Keung 蕭文強 Figures and history on either side, rendered dynamically

Figures and history on either side, rendered dynamically —— GeoGebra to go hand in hand with ancient or medieval Chinese mathematical texts in the mathematics classroom

OR Chi Ming, GeoGebra Institute of Hong Kong
SIU Man Keung, Department of Mathematics, University of Hong Kong

左圖右史，別有「動」天——在數學課堂上結合 GeoGebra 與中國古代數學史素材

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Abstract：Four decades ago the topic of HPM (History and Pedagogy of Mathematics) would be a relatively new venture. With the hard work of many researchers and teachers in the intervening years this is no longer the case. For many years now various authors in different parts of the world have written on the important role played by the history of mathematics in mathematics education. The 10th ICMI Study focuses on the role of history of mathematics in the teaching and learning of mathematics, with its work reported in the study volume History in Mathematics Education: The ICMI Study published in 2000.

Now that enough has been said on a "propagandistic" level, rhetoric has played its part so that discussion should better be channeled to actual implementation. With the dynamic geometric software GeoGebra becoming more and more commonly employed in the school classroom it seems feasible to think about how this can be utilized for the benefit of teaching and learning mathematics at the secondary and even primary school level by combining it with suitably selected historical material from mathematical texts in the past. To fit with the theme of this Workshop on Mathematical Texts in East Asia Mathematical History the presentation will confine attention to texts in ancient or medieval Chinese mathematics.

It is to be recognized that the contextual issue of mathematical texts in the past is an intricate one, involving the motive for which the text was originally written, the readership the text was intended and the use of the text at different periods in history. Such issues have already drawn the attention of many historians of science and mathematics. In this presentation we focus instead on the pedagogical aspect and take the view of a teacher in the mathematics classroom.

Li Zhaohua 李兆华 Collating and Notes on Liu Yueyun's Ceyuan Haijing Tongshi

Collating and Notes on Liu Yueyun's Ceyuan Haijing Tongshi
《測圓海鏡通釋》補證

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Abstract: *Ceyuan Haijing*（《測圓海鏡》，1248）research is one of important branches of mathematics in China during the late of Qing dynasty. In the period, over ten books are published by scholars engaged in the branch, one of them is Liu Yueyun's *Ceyuan Haijing Tongshi*（劉嶽雲《測圓海鏡通釋》, 1896）. Because of loss of his manuscript, just an incomplete copy was printed and passed to us. It is difficult to understand precisely some of conclusions contained in the copy. This paper corrects some of error words and affords proof to main conclusions. Meanwhile the paper points out significance of the treatise on the investigation on *Ceyuan Haijing*.

Keywords：*Ceyuan Haijing Tongshi* (an explanation of *Ceyuan Haijing*), *Gougu Ceyuan Shu*　(method for finding diameter of a circle in contact with nine right triangle), collate, proof

JI Zhigang 纪志刚 The Translation of *Tongwen suanzhi* and its Mathematics Knowledge Sources

The Translation of *Tongwen suanzhi* and its Mathematics Knowledge Sources

JI Zhigang

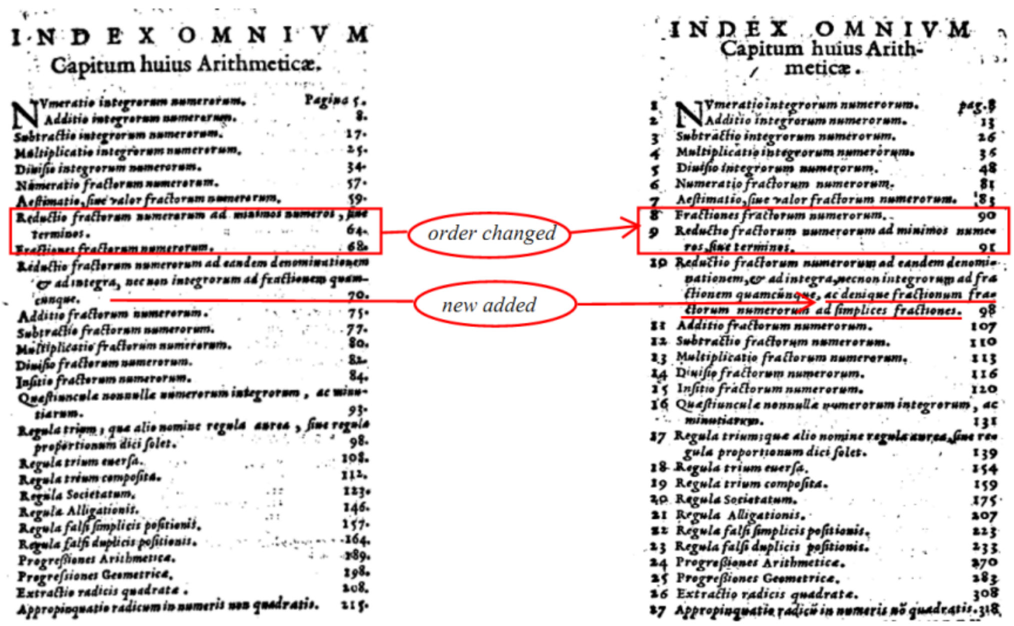
(School of History and Culture of Science, Shanghai Jiao Tong University)

Abstract：It is well know that the *Tongwen suanzhi* (同文算指,1614) was translated from Clavius’ *Epitome Arithmaticae Practicae* (EAP,1583). EAP was republished 7 times from 1584 till 1614. Among them, the 1585 edition and 1607 edition were revised versions. Which one was the original edition that the *Tongwen suanzhi* translated from? If you judge it only by the content pages, the 1607 edition seems to be the original source. But it is wrong.

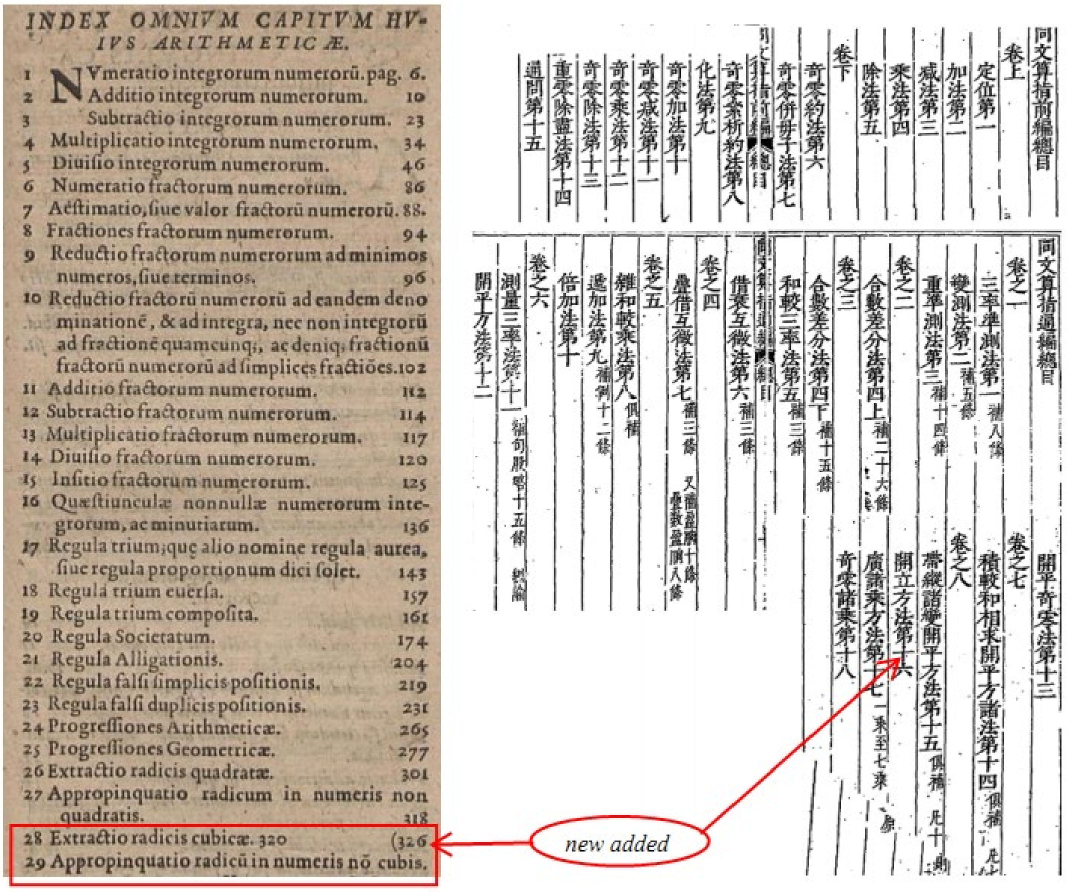
As a Chinese translation work, Li Zhizao made his great effort to translate the Latin mathematics work into classical Chinese. There are many new terms which Li made. For example: 紐數、三率法、變測法、借衰互徵法、疊借互徵法、雜和較乘法， ect. But some criticisms said that Li counted the classical but forgot his own ancestors （數典忘祖, Qian Baocong: *A Folk History of Chinese Mathematics*). In fact, we will show that Li did not ignore the Chinese classical mathematics, and these new terms might be better translations.

