Title and Abstract

1. A.G.Aleksandrov Novosibirsk State University

Title：Differential forms on non-complete intersections

Abstract：

Let $\Omega_X^p, p \geq 0$, be the family of the sheaves of regular differential forms on a variety X. Then the exterior differentiation d endows the family with a structure of an increasing complex$(\Omega_X^{\cdot}, d)$; it was first considered and studied by Henry Poincaré (however it is often called the De Rham complex). On the other hand, one can endow this family with a structure of a complex in other ways. For example, we can consider an increasing complex $(\Omega_{X/k}^{\cdot}, \delta)$, the differential operator δ being the exterior multiplication by a Kähler differential $\omega \in \Omega_X^1$, i.e. $\delta = \wedge\, \omega$; such complex was introduced and studied by George de Rham in 1954. Indeed, the first complex should be named the Poincaré complex of X, while the second one – the De Rahm complex. Next, if there exists a non-trivial regular vector field ν on X, then the contraction ι_ν along the field ν equips the family $\Omega_X^p, p \geq 0$, with the structure of a decreasing complex $(\Omega_X^{\cdot}, \iota_\nu)$ which can be regarded as a contravariant version of the complex $(\Omega_{X/k}^{\cdot}, \delta)$. In 1973 the complex $(\Omega_X^{\cdot}, \iota_\nu)$ has been used by J.Carrel and D.Liebermann in their proof that the Hodge numbers $h^{pq}(X)$ of a Kähler manifold X vanish as soon as $|\, p - q \,|$ is greater than the dimension of the zero-locus of the field ν.

Similar complexes are constructed for the family of the sheaves of regular meromorphic differential p-forms $\omega_X^p, p \geq 0$, introduced by D.Barlet and E.Kunz in 1976. In the case when M is a manifold and $X \subset M$ is a reduced hypersurface or a complete intersection then the sheaf ω_X^p for any $p \geq 0$ can be considered as the image of the residue map defined on $\Omega_M^{p+1}(log X)$, the module of the logarithmic or the multi-logarithmic differential forms with poles on X, respectively. In 1983 the author has computed explicit expressions for the Poincaré series of $\Omega_X^p, \omega_X^p, H_{\{0\}}^*(\Omega_X^p), p \geq 0$, in the case of graded isolated complete intersection singularities of positive dimension, which imply useful formulas for calculating a few important invariants of the singularity.

Now our aim is to discuss basic properties of the above complexes in a more general setting and, making use of certain relations and their interconnections, to calculate some topological

and analytical invariants of isolated non-complete intersection singularities laying emphasis on the graded case and on some interesting applications.

2. Javier Bobadilla BCAM, Bilbao, Spain

Title: Primitive cohomology of smooth projective complete and non-complete intersections

Abstract:

Work in progress.

Let X be a smooth projective manifold. Let $F_1, \dots F_r$ be a set of homogeneous polynomials, all of them of the same degree (sufficiently high) defining X as a subscheme of the projective space. Let c be the codimension of X. Pick $G_1, \dots G_\epsilon$, c generic linear combinations of $F_1, \dots F_r$ The complete intersection $Z_0 \coloneqq V(G_1, \dots, G_\epsilon)$ contains X as an irreducible component. Let

$$Z_t := V(G_{1,t}, \dots, G_{c,t})$$

be a 1-parameter smoothing of Z_0. Our aim is to compare the intermediate primitive cohomology of X (for a certain polarsation) with the intermediate cohomology of Z_t..

If $\dim(\mathrm{X}) \leq 3$ we find a natural embedding of the intermediate primitive cohomology of X into the intermediate cohomology of Z_t. For $\dim(\mathrm{X}) \geq 4$this embedding does not exist in general. We find a necessary and sufficient condition: we define two polynomials P and Q on the Chern classes of the tangent bundle of X and on the polarsation given by the embedding, and the cohomology embedding holds if and only if Q is a multiple of P in the cohomology ring of X.

The condition is satisfied immediately if X is a complete intersection, but also if the codimension of X is sufficiently low. This may be seen as a supporting evidence of Hartshorne conjecture on smooth varieties of small codimension being complete intersections, and perhaps as a tool to address it.

In the case that the condition is not satisfied, the piece of the primitive cohomology that does not embed is shown to embed into the intermediate primitive cohomology of a manifold of dimension 4 less that the original one, setting up in this way an inductive scheme to analyze it.

