## Bjorn Berntson <br> bkberntson@gmail.com <br> University College London

Title: Singularity structure of delay-differential Painlev́e equations
Abstract: Painlev́e type delay-differential equations been obtained by various methods. One particular feature these equations share with the continuous Painleve equations is the existence of elliptic solutions in particular limits. I will discuss the relationship between this phenomenon and the singularity analysis of such equations.

## Adrian Stefan Carstea

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## Department of Theoretical Physics, IFIN-HH Bucharest

Title: Lattice supersymmetric KdV equation and super-QRT mappings
Abstract: Two integrable discretizatios of supersymmetric KdV equations are constructed using Hirota bilinear formalism. The integrability is established by constructing 3 -supersoliton solution which displays two different type of interactions between supersolitons. Travelling-wave reduction is also performed and the emergent super-QRT mapping is studied using singularity analysis.

## Chuan-Tsung Chan <br> ctchan@go.thu.edu.tw <br> Department of Applied Physics, Tunghai University

Title: Connection Preserving Deformation Approach to the q-Difference Equations.
Abstract: In a seminal work, Jimbo and Sakai have derived the q-Difference Painleve VI equation based on the connection preserving deformation (CDP) approach. Such an approach is closely related to the isomonodromy deformation approach in the study of continuous Painleve equations and leads to algebraic expressions which allows geometric analysis and classifications. In this talk, I shall introduce some basic ideas behind this approach and study the generalized cases of the Lax pairs with non-diagonalizable leading matrices.

## Anton Dzhamay <br> Anton.Dzhamay@unco.edu <br> University of Northern Colorado

Title: Higher-Rank Two-Point Schlesinger Transformations and their Hamiltonian Structure
Abstract: Schlesinger transformations are discrete isomonodromic transformations of a Fuchsian system that change the characteristic indices of the system. It is well-known that when the space of accessory parameters of the system is two-dimensional, such transformations can be described by difference Painlevé equations, and we are interested in studying such reductions. In our previous work we have studied such transformations given by the rank-one multiplier matrix and we gave both the evolutionary and the Hamiltonian representation of the dynamic. The transformations we considered change characteristic indices of the system at two of its poles, and only one index at each pole is affected. However, to study reductions to difference Painlevé equations with the affine Weyl symmetry group $E_{n}$, with $\mathrm{n}=7$ or 8 , we need to consider Fuchsian systems that have characteristic
indices with multiplicities. This requires generalizing the Schlesinger transformation equations for multiplier matrices of higher rank. In this talk I explain how to do that and again describe the resulting discrete dynamical system using both the evolutionary and the Hamiltonian formalisms.

## Claire Gilson

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## School of Mathematics and Statistics, University of Glasgow

Title: A graphical approach to the ultra-discrete KdV equation with real site values
Abstract: The ultra-discrete KdV equation is an equation with both discrete independent and dependent variables. The solutions of which exhibit solitonic behaviour. In this talk we shall look at the ultra discrete KdV equation and some simple variants, in the case where the dependent variable values are permitted to take any real value. A direct graphical approach will enable us to identify the solitons and other material within the system, as well as understand the time evolution

## Jarmo Hietarinta <br> hietarin@utu.fi

## Department of Physics and Astronomy, University of Turku

Title: Searching for integrable systems
Abstract: It is well known that integrable systems (however defined) are rare and special. Since integrable systems are also applicable, it is of great interest to find and classify them. I will give a very personal view of searching for integrable systems. I will also point out some open problems; they are concrete and simple to define but quickly lead to computational problems that still seem to be too hard for today's computer algebra systems. The following classes of equations are discussed:

- Two-dimensional standard type Hamiltonians, their quantum versions, as well as generalizations that have no classical counterpart.
- Quantum Yang-Baxter equations, and the higher dimensional tetrahedron equations, as well as permutation type solutions for $N$-simplex equations in general.
- Equations defined in terms of Hirota's bilinear formalism, in which integrability shows up as the existence of three-soliton solutions; this applies to both continuous and discrete equations.
- Singularity confinement as an indicator of integrability of discrete systems.
- Discrete two-dimensional equations and the property of multidimensional consistency, in particular consistency-around-a-cube.