In his preface to *Tongwen suanzhi*, Li Zhizao said ‘some problems were adopted from the *Nine Chapters* to supplement’(間取九章補綴). After a considerable reading through the whole books, we can give a list of Chinese mathematics works which might have been referred to by Li Zhizao, including Cheng Dawei’s *Unified lineage of Mathematical Methods* (算法統宗), Zhou Shuxue’s *The Divine Complete Writings on Astronomy and Mathematics*（神道大編歷宗算會）, Gu Yingxiang’s *Classified Methods of the Sea Mirror of Circle Measurement* （測圓海鏡分類釋術）and *Mathematical Procedures on Base and Altitude*（句股算術）, Xu Guangqi’s *Meaning of Measurement Methods* （測量法義）and *Gou-Gu Yi*（勾股義）. Surprisingly some existing problems were brought from Michael Stifel’s *Arithmetica Integer* (1544). This paper will point that Clavius’ *Geometria Practica* (1606) is also a source.

Appendix I: Contents pages of EAP 1583, 1585 1607 and *Tongwen suanzhi*



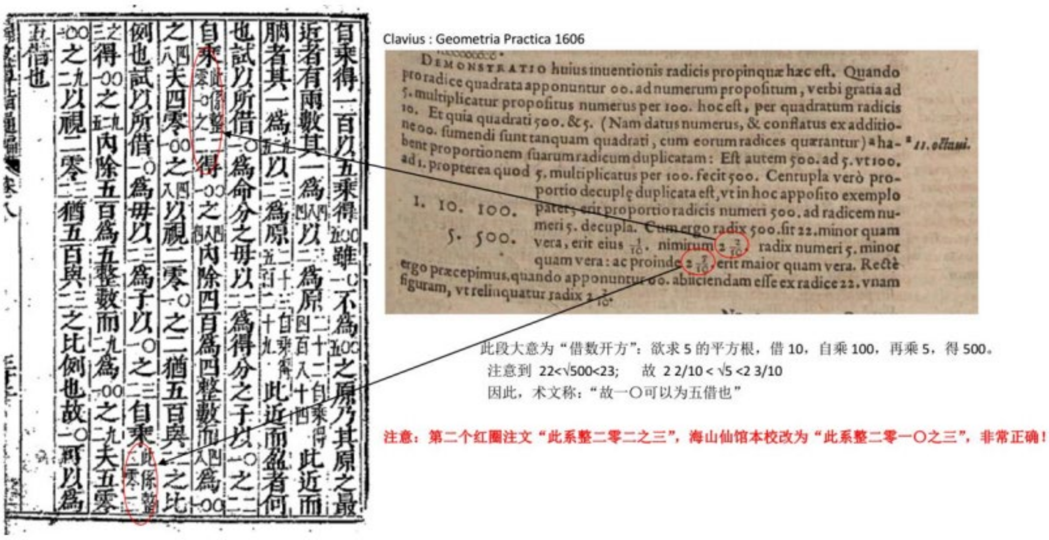
EAP 1583 Edition　　*EAP* 1585 Edition



EAP 1607 Edition

Tongwen suanzhi

Appendix II: the same problem in *Geometria Practica* and *Tongwen suanzhi*



Cheng Chun Chor Litwin 郑振初 Equations of Inscribed Circle in *Ceyuan Haijing*

Equations of Inscribed Circle in *Ceyuan Haijing*

CHENG Chun Chor, Litwin

Abstract: The book "*Sea mirror of Circle measurements*" (测圓海鏡 *Ceyuan Haijing*[16]) investigate cases of circles inscribed in a right angle triangle. To obtain the equation of the radius of a circle inscribed in the right angle triangle with two pieces of information (two events) is an academic activity in the mathematics circle in the Ching Dynasty. This paper discusses the development of the mathematics content written by the scholar Wang Ji Tong (王季同 1874-1947) in the Ching dynasty on the content of "*Sea Mirror*" in his work "*Formula in Nine Chapter*" (*Jiurong Gong Shi* 九容公式). Wang investigated the problems of inscribed circles by algebraic knowledge and construct equation leading to solve the diameter of the circle.

[16] We will address the book as "Sea Mirror" in this paper.

DONG Jie 董杰 A Study of Xue Fengzuo's Method for Building the Trigonometric Tables

A Study of Xue Fengzuo's Method for Building the Trigonometric Tables

DONG Jie

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Inner Mongolia Normal University, Huhhot 010022,China）

Abstract: In the late Ming Dynasty, ***Dace*** (大测，1631，Great Measurement) introduces the West method for building the trigonometric table into China. It is very difficult for the Chinese mathematicians to master the method because of some formulae without proofs and the method introduced too sweeping. ***Xue Fengzuo***（薛凤祚，1599-1680）took the Pythagorean Theorem as the core. According to graphic relations of a right-angle consisted of sine line, cosine line, and the radius, he structured a new method which had highly programmed characteristics to build the trigonometric tables. Compared to ***Dace***，Xue Fengzuo's trigonometric tables are more precise. His methods are more simple, intuitive and effective. His treatise does not follow the Dace. The method of Xue is the achievement which under the guidance of the supplement traditional Chinese scientific system with Western mathematics knowledge.