3. Baohua Fu Chinese Academy of Science

Title：Generic singularities of nilpotent orbit closures

Abstract:

According to a well-known theorem of Brieskorn and Slodowy, the intersection of the nilpotent cone of a simple Lie algebra with a transverse slice to the subregular nilpotent orbit is a simple surface singularity. At the opposite extremity of the nilpotent cone, the closure of the minimal nilpotent orbit is also an isolated symplectic singularity, called a minimal singularity. For classical Lie algebras, Kraft and Procesi have worked out all generic singularities of nilpotent orbit closures. I'll report a joint work with D. Juteau, P. Levy and E. Sommers, where we worked out the exceptional cases.

4. James Fullwood The University of Hong Kong

Title: On tadpole relations via Verdier specialization

Abstract:

S-duality between two regimes of string theory referred to as ‘F-theory' and `type IIB' predicts a linear relation among the Euler characteristic of an elliptic Calabi-Yau fourfold and the Euler characteristics of certain divisors in a particular Calabi-Yau threefold. Such relations are often referred to in the physics literature as `tadpole relations'. It has been found that these tadpole relations coming from the equivalence between F-theory and type IIB may be shown to hold by integrating Chern class identities which hold in a much broader context than physical one. In this talk, using the construct of Verdier specialization we give a top-down explanation for the existence of such Chern class identities, yielding a purely mathematical explanation of the aforementioned tadpole relations predicted by physicists.

5. Gert-Martin Greuel University of Kaiserslautern

Title: On simultaneous normalization of families of isolated singularities

Abstract:

In my talk I give an overview of classical and recent results on simultaneous normalization of families of algebraic and analytic varieties. The concept of simultaneous normalization was introduced by Teissier in 1978. He proved that a 1-parameter family of reduced curve singularities is equinormalizable iff the delta-invariant of the curves is constant. The same result holds for higher dimensional normal parameter spaces, but the proof turned out to be much more involved. After an incomplete proof by Raynaud and Teissier it was finally achieved by Chiang-Hsieh and Lipman in 2006. Chiang---Hsieh and Lipman proved also an analog result for

families of projective varieties with the delta invariant replaced by the Hilbert polynomial of the normalization. This result was generalized in 2011 by Kollar, again for algebraic families of projective varieties.

The case of families of isolated analytic singularities remained open so far and I report on new results and conjectures for this situation. For non-reduced, isolated singularities we replace the classical delta invariant of a reduced curve singularity by a new invariant, which takes the nilpotent elements into account and which we continue to call delta. While Chiang-Hsieh, Lipman and Kollar consider only equidimensional singularities, we do not need this restriction. Instead, We introduce a necessary geometric obstruction to simultaneous normalization, which has not been considered so far. This is partly joint work together with Le Cong Trinh.

6. Adam Harris University of New England

Title: Involutive Deformations of Normal Surface Singularities

Abstract:

A deformation of a complex manifold is said to be “involutive" if the Kodaira-Spencer deformation tensor is $\bar{\partial}$-closed and has vanishing Frolicher-Nijenhuis bracket. It therefore represents a cohomology class which induces a new (i.e., automatically integrable) complex structure. We show that for non-compact complex surfaces, such as the regular part of a normal surface-singularity, such deformations are numerous. We discuss examples and sufficient conditions for Stein completion of deformations of the regular part when stability of a complex Euclidean embedding is not pre-determined.

7. Stanislaw Janeczko Wydział Matematyki i Nauk Informacyjnych

Title：Differential forms on singular varieties, symplectic invariants"

Abstract:

There are studied germs of singular varieties in a symplectic space. And especially the so-called "ghost" symplectic invariants which are induced purely by singularity. Algebraic restrictions of differential forms to singular varieties are introduced and we show that the "ghost" invariant is exactly the invariant of the algebraic restriction of the symplectic form.

This follows from the generalization of Darboux-Givental theorem from non-singular submanifolds to arbitrary quasi-homogeneous varieties in a symplectic space. Using algebraic

restrictions we introduce new symplectic invariants and explain their geometric meaning. We prove that a quasi-homogeneous variety N is contained in a non-singular Lagrangian submanifold if and only if the algebraic restriction of the symplectic form to N vanishes. We show that the method of algebraic restriction is a powerful tool for various classification problems in a symplectic space. We illustrate this by complete solutions of symplectic classification problem for the classical A, D, E singularities of curves and for the regular union singularities.