## Kenji Kajiwara <br> kaji@imi.kyushu-u.ac.jp <br> Kyushu University

Title: A discrete model of vortex filaments
Abstract: It is well known that smooth space/plane curves admit integrable deformations governed by various soliton equations. Among them, the binormal flow of the smooth space curves is wellknown as a model of dynamics of vortex filaments in fluids. It describes a dynamics driven by the selfinduction of vortex filaments through the Biot-Savarts law under the local induction approximation. Under this flow, the complex curvature (sometimes referred to as the Hasimoto transformation) satisfies the nonlinear Schrödinger equation (NLS).

In this talk, we present a discrete model of dynamics of vortex filaments in fluids. Namely, we formulate a discrete deformation of discrete space curves whose (discrete counterpart of) complex curvature satisfies the discrete NLS proposed by Hirota-Ohta-Tsujimoto. We also construct an explicit formula of both smooth and discrete curves in terms of the tau functions of two-component of KP hierarchy. This presentation is based on the collaborated work with Sampei Hirose, Junichi Inoguchi, Nozomu Matsuura and Yasuhiro Ohta.

## Peter van der Kamp <br> P.VanDerKamp@latrobe.edu.au <br> La Trobe University

Title: Laurentification
Abstract: A recurrence relation has the Laurent property if it generates Laurent polynomials in the initial values. Ultra-discretising difference equations yields bounds on the degrees and on the multiplicities of divisors. Recursive factorisation can then be used to prove polynomial upper bounds on growth of degrees. It also enables us to obtain recurrence equations for the divisors of the homogenised equation. When the difference equation is singularity confining, the order of the obtained recursion is fixed. Such recurrences are likely to have theLaurent property.

## Qing-Ping Liu

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## China University of Mining and Technology (Beijing)

Title: Supersymmetric Integrable Systems: Darboux-Backlund Transformations and Discretizations
Abstract: In this talk, we will review our recent results on the construction of Darboux-Backlund transformations and discretizations for some well-known supersymmetric integrable systems. The examples include supersymmetric KdV, supersymmetric MKdV and supersymmetric NLS. This is a joint work with Decio Levi and Ling-Ling Xue.

# Sen-Yue Lou <br> lousenyue@nbu.edu.cn <br> Ningbo University and East China Normal University 

Title: Discrete Alice-Bob systems and multiple soliton solutions: AB-Toda system
Abstract: The usual differential-difference and pure difference systems are extended or restricted to the Alice-Bob forms. The details on the AB-Toda system are discussed especially on its Lax pair and multiple soliton solutions.

## Kenichi Maruno <br> kmaruno@waseda.jp <br> Waseda University

Title: Validated computation and integrability of difference equations
Abstract: Validated computation (validated numerics), which means rigorous numerical computation with guaranteed error, recently became one of powerful tools for various fields in mathematics. We propose a method to perform validated computation for nonlinear difference equations including discrete integrable systems. Based on this method, we discuss the relationship between the growth of interval widths and integrability of difference equations.

## Nobutaka Nakazono

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## School of Mathematics and Statistics, University of Sydney

Title: A reduction from a 4-cube to a rhombic dodecahedron associated with q-Painlevé equations
Abstract: In this talk, I will present a reduction from a system of partial difference equations known as ABS equations (Adler, Bobenko and Suris 2003) to ordinary difference equations known as discrete Painlevé equations. We show that a reduction from a 4-dimensional hypercube (4-cube) to a rhombic dodecahedron causes the reduction from the system of ABS equations to q-discrete Painlevé equations, which are A4(1)-surface type in Sakai's classification (Sakai 2001). Moreover, I also present Lax pairs of the q-Painlevé equations that are constructed using this reduction.

This work has been done in collaboration with Prof. Nalini Joshi and Dr Yang Shi and supported by an Australian Laureate Fellowship \# FL120100094 and grant \# DP130100967 from the Australian Research Council.