Keywords: the method for building the trigonometric tables, Xue Fengzuo，Dace, Chinese mathematical traditions , integration between Chinese and western science

Gao Hongcheng 高红成 A Textual Research on the Original English Text of Yuanzhui Quxianshuo Translated by Li Shanlan and Joseph Edkins

A Textual Research on the Original English Text of Yuanzhui Quxianshuo Translated by Li Shanlan and Joseph Edkins

李善蘭、艾約瑟合譯《圓錐曲線說》英文底本考

Gao Hongcheng

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Abstract: *Yuanzhui Quxianshuo*（《圓錐曲線說》），3 volumes， cooperatively translated by Li Shanlan（李善蘭)and Joseph Edkins，the earliest version which today can see，is attached to *Zhongxue*(《重學》）published in 1866 at Jinling Publishing Bureau（金陵書局). The content consists of pure mathematics, and the academia generally considers that it was the explanation to the mathematical content in *Zhongxue*. In recent ten years, studies on version, content, impact and the original English version of *Zhongxue* have made great progress. But the original English version of *Yuanzhui Quxianshuo* is uncertain still. In the Late Qing，it was an important text by which people studied the knowledge of conic sections. It is not easy to determine the original English text of Chinese Version of western science works in the Late Qing. There is not the author's information in the book , *Yuanzhui Quxianshuo*，only with the names of translators，and it is difficult to ascertain its original English text. But it is the mathematical works and there is a certain logic and some unique characteristics，such as proof method，the unique data， which are the key evidences to compare and confirm texts. With the help of the Internet and academic friends，the paper finds an English version almost one-to-one correspondence to the Chinese. This paper comes to conclusion that the *Yuanzhui Quxianshuo* was translated from *Conic Sections*, in *A Course of Mathematics* by Charles Hutton（1737-1823），an English mathematician.

Keywords：*Yuanzhui Quxianshuo* the original English text *Conic Sections*， *A Course of Mathematics* Charles Hutton

DENG Liang 邓亮 A Primary Study on Lin Chuanjia's *Weiji Jizheng*

A Primary Study on Lin Chuanjia's *Weiji Jizheng*

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Abstract: Lin Chuanjia is a famous scholar, literary historian, and educationalist during late Qing dynasty and early period of the Republic of China. This article outline his life and family, research out some early life details; analyses the content of his mathematics work named *Weiji Jizheng*, especially for chapter I kaogu (textual research), which mainly explains the thought of western learning originated from China on calculus; points out the source of his mathematical thought, and compares the similar views of other scholars of the same period.

Keywords: Lin Chuanjia, *Weiji Jizheng*, calculous, western learning originated from China

林传甲《微积集证》初探

邓亮
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摘要：林传甲是清末民初著名学者、文学史家、教育家。文章概述了其生平家世，补充考证出若干早期生平细节；分析了其数学著作《微积集证》的结构与内容，尤其是卷一考古所体现的微积分西学中源说；指出其数学思想的来源，并与同期其他学者的相似言论进行比较。

关键词：林传甲，《微积集证》，微积分，西学中源说

SASAKI Chikara 佐佐木力 Tohoku-Göttingen（東北月沈原）: An Academic Home of Modern Chinese Mathematics

Tohoku-Göttingen（東北月沈原）: An Academic Home of Modern Chinese Mathematics

SASAKI Chikara 佐佐木力
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Abstract: "Tohoku University 東北大学 in Sendai is the third national university founded legally in 1907, and opened actually in 1911, after the University of Tokyo 東京大学 established in 1877, and Kyoto University 京都大学 in 1897. Tohoku University had a remarkable academic policy which admitted foreign and female students, and Japanese students who had studied in high schools a discipline different from a discipline for application, thanks to its first President Sawayanagi Masataro 澤柳政太郎 (1865-1927). Thanks to this liberal policy of admission, some talented Chinese students entered the University, for example, Chen Jiangong 陳建功 (1893-1971) and Su Buqing 蘇步青 (1902-2003).

Tohoku University emphasized original research rather than 'Brotstudium', following Göttingen University in Germany. The mathematics department at Tohuku University is called "the Mathematical Institute," following Göttingen's "Das Mathematisches Institut." Actually, Fujiwara Matsusaburo 藤原松三郎 (1881-1946), one of the two founding professors of the Mathematical Institute, and its younger associate professor Kubota Tadahiko 窪田忠彦 (1885-1951) studied at Göttingen. They were academic advisers of Chen Jiangong in Fourier series and Su Buqing in differential geometry, respectively. Final evaluation reports of D. Sc. Theses of both Chen and Su were recently discovered and will be introduced in this lecture."

Seisho Yoshiyama 吉山青翔 A Study of Zhang Shenfu's Letters to Yoshio Mikami:A Scene in the Exchange History for Study of the East Asia Mathematical History between Japan and China

A Study of Zhang Shenfu's Letters to Yoshio Mikami:A Scene in the Exchange History for Study of the East Asia Mathematical History between Japan and China

Seisho Yoshiyama(aka: Wang Qingxiang)
(Yokkaichi University, Japan)

张申府致三上义夫书简研究：日中东亚数学史研究交流史的一个断面

吉山青翔(=王青翔)
（四日市大学环境情报学部）

Abstract: Prof. Zhang Shenfu (1883~1986) was one of three founders of Chinese Communist Party, an assistant professor of mathematics and logic, library chief at Peking University, and a professor of philosophy at Tsinghua University in China. He had been exchanging letters with Dr. Yoshio Mikami on the study of East Asia Mathematical History and so on for eight years.

Dr. Yoshio Mikami (1875~1950) was a lecture at Tokyo School of Physics (The forerunner of Tokyo University of Science), an investigating researcher at the Imperial Academy of Japan. He published a lot of papers and books on the east Asia mathematical history，He was also the first one that introduced the Japanese and Chinese mathematical history to the world in English language.

The exchanging of letters between Prof. Zhang and Dr. Mikami started in the year 1918, discontinued in the year 1925 .

In the year 1918，Prof. Zhang mailed Dr. Mikami 20 yuan of Chinese money as the prices of books by asking of Prof. Li Yan. This is the starting of the exchanging of letters. In the year, Prof. Zhang worked as an assistant professor and the library chief where Mao Zedong (=the first president of China People's Republic) worked as an editor of catalog of books at the Peking University,

The exchanging of letters discontinued in the year 1925. In this year, Prof. Zhang replied Dr. Mikami's letter of Sept. seventh 1925, this was the last letter.

The exchanging of letters was chiefly exchanging of the original texts and views on the east Asia mathematical history.

ZHANG Hong 张红 Mathematics exchanges between China and Japan and mathematics education modernization process in Sichuan

Mathematics exchanges between China and Japan and mathematics education modernization process in Sichuan

ZHANG Hong
(Sichuan Normal University)

Abstract: Japanese mathematics and Chinese mathematics have close relationship since the 6th century. China set up Chinese mathematics educationin in Traditional Chinese（国学），began in the Sui dynasty (A.D. 581-618) , and continued to the Tang dynasty and the northern Song dynasty . In the 7th century, at the beginning of the Japanese mathematics system establishment is mainly influenced by the Chinese mathematics education. In the end of the 19th century and early 20th century, a large number of Chinese students studying in Japan came back to China, and some Japanese teachers came to China, which made Chinese universities and middle schools as reference to Japan. In this report, I will discuss the establishment process of the modern mathematics education system in Sichuan under the background of the communication between China and Japan. In the early stage of the process, we learned product of Japanese mathematics education system, and at this late stage, we studied European and American and perfected our mathematics education system gradually.