8. Chung-Hyuk Kang Seoul National University

Title: Irreducibility criterion of the Weierstrass polynomials of two complex variables

Abstract:

In this talk, we will discuss a necessary and sufficient condition for the Weierstrass polynomials of two complex variables to be irreducible in, the ring of convergent power series at, using the following statements:

1、Write down the fundamental lemma for irreducibility criterion of any element in.

2、Find the construction of the standard Puiseux irreducible Weierstrass polynomials from the irreducible Weierstrass polynomials, using the similar method as it has been seen in the construction of the standard Puiseux expansions from the Puiseux expansions.

3、Write down a new theorem, called “The division algorithm for the Weierstrass polynomials”, which can be proved by the Weierstrass division theorem.

It can be clearly shown by the Weierstrass preparation theorem that to find irreducibility criterion of the Weierstrass polynomials of two complex variables is equivalent to find irreducibility criterion for germs of analytic functions of two complex variable which has been researched by S.S. Abhyankar and T.C. Kuo.

9. Henry Laufer Renaissance Technologies

Title: Seasonality in United States Home Prices

10. Ignacio Luengo Universidad Complutense de Madrid

Title: Common dicritical of jacobian pairs

A jacobian pair in the plane is a pair of polynomials (f, g) such that its jacobian is $\mathrm{jac}(f, g) = 1$ and $\emptyset = (f, g)$ is not an automorphism. We will describe the tree of common divisor of f and g for a jacobian pair and compute from this tree

$$\int_{\mathrm{K}} \chi\left(f^{-1}(\mathrm{t})\right) \mathrm{d}\chi$$

This computation allows us to prove that in certain cases there are no jacobian pairs.

11. Laurentiu Maxim University of Wisconsin-Madison

Title： Equivariant invariants of external and symmetric products of quasi-projective varieties

Abstract:

I will start by revisiting formulae for the generating series of genera of symmetric products (with suitable coefficients), which hold for complex quasi-projective varieties with any kind of singularities, and which include many of the classical results in the literature as special cases. Important specializations of these results include generating series for extensions of Hodge numbers and Hirzebruch genus to the singular setting and, in particular, generating series for intersection cohomology Hodge numbers and Goresky-MacPherson intersection cohomology signatures of symmetric products of complex projective varieties. In the second part of the talk, I will describe a generating series formula for equivariant invariants of external products, which includes all of the above-mentioned results as special cases. This is joint work with Joerg Schuermann.

12. Alejandro Melle Universidad Complutense de Madrid

Title: On irreducible free and nearly free divisors in the projective plane.

Abstract:

I will report on our work in progress on the study of irreducible free divisors and nearly free divisors in the projective plane.

The notion of free divisor introduced by K. Saito in the study of discriminants of versal deformations of isolated hypersurface singularites.

The notion of nearly free divisors have been recently introduced by A. Dimca and G. Sticlaru.

(This is a joint work with E. Artal, L. Gorrochategi and I.Luengo).

13. Andras Nemethi Alfréd Rényi Institute of Mathematics

Title： From Laufer's computation sequences to lattice cohomology

Abstract:

Computation sequences introduced by Laufer were key technical ingredients in the study of geometrical properties of rational and elliptic complex surface singularities. They make the ideal geometric choices among the abundant combinatorial pathes of integral cycles.

In the talk I would like to present the way how they lead naturally to the definition of the `graded roots',`path lattice cohomology' and `lattice cohomology' associated with any resolution graph of a normal surface singularity. Moreover, the generalizations of these sequences provide crucial theorems in lattice cohomology theory (like the `Reduction Theorem', where they are used in the construction of contractions and contractibility of some spaces associated with latticial cubes of integral cycles). In this way Laufer's sequences appear as (minimalising) flows in the space of cycles.

14. Leslie Saper Duke University

Title： L^2-cohomology of projective algebraic varieties

Abstract：

The intersection cohomology of a complex projective variety X agrees with the usual cohomology if X is smooth and satisfies Poincare duality even if X is singular. It has been conjectured in various contexts that the intersection cohomology may be represented by the L^2-cohomology of a Kaehler metric defined on the smooth locus of X. In this talk we review work in this area and report on some new work in progress.

15. Jose Seade The National Autonomous University of Mexico

Title: Remarks on Laufer's formula for the Milnor number.