## Maciej Nieszporski

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Katedra Metod Matematycznych Fizyki, Wydzial Fizyki, Uniwersytet Warszawski
Title: In the zoo of discrete integrable equations and correspondences
Abstract: A link beetwen involutive rational maps and difference integrable systems allows us to unify difference integrable equations and corresspondences.I will show that the link can serve as classification tool for the systems.I will present the menagerie of integrable systems that revealed while we were investigating the link, including correspondences and difference equations on fourpoint, six-point and seven-point stencils.

Frank Nijhoff<br>frank.nijhoff@gmail.com<br>University of Leeds

Title: Elliptic Lax pairs and isomonodromic deformation problems associated with integrable lattice equations


#### Abstract

In recent work with N. Delice and S. Yoo-Kong we proposed a general family of higherrank Lax pairs associated with a class of integrable systems associated with an elliptic curve. We also present a general class of isomonodromic deformation problems on such curves and demonstrate how to resolve the relevant compatibility conditions using some (perhaps novel) elliptic identities.


## Jonathan Nimmo <br> Jonathan.Nimmo@glasgow.ac.uk <br> University of Glasgow

Title: An explicit formula for ultradiscrete bound states
Abstract: In two papers (Willox et al. 2010,1012), a procedure to solve the Cauchy problem for the ultradiscrete KdV equation in which the dependent variable takes arbitrary real values was described. A key step in this procedure is the calculation of the bound state ("square integrable" eigenfunction) for the eigenvalue corresponding to the fastest soliton. In this talk, we will present an explicit general formula for this bound state, expressed in terms of the time update and downdate of the initial data. Illustrative examples will be presented and some aspects of the proof of this result will be discussed.

> Yasuhiro Ohta
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> Department of Mathematics, Kobe University

Title: Quaternion Pfaffian - an extension of quaternion determinant
Abstract: Pfaffian with quaternion elements is introduced by following Moore's definition of quaternion determinant for self-dual quaternion matrix [E. H. Moore, Bull. Am. Math. Soc. 28 (1922) 161]. Some properties of quaternion Pfaffian are also discussed.

## Linyu Peng <br> 1.peng@aoni.waseda.jp <br> Department of Applied Mechanics and Aerospace Engineering, Waseda University

Title: The dierence variational bicomplex and its applications
Abstract: In this talk we discuss the difference variational bicomplex with respect to difference systems. The bicomplex, a double co-chain complex, enables us to consider, for instance symmetries, conservation laws, and inverse problems, in a more fundamental sense. In particular, we will show how the bicomplex would be powerful in the understanding of Noether's theorem, multisymplectic integrators and multidimensional consistency.

# John Roberts <br> jag.roberts@unsw.edu.au <br> School of Mathematics and Statistics, UNSW 

Title: Birational maps preserving biquadratic fibrations
Abstract: In a recent paper (J. Phys. A: Math. Theor. 48 (2015) 08FT02), Jogia and I gave some characterisation of birational maps that send biquadratic fibrations to themselves, but most generally not preserving each fibre. We concentrated on the symmetric case, which can be written as a second order difference equation, and presented the generalisation of a symmetric QRT map. In this talk, I present recent extensions on this topic done with Dinh Tran where we consider the asymmetric case.

Junkichi Satsuma
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Department of Mathematical Engineering, Musashino University
Title: Application of Ultradiscretization to Filtering Theory
Abstract:A new filtering method for nonlinear systems is proposed through the procedure of ultradiscretization. Discrete-time Kalman filter is readily applied to the obtained ultradiscretized system by imposing some conditions on system variables and parameters. Some numerical experiments are given to show the efficiency of the method.