中日数学交流与数学教育在四川的现代化过程

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摘要：日本数学与中国数学自公元 6 世纪以来就有密切的关系。中国在国学中设立专门的数学教育始于隋代（公元 581-618），并延续至唐代和北宋。公元 7 世纪，日本算学制度建立之初，主要是受了中国数学教育的影响。19 世纪末和 20 世纪初，大批的留日学生、日本教习使得中国的大学和中等学校不可避免地以日本为参照。本报告讨论在中日交流背景下，近代数学教育制度在四川的建立过程。这一过程的初期，是学习日本数学教育制度的产物，并在后来学习欧美的过程中得以完善。

关键词：数学交流 数学教育 四川 留日学生 日本教习 欧美留学生

Guo Shirong 郭世荣 Japanese Scholar Riken Fukuda's Understanding and Adaption of the Structure of the Astronomical Treatise *Tantian* (談天)

Japanese Scholar Riken Fukuda's Understanding and Adaption of the Structure of the Astronomical Treatise *Tantian* (談天)
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Abstract: The astronomical treatise *Tantian* was published in China in1859. It is a Chinese translation of British astronomer John F.W. Herschel's (1792~1871) *Outlines of Astronomy* (edition IV, 1851), translated by Alexander Wylie (1815~1887) and Li Shanlan (1811~1882). The Chinese version was introduced into Japan shortly after its publication. Japanese scholar Riken Fukuda (1815~ 1889) published his new version of the work, not only adding his Japanese phonetic notations to text but also changing its structure of the work.

Comparing with the original Chinese text, there are two kinds of changes in Fukuda's adaption text. Firstly, the counting way of the years and the metric units were changed according to Japanese manner, which was intelligible and trivial. Secondly, the order of the chapters was changed, although the Chinese text itself keeping adoption without one word changed. It is worth to be paid attention.

The present paper deals with three problems. First, the change of the order of chapters reflects Fukuda's idea of compilation of astronomical texts and his understanding of the logical structure to the original work. Second, is the adaption reasonable? Third, what is its influence for the reader's understanding of the contents and structure of the work?

Keywords: *Tantian*, *Outlines of Astronomy*, Riken Fukuda, adaption, Herschel

日本学者福田泉对《谈天》结构的理解与改编
郭世荣
(内蒙古师范大学科学技术史研究院)

摘要：1859 年，由英国教士伟烈亚力（Alexander Wylie，1815~1887）和中国学者李善兰(1811~1882)根据英国天文学家侯失勒 (John F.W. Herschel, 1792~1871)的《天文学纲要》(*Outlines of Astronomy*)第 4 版汉译的天文学著作《谈天》出版，该书很快便传到日本。日本学者福田泉(1815~1889)于 1861 年出版日文训正本《谈天》，不仅加日文了日文训点，而且对全书顺序做了改编。

福田泉对《谈天》做了两方面的改编，包括为了适应日本实际情况的简单改编，如纪年和单位转换等，这些都不是本质的。值得特别注意的是他对《谈天》结构也做了改编，在对汉译文"不妄改一字"的前提下，对全书各章的顺序做了全面调整。

本文主要探讨三个问题：第一，福田泉对《谈天》编排顺序的调整代表了福田氏对天文学著作编写的一种思路，反应了他是如何理解《谈天》的内容和逻辑结构的。第二，这种调整是不是合理。第三，改编对于读者理解原著造成什么样的影响。

关键词：谈天 天文学纲要 福田泉 改编 侯失勒

XU Zelin 徐泽林 The view on mathematics in Chinese-character cultural circle from *Sūdo Shōdan*

The view on mathematics in Chinese-character cultural circle from *Sūdo Shōdan*

XU Zelin
(Donghua University, Shanghai, 201620)

Abstract: Nishimura Tōsato(西村遠里,1718～1787) was a famous astronomer in Edo period, one of his conversations with his friends--Mizu Yukinaga(水之長)、Sugi Tomokazu(杉知一)--on mathematical problems was recorded in his book *Sūdo Shōdan*(《数度霄谈》), also called *Sūgaku Yawa*(《数学夜话》),it's ranged over many topics such as the origin and function of Mathematics, relationship between mathematics and calculation, mathematics and Measurements, mathematics and astronomy, mathematics and Yi-ology(divination), the superiority and inferiority of the mathematics in China and Japan, etc. All the viewpoints in this book are dialectical discoursesin the Confucian classical context, it is a reflection of traditional view on mathematics in Chinese-character cultural circle, and its ideological origins and reference source can be find in Chinese classical literatureand mathematical documents.Aida Yasuaki(会田安明,1747~1817), a wasan mathematician,gave his comments on those discourses from the stand point of mathematician in his book *Sūgaku Yawa Hyōrin*(《数学夜話評林》), although he pointed out the suspicion and irrationality of some of viewpoints, he didn't go beyond the barriers of the view on mathematics in classical literature. These above-mentioned two books demonstrate that the eastern Asian society have arrived at the common understanding of the nature of mathematics, which was caused by theformation and influence of the Han culture.

Key words: Nishimura Tōsato; *Sūdo Shōdan;* Aida Yasuaki; *Sūgaku Yawa Hyōrin*; view on mathematics; Chinese-character cultural circle

从《数度霄谈》看汉字文化圈的数学论

徐泽林
（东华大学，上海，201620）

摘要：江户时代著名的天文学家西村远里（1718～1787）的《数度霄谈》（1778），又名《数学夜话》，记录了他与两位朋友水之長、杉知一关于数学问题的一次谈话，其话题涉及到数学的起源、数学的功用、数学与算学的关系、数学与度量衡、数学与天文历法、数学与易学（术数）、中日数学之胜劣等内容，书中观点都是在中国儒家经典语境下的思辨性论述，它们在中国古典文献和数学文献中都能找到思想渊源和文献出处，反映的是汉字文化圈传统数学观。和算家会田安明（1747～1817）著《数学夜話評林》，从数学家的立场对这些论述进行了评论，虽理性地指出了其中某些观点的可疑性和不合理性，但整体上仍然与西村遠里一样，没有超越古典文献关于数学论的传统藩篱。上述二书内容表明，东亚传统社会对数学本质认识具有共同的文化认同，这是由汉文化的形成和影响所造成的。

关键词：西村遠里 《数度霄谈》 会田安明 《数学夜話評林》 数学论 汉字文化圈

Feng Lisheng 冯立昇 Great Tradition and Little Tradition：A Framework for Studying History of Mathematics in China

Great Tradition and Little Tradition：A Framework for Studying History of Mathematics in China

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Abstract: The concepts of Cultural Anthropology, "great tradition" （大传统） and "little tradition"（小传统), have been prevalent in the study of the relation between high culture（or elite culture）and low culture（or popular culture）. As a theoretical framework, we shall inquire how useful the concepts are for studying Chinese traditional mathematics with a new approach, and understanding the Historical development and cultural features of Chinese mathematics. This article analyses the phenomenon of "great tradition" and "little tradition" in the history of Chinese mathematics, and discusses the interaction of these two kinds of mathematical traditions in different periods and its influence on the development of mathematics. The author considers that the little tradition of Chinese mathematics based on the practical arithmetic and computational skills has been formed in the Pre-Qin period, and influence to modern China; however the great tradition of Chinese mathematics established on the structuration and systematization has been formed later than the little tradition of Chinese mathematics and easy to lose. *Nine chapters on the Art of Mathematics*（九章算术)and its companion and commentary play an important role on the formation and transmission of traditional mathematics. The positive interaction between these two kinds of traditions promoted the comprehensive prosperity of Chinese mathematics during the Song and Yuan dynasties, but the little tradition of Chinese mathematics, especially for the business mathematics and abacus calculation（珠算）, have been further prosperity, and played an important role in the society of the Ming Dynasty, just because the great tradition of Chinese mathematics, such as the Tianyuan method （天元术）and rod arithmetic（筹算）, have been lost in the that time. Owing to the loss of the great tradition of Chinese mathematics and the input of western advanced mathematics, mathematicians in the Qing Dynasty effort to rebuild the great tradition of Chinese mathematics through the collation and intensive study on the ancient mathematics books, and have great influence on the communication and fusion between Chinese and Western mathematics.