Abstract:

The classical formula of Laufer for the Milnor number of a normal two dimensional hypersurface singularity expresses the Milnor number in terms of invariants of a resolution. In this talk we will discuss how this interesting formula relates to the classical Rohlin's signature theorem for manifolds of dimension 4, and discuss refinements for compact complex surfaces.

16. Shunsuke Takagi University of Tokyo

Title: On the tautness of F-singularities

Abstract:

F-singularities are classes of singularities in positive characteristic defined via the Frobenius morphism. I will explain a result of Y. Tanaka on the tautness of two-dimensional F-singularities.

17. Shengli Tan East China Normal University

Title： Singularities and topological invariants of an ordinary differential equation

Abstract:

The main purpose of this talk is to study the ordinary differential equation

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)}$$

by using Seidenberg’s resolution of singularities. As an application, we will first define some topological invariants, the Chern numbers and, for such an ODE on any algebraic surface. Poincaré and Painlevé proposed some problems on the algebraic integrability of these ODEs. We will give positive answers to these problems for ODEs with. Finally, we will discuss the birational and biregular classification of ODEs by using their Chern numbers.

18. Tadashi Tomaru School of Health Sciences, Gunma University

Title： The maximal ideal cycles over normal surface ingularities with C^*-action

Abstract:

Let $\pi: (\tilde{X}, E) \to (X, o)$ be a good resolution of a normal complex surface singularity, where $E = \bigcup_{i=1}^{r} E_i$ is theirreducible decomposition of E. The maximal ideal cycle is a cycle $M_E :=$ $min\,\{(f \circ \pi)_E | f \in m, f \neq 0\}$ on E(S.S.T. Yau [12]), where m is the maximal ideal of $O_{(x,o)}$ and $(f \circ \pi)_E = \sum_{i=1}^{r} v_{E_i}(f \circ \pi)E_i$ ($v_{E_i}(f \circ \pi)E_i$ is the vanishing order of $f \circ \pi$ on E_i). In [11], Ph. Wagrich showed that if $mO_{\tilde{x}}$ is locally principal, then $mO_{\tilde{x}} = O_{\tilde{x}}(-M_E)$ and $mult(X, o)$ is equal to $-M_E^2$. In the following, let M_0 and Z_0 be the maximal ideal (resp. fundamental) cycle for the minimal resolution. Then, $mult(X, o) \geq -M_0^2 \geq -Z_0^2$.

For the maximal ideal cycle M_E and the Artin's fundamental cycle Z_E, we have always $M_E \geq Z_E$ for any resolution. In [6], H. Laufer showed that if (X, o) is a hypersurface singularity defined by $z^2 = y(x^4 + y^6)$, then $M_0 \geq Z_0$ for the minimal resolution of (X, o). For rational singularities, M. Artin [1] showed that $mult(X, o) = -Z^2$(so $M_E = Z_E$ for any resolution). In his famous paper [5], H. Laufer introduced the notion of minimally elliptic singularities. For such singularities with$Z^2 \leq -2$, he proved that $mult(X, o) = -Z^2$ (so $M_E = Z_E$ f for any resolution). In [12], Yau introduced the notion of maximally elliptic singularities. In [7], M. Tomari generalized the Laufer's result above to maximally elliptic singularities.

For Kodaira singularities (defined by U. Karras [2]), we have always $M_0 = Z_0$. In [9], we proved two sufficient conditions to be Kodaira singularities for hypersurface singularities defined by $z^n = f(x, y)$ (also see [10]). Recently, K. Konno and D. Nagashima [3] gave a necessary and sufficient condition to be $M_0 = Z_0$ for Brieskorn hypersurface singularities. Also, F.N. Meng and T. Okuma [4] generalized it to Brieskron complete intersection singularities.

In last year, for normal surface singularities with $\mathbb{C}^*$-ACTION, Masataka Tomari and I [8] considered such problem and obtained some results. We report our results, and also explain that the importance of the maximal ideal cycle when we consider the sequences of singularities defined by $z^n = f(x, y)$ ($n = N_0, N_0 + 1, N_0 + 2, \ldots, where\ N_0 := ord(f)$).

REFERENCES

[1] M. Artin, On isolated rational singularities of surfaces, Amer. J. Math. 88(1963), 1129-138.