## Yang Shi <br> yshi7200@gmail.com <br> The University of Sydney

Title: Discrete integrable systems and reflection groups
Abstract: We present a method of constructing discrete integrable systems with crystallographic reflection group (Weyl) symmetries. Our approach exploits the combinatorial and the geometrical properties of the Weyl groups. Discrete integrable systems are associated with space-filling polytopes from the geometric representation of the finite Weyl groups in the n-dimensional Euclidean space Rn . The "multi-dimensional consistency" property of the discrete integrable system is shown to be inherited from the combinatorial properties of the polytope; while the dynamics of the system is described by affine translations of the polytopes on the weight lattice of the Weyl group. Different classes of discrete integrable systems result depending on the symmetry and combinatorics of the construction. We obtain some well-known systems such as the multi-dimensional consistent systems of ABS type quad-equations and their reductions to some discrete Painlevé equations from Sakai's classification, thus clarifying the relationship between quad-equations and discrete Painlevé equations in terms of their symmetry groups.

This work is a collaboration with Nalini Joshi and Nobutaka Nakazono.
This research was supported by an Australian Laureate Fellowship \# FL 120100094 and grant \# DP130100967 from the Australian Research Council.

## Daisuke Takahashi <br> daisuket@waseda.jp <br> Department of Applied Mathematics <br> Faculty of Science and Engineering <br> Waseda University

Title: Complexity of max-min-plus equation and binary decision diagram
Abstract: We discuss the complexity of solutions to digital evolutional equations. The equations can be described by max-min-plus form and we can solve them directly using formulas on max operations. There are various levels of complexity for the expression of solution and we define the complexity by the order of number of terms included in the solution.

On the other hand, binary decision diagram (BDD) widely used in the information theory is another tool to solve digital evolutional equations. We can evaluate the complexity of solutions using the software package to search the minimum complexity of BDD.

In our talk, first we explain about the complexity of solutions to max-min-plus equations. Then we show the effectiveness of binary decision diagram to evaluate the complexity.

## Tomoyuki Takenawa

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Tokyo University of Marine Science and Technology
Title: Fiber-dependent deautonomization of integrable 2D mappings
Abstract: Although it is well known that the QRT mappings, two-dimensional mappings preserving a rational elliptic fibration, can be deautonomized to discrete Painlevé equations, it has not been well studied that how this procedure depends on the choice of the fibers. In this talk we establish the way of deautomization for the pairs of a QRT mapping and a fiber.

Tetsuji Tokihiro
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## Graduate School of Mathematical Sciences, the University of Tokyo

Title: On higher dimensional analogue of Hietarinta-Viallet equation
Abstract: A two dimensional analogue of the Hietarinta-Viallet equation is proposed. The equation is a two dimensional lattice equation (an extended discrete KdV equation) and is equipped with both integrable and chaotic features. We prove its co-primeness, which is the algebraic reinterpretation of singularity confinement, and show that its reduction gives a hierarchy of "quasi-integrable" equations including the Hietarinta-Viallet equation and its extensions whose algebraic entropy is rigorously obtained.

# Teruhisa Tsuda <br> tudateru@econ.hit-u.ac.jp <br> Hitotsubashi University 

Title: Hermite-Pade approximation, isomonodromic deformation and hypergeometric integral
Abstract: This talk is based on a joint work with Toshiyuki Mano (arXiv:1502.06695). We develop an underlying relationship between the theory of rational approximations and that of isomonodromic deformations. We show that a certain duality in Hermite's two approximation problems leads to the Schlesinger transformations, i.e. transformations of a linear differential equation shifting its characteristic exponents by integers while keeping its monodromy invariant. Since approximants and remainders are described by block-Toeplitzs determinants, one can clearly understand the determinantal structure in isomonodromic deformations. We demonstrate our method in a certain family of Hamiltonian systems of isomonodromy type including the sixth Painleve equation and Garnier systems; particularly, we present their solutions written in terms of iterated hypergeometric integrals. An algorithm for constructing the Schlesinger transformations is also discussed through vector continued fractions.

## Claude-Michel Viallet <br> viallet@lpthe.jussieu.fr <br> Centre National de la Recherche Scientifique Sorbonne Universités Univ. Pierre et Marie Curie

Title: Singularity, integrability, Laurent property and all that
Abstract: Using specific examples, we go back to the singularity/entropy analysis of maps in dimension two and higher. An interesting example is Somos4, for which the evolution is given by a map in four dimensions. This map is integrable, non confining, and has the Laurent property (we give an alternative proof of the Laurent property). We describe an example in dimension three which is integrable and confining, but does not have the Laurent property, and recall an example of a confining non integrable order two recurrence which has the Laurent property. The basic tool we use is the analysis of factorization properties of the iterates of the maps, these properties being a direct consequence of the presence of singularities.