大传统与小传统：中国数学史研究的一个新视角

冯立昇
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摘要：大传统(great tradition)和小传统(little tradition)是文化人类学中流行的概念与方法，用于高、低层文化（或精英与大众文化）关系的研究。这对概念及其理论也为我们考察中国传统数学提供了一个新的视角，对于中国数学史研究具有重要的启发意义，有助于我们重新认识和理解中国数学的历史发展及其文化特点。本文考察了中国数学史上的"大传统"与"小传统"现象，并对不同时期大、小数学传统的互动关系及其对数学发展的影响进行了探讨。

作者认为，以实用算术和计算技能为基础的数学小传统，其历史源远流长，在先秦时期已经形成并且影响至现代；而建立在知识的条理化和系统化基础上的数学大传统，其形成时间则要晚于数学小传统且容易流失，《九章算术》及其注释对数学大传统的形成及其知识传承发挥了重要作用。宋元时间两种数学传统的良性互动对数学的全面兴盛起到了重要的促进作用，而属于“大传统”之数学知识的天元术和筹算在明代的失传促成了“小传统”的商业数学和珠算的进一步繁荣，“小传统”数学在明代社会扮演了重要角色。面对数学大传统的流失和西方先进数学的输入，清代数学家通过对古代数学著作的整理与深入研究，进行了再建数学大传统的努力，并对中西数学的会通与融合产生了重要影响。

GUO Shuchun 郭书春 关于《中华大典·数学典》

关于《中华大典 · 数学典》

郭书春
（中国科学院自然科学史研究所研究员，《数学典》主编）

《中华大典》系中国国务院批准的重大文化出版工程，被列为国家文化发展纲要的重点出版工程项目，前新闻出版署将其列为“十一五”国家重大出版工程规划之首。《中华大典 •数学典》是其二十几个典之一，于 2006 年春启动。经过 30 位同仁近十年不懈努力，现已编纂完成，将于 2016 年上半年由山东教育出版社出版。今借此从事东亚数学典籍研究的各位同仁聚会之际，简要介绍《数学典》的情况，请批评指正。

一.《数学典》的编纂

2006 年春，由吴文俊、任继愈、席泽宗三位大师推荐，我勉为其难，主持《数学典》的编纂，随即向全国数学史同仁发出约请通知，获得热烈回应，组织了编委会。同时，我们根据《中华大典》的有关规定，结合《数学典》的具体情况，起草了“《中華大典•數學典》編纂方案及校點條例”，并在 2006 年 12 月 26 日召开了第一次编委会会议，任继愈、吴文俊、席泽宗、于泳湛、伍杰等先生和山东教育出版社、山东出版集团的负责人参加了会议。由此正式开始了《数学典》的编纂工作。

《数学典》的编纂有许多有利条件。

一是《数学典》的编委会相当强，记得在编委会第一次会议上我说过：“环视左右，除了鄙人之外，全都是博士、硕士。”当然，10 年来编委会也在不断调整，由于各种原因，有的先生退出了，也有不少先生主动要求参加。目前完成《数学典》编纂的 30 位先生，大都是中国数学史领域的学术带头人和科研骨干。他们功底深厚，学风严谨，工作认真，一以贯之。与有的典“铁打的军营流水的兵”，一个总部换几拨人来编不同，《数学典》各总部都是一位（个别的有 2 位）先生从头编到底，责任心强。完成编纂的各位先生都是在编纂经费相当低的时候参加的，可谓是不计名利，为了数学史事业的发展无私贡献。

二是山东教育出版社对这项工作非常重视。陆炎总编辑日理万机，时时关心《数学典》的工作。责任编辑韩义华先生年逾花甲，编辑经验丰富，工作非常认真负责，常常发现总部主编和分典主编没有发现的问题，提出真知灼见的编纂意见。山东教育出版社为《数学典》争取到相当丰厚的经费，是各典中最高的，并且破例在各总部标题下印出总部主编，而不像其他典那样只在分典说明中提及。

三是《中华大典》主编任继愈、副主编吴文俊、席泽宗和工作委员会、办公室的关心支持，后来是国家出版基金委的资金保障。

四是中国科学院基础局和自然科学史研究所、内蒙古师范大学、清华大学等有关单位的领导和图书馆的强有力的领导及经费、人力、物力、图书的支持。

当然，我们的困难也很多。首先是《数学典》编纂人员都十分忙。在编纂《中国科学技术史 •数学卷》时，我说过：“从事任何一项课题，你请来的都是忙人，不忙的你也不敢请。”本典的编纂人员有司局级、师级领导干部和高等学校的学院院长、研究所所长，许多先生承担国家和省部级的研究课题，担负相当繁重的行政、教学和科研任务，不可能全力投入《数学典》的编纂。

其次，编纂工作对大多数编纂人员实际上是边干边学的过程。因为我们大都是理工科出身，古籍校勘、标点和版本等知识先天不足，需要在编纂中不断学习这些知识。

第三，中国数学史尽管是研究基础相当好的学科，但是我们工作的主要对象数学典籍，95%以上是明末至清末的，除了二三十部重要典籍外，大多数研究不够甚至从未研究过。必须对这些著作的数学内容、成就有一个基本的了解，才能开始编纂。因此，编纂工作实际上是边研究边编纂的过程。这大大增加了编纂的难度，当然影响进度。

在《数学典》立项之前，其主要编纂人员于 2004 年承担了中国科学院重大科研项目《中国科学技术史•数学卷》的撰著任务，《数学卷》申请到的科技部学术专著出版基金要在 2009 年结题，我们不得不在 2009 年将主要精力转入《数学卷》的撰著。

我们的编纂工作也走过弯路，主要是开始工作后不久，为了加快进度，在大典办公室的动员下试图进行数字化编纂。但是办公室的数字化程序迟迟不能用，耽误了我们大约二年的时间。发现这条路走不通，我们没有再听大典办公室个别人“狼来了”式的承诺，毅然转入常规编纂。好在当时为了做数字化编纂，在大典办公室和中国科学院自然科学史研究所图书馆协助下，将 1500 万字的数学古籍做了数字化数据，同时花半年的时间，整理了《数学典拓展库书目》(22 万字)，为后来的编纂工作提供了方便。

编委会全体同仁与出版社的同志群策群力、同心同德，经过近 10 年的艰苦卓绝的努力，终于完成了这一艰巨工作。当然，由于上面谈到的原因，已经编成的《数学典》不可避免地在书目的选定、版本的取舍、标点和校勘，甚至在各分典、总部字数的分配上都会存在若干缺点和不足，欢迎各位批评指正。

在《数学典》即将付梓之时，我们特别怀念李俨、钱寳琮、任继愈、严敦杰、席泽宗、李迪等先生。李俨、钱寳琮是中国数学史学科的奠基人，他们开辟草莱，筚路褴褛的著述使我们能够从宏观上把握中国数学史的发展历程，李俨的中算藏书，海内外独步，去世时赠自然科学史研究所图书馆，为我们的编纂提供了得天独厚的条件。严敦杰、李迪是中国数学史事业的主要继承者，李俨的藏书主要是严敦杰帮助采购的，李迪桃李满天下，《数学典》的大半编纂者是李迪门下或再传弟子。著名学者、《中华大典》总主编任继愈、副主编席泽宗生前不断亲自指导《数学典》的编纂，经常耳提面命。特别要感谢当代中国数学泰斗、《中华大典》副主编、《数学典》名誉主编吴文俊先生，他四十年来一直倡导支持中国数学史研究，十年来一直关心《数学典》的编纂。还要感谢《数学典》副主编郭世荣、冯立昇和参加《数学典》编纂工作的各位同仁、感谢《中华大典》工作委员会、办公室的各位先生，感谢山东教育出版社陆炎总编辑和韩义华先生，感谢中国科学院基础局、规划局和中国科学院自然科学史研究所、内蒙古师范大学、清华大学的领导和图书馆。