[2] U. Karras, On pencils of curves and deformations of minimally elliptic singularities, Math. Ann. 247(1980), 43-65.

[3] K. Konno and D. Nagashima, Maximal ideal cycles over normal surface singularities of Brieskorn type. Osaka J. Math. 49(2012), no.1, 225-245.

[4] F. N. Meng and T. Okuma, The maximal ideal cycles over Complete intersection surface singularities of Brieskorn type. Kyushu J. Math. 68(2014), no.1, 121-137.

[5] H. Laufer, On minimally elliptic singularities. Amer. J. Math. 99(1977), no.6, 1257-1295.

[6] H. Laufer, On normal two-dimensional double point singularities. Israel J. Math. 31(1978), no.3-4, 315-334.

[7] M. Tomari, A p_q-formula and elliptic singularities. Publ. Res. Inst. Math. Sci. 21(1985), no.2, 297-354.

[8] M. Tomari and T. Tomaru, The maximal ideal cycles over normal surface singularities with C^*-action (preprint)

[9] T. Tomaru, On Kodaira singularities defined by $z^n = f(x, y)$, Math. Z. 236(2001), 133-149.

[10] T. Tomaru, Pencil genus for normal surface singularities, J. Math. Soc. Japan, Vol.59(1), (2007), 35-80.

[11] Ph. Wagreich, Elliptic singularities of surfaces, Amer. J. Math. 92(1970), 421-454.

[12] S.S.-T. Yau, On maximally elliptic singularities, Trans. Amer. Math. Soc. 257(2), (1980), 269-329.

19. Chengyang Xu **Beijing University**

Title：dual complex of a singular pair

Abstract:

The dual complex is the combinatorial datum which characterizes how varieties intersect with each other. We will discuss the recent progress on studying it, especially the results which are obtained by using minimal model program.

20. Stephen Yau **Tsinghua University**

Title：Non-existence of Negative Weight Derivations on Isolated Singularities: Halperin, Wahl and Yau Conjectures.

Abstract:

Let $R = C\{x_1, x_2 \dots, x_n\}/(f)$ where f is a weighted homogeneous polynomial defining an isolated singularity at the origin. Then R and Der(R, R) are graded. It is well known that Der(R, R) does not have a negatively graded component. Wahl conjectured that this is still true for $R = C\{x_1, x_2 \dots, x_n\}/(f_1, f_2, \dots, f_m)$an isolated, normal and complete intersection singularity

and $f_1, f_2, \dots, f_m$ are weighted homogeneous polynomials with the same type $(w_1, w_2, \dots, w_n)$. Motivated by rational homotopy theory, Halperin conjectured that there are no negative weight derivations on R if the dimension of R is zero which is one of the most important open problems in rational homotopy theory. On the other hand, Yau conjectured that the moduli algebra $A(V) = C\{x_1, x_2, \dots, x_n\}/(f_{x_1}, f_{x_2}, \dots, f_{x_n})$ has no negatively weighted derivations, for any weighted homogeneous polynomial f defining an isolated singularity at the origin. By assuming the truth of this conjecture, he gave a characterization of weighted homogeneous hypersurface singularities using only the Lie algebra L(V) of derivations on A(V). We give a positive answer to the Halperin and Wahl Conjectures (without the condition of isolated complete intersection singularity) for any 'large' quasi-homogeneous isolated singularities (i.e. the degree of $f_i, 1 \leq i \leq m$ are bounded below by a constant). We also prove that the Yau Conjecture is true for all fewnomial singularities with multiplicity at least 5.

This is a joint work with Chen Hao and Huaiqing Zuo.

21. Qilin Yang Sun Yat-Sen University

Title：On Fano threefolds with semi-free C^*-actions, I

Abstract:

Let X be a Fano threefold and $C^* \times X \to X$ an algebraic action. Then X has a S^1-invariant Kähler structure and the corresponding S^1-action admits an equivariant moment map which is at the same time a perfect Bott-Morse function. We will initiate a program to classify the Fano threefolds with semi-free C^*-actions using Morse theory and the holomorphic Lefschetz fixed point formula as the main tools. In this paper we give a complete list of all possible Fano threefolds without "interior isolated fixed points" for any semi-free C^*-action. For the actions whose fixed point sets have only two connected components, and in a few other cases, we give the realizations of the semi-free C^*-actions.

This is a joint work with Dan Zaffran （dzaffran@fit.edu）.