## Ralph Willox <br> willox@ms.u-tokyo.ac.jp <br> Graduate School of Mathematical Sciences, the University of Tokyo

Title: A direct linearization of a Box \& Ball system with arbitrary carrier capacity
Abstract: A couple of years ago a novel, inverse scattering-type, technique was discovered for solving the Cauchy problem for the ultradiscrete KdV equation over the real numbers. When restricting the initial values for this system to the set $\{0,1\}$, the action-angle variables that arise in that approach turn out to be related in a rather simple way to those that can be obtained from an algorithm proposed by T. Takagi for calculating the Kerov-Kirillov-Reshetikhin bijection for rigged-configurations associated with $A_{1}^{(1)}$-type crystals.
I will explain a simple method to obtain these rigged configurations from initial states for the socalled 'Box\&Ball' system with arbitary (i.e. infinite or finite) carrier capacity, and I will discuss the linearization of the time evolution of this Box\&Ball system in terms of the action-angle variables obtained from the rigged configurations. If time permits, I will also describe the connection of these
action-angle variables with those obtained from the ultradiscrete inverse scattering approach.
This talk is based on joint work with S. Kakei (Rikkyo University), S. Tsujimoto (Kyoto University) and J.J.C. Nimmo (University of Glasgow).

## Pavlos Xenitidis

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School of Mathematics, Statistics and Actuarial Science University of Kent
Title: $\mathbb{Z}_{N}$ grading, Lax pairs and integrable lattices


#### Abstract

In this talk first we will consider a family of $N \times N$ matrices depending linearly on a spectral parameter $\lambda$ and having a particular $\mathbb{Z}_{N}$ graded structure. Then these matrices will be employed in the construction of Lax pairs which lead to integrable systems of difference equations, differential-difference equations, as well as to Yang-Baxter maps. Finally integrability properties of some of the newly found systems will be discussed.


## Sikarin Yoo-Kong <br> syookong@googlemail.com <br> King Mongkuts University of Technology Thonburi, Thailand

Title: On the Lagrangian 1-form formalism
Abstract: The Lagrangian multi-form formalism has become one of the main research topics in the integrable systems both discrete-time and continuous-time cases, after pioneer works were initiated by Lobb and Nijhoff. In this talk, we focus on the simplest case on this context called the 1form structure. An overview of the formalism both discrete-time and continuous-time cases will be presented together some new results. Some open questions will be also discussed at the end of the talk.

Guofu Yu
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Shanghai Jiaotong University
Title: The generalization and integrable discrete analogue of the coupled integrable dispersionless system
Abstract: In this talk, the coupled integrable dispersionless (CID) equations will be linked to the first negative flow of AKNS hierarchy. The vector generalization, complex version and integrable discrete analogue of the CID equations will be analyzed.

## Cheng Zhang <br> mathphyzc@163.com <br> Shanghai University

Title: On an non-commutative extension of $H 1$ equation
Abstract: In this talk, we will focus on an non-commutative extension of the discrete potential KdV equation, known $H 1$ equaiton in the ABS classification. Motivations to the non-commutative setting will first be presented. Then we will focus on some integrability properties of the equation. Special solutions to the equation will be discussed at the end. This work is a collaboration with Yingying Sun, Dajun Zhang and Sunlin Zhao.

> Zuo-Nong Zhu
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> Department of Mathematics
> Shanghai Jiao Tong University

Title: On integrable nonlocal equation
Abstract: In this talk, we focus on the topic of integrable nonlocal equations. We will discuss gauge equivalences for integrable nonlocal NLS equation introduced by Ablowitz and Musslimani. We also investigate soliton solutions for a coupled integrable nonlocal NLS equation and an integrable nonlocal mKdV equation. This is a joint work with Li-yuan Ma, Cai-qin Song, and Jia-liang Ji.