二.《数学典》的宗旨

《数学典》在保留中国古代数学的特色基础上，运用现代数学的观念和方法，对远古到清末（1911 年 12 月 31 日前）在中国疆域范围内产生的汉文典籍、文献资料、出土文物等中有关数学的资料进行系统的整理、分类、汇编，以期为中国科学史和文化史、数学和数学史的研究者、爱好者提供准确、全面、可信的学科资料。

《数学典》在编纂中坚持“质量第一”的原则：内容全面而没有重大脱漏，分类科学而基本上没有交叉重复，取材精当而防止拣小失大，版本精善而摈弃粗制滥造，校点得当而避免错校误改，力图使之成为一部系统、准确、严谨、权威的原始资料汇编。

资料的选编力求体现全面性、科学性、系统性和实用性。

所谓全面性就是资料的选编覆盖了清末以前整个中国数学发展的各个时代，各个分支，各个方面，没有漏收主要的典籍、重要的数学家与成就，同时对不同学术观点兼收并蓄。由于明末之前的数学典籍存世不多，除重复者及个别意义不大的注疏外，基本上做到了有闻必录。

所谓科学性就是资料的选编力求科学准确地体现中国古代数学的思想、方法、成就、典

籍、数学家及各分支的发展情况。所用资料的底本，尽可能使用善本。凡有原本者，不用后世类书的引文。

所谓系统性就是力求系统反映中国古代数学思想、数学方法的真实情况，数学各个分支的发展史，既展现中国古代数学的整体情况，又使读者可系统了解中国数学各分支的发展情况。

所谓实用性就是便于读者使用。

三.《数学典》的结构

（一）经纬目的设置

《数学典》采取以经目为纵，纬目为横，经、纬相结合的编排方式。经纬目的设置体现了中国古代数学的特点，突出全面性、科学性、系统性和实用性。

《数学典》下设《数学概论》《中国传统算法》《中西算法会通》《数学家与数学典籍》四个分典，分典之下设总部，总部下设部和分部，这是经目。

部或分部之下设纬目。纬目下集录古代典籍关于中国古代数学的论述。《〈中华大典〉编纂工作总则》规定的纬目是题解、论说、综述、传记、纪事、著录、艺文、杂录、图表九项。我们认为，这九项是针对文史各典设置的，对《数学典》并不都合适，数学以算法为主，而算法很难归入以上九项。即使对文史各典，论说和综述也很难区分。我们向《中华大典》编委会与工委会提出并获得批准：《数学典》的纬目设置要作变通。各分典的纬目分别是：

《数学概论分典》：题解、综论、纪事、艺文、杂录、图表等；

《中国传统算法分典》、《中西算法会通分典》：题解，算法，综论，纪事，图表等；

《数学家与数学典籍分典》：传记或著录、综论、著录、艺文等。

题解：收录对该部学科名称、概念的涵义与特点等作总体介绍、界定的资料。

算法：集录了历代数学著作中的“术”、“法”、“草”等，对“术”、“法”等的正确性的论证及例题。

综论：收录有关学科或事物的性状、制度、范畴、特点及学科地位、发展情况等内容，顾及了不同的学派及观点。

纪事：收录了该部学科或事物的有关具体活动和事例的资料。

传记：收录了有关数学家的传记资料。

著录：收录了重要数学家与数学典籍的有关著作资料，如专集、序跋、重要史籍、藏书题记，对数学典籍的内容的介绍、评述，以及典籍的成书过程、版本源流等。

艺文：收录有关学科或事物的属于文学欣赏性的散文、韵文、诗词等。

杂录：凡未收入“题解”、“综论”、“纪事”、“传记”、“著录”、“艺文”，而又有较高参考价值的资料，一般收入此目。

图表：图表分为图与表。本典的图大都随文，以免图、文割裂，不知所云。表主要指“算表”，集录三角函数表、对数表等。

（二）各分典的内容及总部

《数学典》所属四个分典的字数、内容及总部是：

1．数学概论分典　该分典约 140 万字。收录了中国古代数学著作的序跋、数学典籍的注疏、二十四史《律历志》《艺文志》及其他文史典籍中对数学的起源、内容、意义和功用，对数学教育、中外交流，对数学与其他学科的关系等有若干精辟的论述。分为算学的起源与发展、算学的功能、记数法和计算工具、数学教育与考试、数学与度量衡、数术与算学、数学游戏、中外数学交流、中西数学关系与比较等 9 个总部。

2．中国传统算法分典　该分典约 500 万字，集纳了自远古至清末中国传统数学的主要成就，分成分数和率、筹算捷算法和珠算、盈不足术、面积、体积、开方、句股测望、方程

术、天元术和四元术、垛积招差、不定问题、极限和无穷小分割方法、数学与天文历法等13个总部，每个“总部”再按不同的专题，分成若干“部”，汇集数学或其他典籍中的数学成就。

3．中西算法会通分典 该分典约550万字，分成算术、对数、数论、几何、画法几何、三角、代数、圆锥曲线、微积分等9个总部。明末西方数学传人中国，开始了中西数学会通和中国数学逐步西化的阶段。这时中国已经失去数学强国的地位，尽管中国传统数学有所复兴，然而与世界数学先进水平的差距越来越大。这一时期从事数学研究的人和著述特别多，超过历史上的任何时期，传世的数学著作占有传本的古代数学著作的95%以上，因此，对此时的资料我们做了精选，而不是有闻必录。

4．数学家与数学典籍分典 该分典拟编150万字。数学家的传记是数学史研究的重要方面。然而二十四史中没有以数学家立传的数学家。数学典籍是数学思想、数学方法和数学成就的主要载体。然而宋元之前的数学著作大部分亡佚。本分典汇集历代典籍中数学家的传记资料，以及对数学典籍的记述和论述。拟分成汉魏南北朝隋唐、宋元、明代、明末至清前期、清中期、清后期等6个总部

（三）序和说明

《数学典序》约一万字，概述中国古代数学的发展概况、典籍、成就、特点、弱点及其在世界文明史、科学史和数学史上的地位，以及本典编纂的特点。

各分典的“说明”说明该分典的主要内容和编纂特点。《数学概论分典》的“说明”还说明各个时期对数学的认识；《中国传统算法分典》与《中西算法会通分典》的“说明”还说明其算法的现代意义及在中国科学技术史、文化史和世界文明史上的地位；《数学家与数学典籍分典》的“说明”还要说明各个时期数学家的作用与地位，数学典籍的特点。

四．文献选编

（一）文献标注

所选编的资料都标注了文献的出处，一般含有朝代、作者、书名、卷次与篇章等。

1.朝代：基本上依传本所题。后人有怀疑但没有确凿的证据者，不予采信。如对注《周髀筭经》的赵爽，钱宝琮认为是三国吴人，但证据不充分。本典仍依明刻本标注“汉•赵爽”。

传本未题朝代的典籍，一般以成书时代为准。

传本未题朝代，后人考证得年代，但难以对应确切的朝代或政权，只好标注最相近的朝代。如《孙子筭经》，钱宝琮认为成书于公元400年前后，为20世纪学术界遵从。公元400年在南方是东晋，本典则标注为晋。

关于《周髀筭经》、《九章筭术》及秦汉数学简牍《数》与《筭数书》等不标注朝代。

2.作者：标名均以原书作者为准。有的传本标注了作者，但在清乾嘉之后疑其系后人伪作，但根据不足者，本典不予采信。例如《数术记遗》，本典依南宋本标注汉·徐岳《数术记遗》。有的传本均未标注作者，后人考得其作者，如有史料佐证，本典予以采信。有的典籍如《孙子筭经》、《夏侯阳筭经》无法确定其作者则标注为“佚名”。但《周髀筭经》《九章筭术》等则不标注作者。

有的著作含有几种内容，经考证，各种内容的作者确凿无疑者，要标注其作者。如南宋杨辉《详解九章算法》含有《九章》本文、刘徽注、李淳风等注释、北宋贾宪《黄帝九章筭经细草》和杨辉的详解五种内容。本典对《九章》本文、三国魏刘徽注、唐李淳风等注释以外的内容，凡大字者，标注为“宋·贾宪《黄帝九章筭经细草》”。而对部分小字则标注为杨辉详解。

3．书名：有通用简称者，本典用其简称，原书名冠有“大唐”、“大清”、“国朝”、“御制”等字样一律不用。“数理精蕴”不作“御制数理精蕴”。

有的数学著作的书名古今异字，遵从其当时用字。例如《九章筭术》是汉代本名，唐李淳风等称作《九章筭经》，清戴震称作《九章算术》。本典涉及《九章筭术》的著述中的"筭"则因时因书而异。清中叶之前一般用《九章筭术》，而戴震整理的及受戴震影响的版本，则用《九章算术》。

有的汉唐算书没有戴震以前的刻本或抄本，戴震从《永乐大典》的辑录本皆作《××算经》，但查凡作《××算经》者，南宋本、大典本皆作《××筭经》，因此《数学典》径直皆作《××筭经》，且不再出校勘符号。如《海岛筭经》，本典对清中叶以前的资料，一律作《海岛筭经》，而对戴震整理的及受戴震影响的版本，则用《海岛算经》。

本典辑录纪传体史书标明书名与篇名，不标注类别。例如："《南齐书》卷五二《祖冲之传》"不作"《南齐书》卷五二《文学·祖冲之传》"。各史合传者可分别标注的本典都分标。例如："《汉书》卷四二《张苍传》"不作"《汉书》卷四二《张周赵任申屠传》"。各史附传在传主前补姓氏分标，例如"《南史》卷七二《祖暅之传》"不作"《南史》卷七二《祖冲之传子暅之》"

《数学典》对类书只限于引用佚书、佚文或异文。例如"唐·欧阳询《艺文类聚·鸟部》　《九章筭术》曰：'五雀六燕，飞集衡，衡适平。'"与传本"五雀六燕"问的文字相异，应该是异文。

随原文引用的注疏，本典写明注疏者时代、姓名及注、疏等字样。同一段文字连续有同一人的几段注疏，自第二段起的标目得省去朝代名。例如：

《九章筭术》卷四《少广》　开方术曰：置积为实。借一筭，步之，超一等。**三国魏·刘徽注**　言百之面十也，言万之面百也。议所得，以一乘所借一筭为法，而以除。**刘徽注**　先得黄甲之面，上下相命，是自乘而除也。除已，倍法为定法。**刘徽注**　倍之者，豫张两面朱幂定袤，以待除，故曰定法……

独立引用的注疏，要先列出被注疏的文献、篇章，不再赘其朝代、作者，后列注疏者的朝代、姓名及注、疏等字样。如："齐同"的题解：

《九章筭术》卷一《方田》三国魏·刘徽注　凡母互乘子谓之齐，群母相乘谓之同。同者，相与通同共一母也。齐者，子与母齐，势不可失本数也。

文献的序、跋、后记，包括题词、题式等，本典根据不同的情况标注。

作者自撰的序跋等在文献与"序"等字之间加符号"·"。他人所撰之序跋等在文献与"序"等字之间不加符号"·"。

（二）避免重复

《中华大典》一般不允许重复。一是各典之间不能有大量重复。一是本典中不允许重复。中国古代数学典籍中有若干重复的内容，一般说来，《数学典》没有重复采编，而提出以下处理方式：

1. 基本相同者，只录最早的文字。例如《九章筭术》卷三《衰分》"女子善织"问：

今有女子善织，日自倍。五日织五尺，问：日织几何?

荅曰：

初日织一寸三十一分寸之十九，

次日织三寸三十一分寸之七，

……

术曰：置<u>一、二、四、八、十六为</u>列衰；副并为法；以五尺乘未并者，各自为实，实如法得<u>一尺</u>。

《孙子筭经》卷中"女子善织"问作

今有女子善织，日自倍。五日织<u>通</u>五尺，问：日织几何?

荅曰：

初日织一寸三十一分寸之一十九，
次日织三寸三十一分寸之七，
……

术曰：各置列衰；副并，得三十一，为法；以五尺乘未并者，各自为实，实如法而一，即得。

下面划线者为相异的文字。可见两者只是个别文字有异，并无本质区别，则只选编《九章筭术》的。

2.既有相同，又有不同者，仅录入不同者。例如《孙子筭经》卷中关于约分、合分、减分、平分的题目分别与《九章筭术》相应部分的第一个题目相同，只是个别字有差别，然而《九章筭术》在答案之后没有此题的演算术文，而在几个例题后有抽象性总术，《孙子筭经》却在答案之后有此题的演算术文。本典删去《孙子筭经》的问题和答案，而将术文插入《九章筭术》的相应题目之后，单独标注，但均退一格。比如关于约分，便标注为：

《九章筭术》卷一《方田》　今有十八分之十二。问：约之得几何?

答曰：三分之二。

晋·佚名《孙子筭经》卷中　术曰：置十八分在下，一十二分在上。副置二位，以少减多，等数得六，为法。约之，即得。

《九章筭术》卷一《方田》　又有九十一分之四十九。问：约之得几何?

答曰：十三分之七。

约分术曰：可半者半之。不可半者，副置分母、子之数，以少减多，更相减损，求其等也。以等数约之。　**三国·魏刘徽注**　等数约之，即除也。其所以相减者，皆等数之重迭，故以等数约之。

3.所论主题相同，而文字差别较大者，则皆录入。例如《九章筭术》的合分术是：

《九章筭术》卷一《方田》　合分术曰：母互乘子，并以为实。母相乘为法。实如法而一。不满法者，以法命之。其母同者，直相从之。

《筭数书》的合分术是：

汉简《筭数书·合分术》　母相类者，子相从。其不相类者，母相乘为法，子互乘母并以为实，如法成一。

两者的方法相同，但文字差异较大，均编入。

4.已经单列的文字在他处引用时可以略去以避免重复

例如《九章筭术》卷四《少广》开方术刘徽的第一段注是关于“开方”的题解，需单列：

《九章筭术》卷四《少广》　开方**三国魏·刘徽注**　求方幂之一面也。

在集录《九章筭术·少广》开方术及其刘徽注时便略去此段注文。

5.对经注不能分离的内容，即使不得不重复经文，也要收录。例如对《九章筭术》的圆面积公式“半周半径相乘得积步”，在“面积总部”和“极限思想与无穷小分割方法总部”都引用了，不得不重复。

6.避免各总部内容的重复

中国古代数学的分类与现今数学不同。一种方法或问题往往含有现今数学的几类内容，其交叉之处不胜枚举。如《测圆海镜》卷二最后一问与卷三—十二的所有问题，既是句股容圆问题，又使用天元术，如果两类都收，势必造成大量重复。本典规定这些内容归天元术和四元术总部，句股测望总部不再收入。

（三）编纂与排印方式

1．编纂顺序：《中华大典》各级纬目汇集的资料，以所采用的古籍为单位，按著作完成的时间顺序排列。对《周髀筭经》、《九章筭术》与秦汉数学简牍难以分先后，本典规定其顺序为：《数》、《筭书》、《筭数书》、《周髀筭经》、《九章筭术》、《筭术》。

2. 编排方式：全典采用繁体字竖排。以一个标注下的一段为单位排印。

3. 分册：《数学典》分为 8 册，分册不破总部。每册 150-200 万字。各册分别是：《数学概论分典》为第一册，《中国传统算法分典》为第二、三、四册，《中西算法会通分典》为第五、六、七册，《数学家与数学典籍分典》为第八册。

五．文献版本、标点和校勘

（一）版本

《数学典》对选用古籍版本的基本要求是：现代影印的精校精刻本，宋元刻善本及明代精校精刻本，清代的精校精刻本，优先选用公认的优秀的近、现代整理圈点本及现代学者的校点整理本。

（二）句读

本典力求做到句读无误，避免读破句。为此，《数学典》遵从《中华大典》粗线条句读的规定，亦即在不影响原意的前提下，对既可长读又可短读，或可连读亦可分读的句子，采取长读法。例如“**《九章筭术》卷一《方田》**　合分术曰：母互乘子，并以为实。”不作“合分术曰：母互乘子，并，以为实。”

如果不读断，会有碍文意，或引起误解者，必须读断，不可长读。例如对南宋本刘徽求圆周长的文字“以半径一尺除圆幂，倍所得，六尺二寸八分，即周数”，不可作“以半径一尺除圆幂，倍所得六尺二寸八分，即周数。”因为这在数学上全错不通。

《数学典》还对某些常用的句型的句读，人名、字号、籍贯的句读等做了规定。

（三）校勘

《数学典》对所选底本中的严重衍脱舛误做了校勘。校勘本着少而精的原则，可改可不改的不改。校勘符号用圆括号和方括号。圆括号用来括住衍误的文字，方括号用来括住校补的字。例如“**《九章筭术》卷九《句股》三国魏·刘徽注**　句股相并〔幂〕而加其差幂，亦减弦幂，为积，盖先见其弦，然后知其句与股。今适等，自乘，亦各为方，(先见其弦，然而后知其句与股，适等者，令自乘，亦令)〔合〕为弦幂。令半相多而自乘，倍之，〔又半并自乘，倍之,〕亦〔合〕为弦幂。”

所汇编的图、表中的字词、数字如有舛误，本典俓直校勘，不赘舛误文字、数字，也不加校勘符号。

《数学典》对避讳、通假与简体字等也做了规定。

（四）标点

《数学典》对编纂的所有资料都用现代标点符号标点。《中华大典》编纂及校点通则规定要尽量使用句号和逗号，少用顿号、冒号和引号，《数学典》基本遵从这些规定。但有时不用顿号会引起误解，则使用了顿号。如中国古代数学表示带分数其整数部分与分数部分的分母连书，如不用顿号，今人便不知所云。又如《九章筭术》卷五《商功》中“其二十步上下棚、除，棚、除二当平道五。”“棚”、“除”是两种设施。刘徽注曰：“棚：阁；除：邪道。”戴震不解此义，以为“棚除”是一个词，在微波榭本《九章算术音义》中杜撰了“棚除　上，薄耕切。……下，迟据切。……”，对“棚”、“除”的释义违背了刘徽注，并强加给唐李籍，以取代《九章筭术音义》的“棚，薄耕切”条。戴震的杜撰蒙蔽了不少人，清末广雅书局翻刻《武英殿聚珍版丛书》本《九章算术》，竟以戴震在微波榭本中的杜撰改动原本。

六．刍议《数学典》编纂对中国数学史研究的意义

《中华大典》的编纂对我国文化建设的重大意义，许多文件和有关人士的讲话都谈过了，此不赘述。这里仅就《数学典》的编纂对中国数学史研究的意义提示几点。

首先，为广大中国数学史工作者提供了中国古代数学各个分支的大量的原始素材，便于他们查阅，会成为中国数学史研究的某种出发点。

其次，广大读者会通过《数学典》的目录了解中国古代数学的各个方面和分支、细目，掌握中国古代数学的概貌。也会通过《数学典序》和各分典的说明了解中国古代数学的发展概况、典籍、成就、特点、弱点及其在世界文明史、科学史和数学史上的地位，了解我们的祖先在各个时期对数学的认识，了解古代算法的现代意义，了解各个时期数学家的作用与地位以及数学典籍的特点，等等。

第三，不少先生对古籍的版本、校勘了解不多，编纂《数学典》实际上是一个边干边学的过程，尤其学习了古籍整理的基本知识，实践了校勘工作，在某种意义上培养了中国数学史研究队伍，对今后从事中国数学史研究大有裨益。

第四，明清数学一直是中国数学史研究的薄弱环节，常被戏称为“明不明，清不清”。编纂《数学典》，实际上是20世纪以来对明末至清末数学典籍从未有过的全面研究。这种研究尽管还是初步的，但对进一步深入研究传统数学在清代的发展，西算在明末至清末的传入及与中算的会通，打下了良好的基础。

第五，我在《数学典》编委会会议上多次讲过，编纂工作给我们提供了不可多得的逐字逐句读原著的机会，做个有心人，在编纂过程中，一定会有不少心得，发现并提出若干新的课题，促进自己的数学史研究。我高兴地看到，已有不少同仁发现了新的课题，在完成各总部的编纂的同时，做出了新的成果。

第六，《数学典》的编纂过程实际上是对中国古代数学典籍的一个全面了解，为今后进一步开展数学古籍的整理奠定了基础，培养了队伍。我在《数学典》编委会会议上透露过，在《数学典》完成之后，准备编纂《中国古代数学典籍汇编》（暂用名）。实际上，早在2011年，上海古籍出版社一位编辑看到《中华读书报》就《中国科学技术史·数学卷》出版对我的采访，来电话希望我主持影印中国古代数学古籍，考虑到这对我们学科是一项极其重要的基本建设，我答应了，但得在完成《数学典》之后。在《数学典》的编纂接近尾声的时候，2015年11月11日我与上海古籍出版社的先生商谈了此事，初步确定编10册。根据1992年编纂《中国科学技术典籍通汇·数学卷》的经验，我们的工作就是定书目，选版本，写提要，诚恳欢迎各位参加。

Appendix I: List 附一：与会人员名录

Name	**姓名**	**Title/职称**	**Employer/单位**	**Mobile Phone/ 手机号码**	**Email/邮箱**
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Appendix II: Address 附二：会议地址

The address of our TSIMF (which the workshop will be held at this place): Tsinghua Sanya International Mathematics Forum, No.100, Tsinghua Road, Phoenix Town, Sanya, Hainan, P. R. China. (In Chinese is:中国海南省三亚市凤凰镇清华路 100 号，清华三亚国际数学论坛). The conference room is near by the dining hall at our TSIMF. Our TSIMF has the facilities of guest rooms for the all participants. We will arrange the pick-up and send-off service at Sanya Phoenix International Airport for the all participants according to their detailed flight information or train information.

There are two ways to go to Sanya International Mathematics Forum from the Sanya Phoenix International Airport:

- By shuttle car of the Forum; if you have provided flight arrival information to us in advance.
- By taxi at Sanya Phoenix International Airport (the ride should cost about ¥15 to ¥25) if you did not provide your flight information to us in time or you want to reach our TSIMF by yourself.

Map of TSIMF's Park 园区平面图