TITLES AND ABSTRACTS

50 Mininutes' Report

1. Tonnesen-Friedman Christina, Union College

Title: Weighted Extremal Kähler metrics on Projective Bundles

Abstract: This talk, based on joint work with Apostolov and Maschler, will be concerned with weighted extremal Kähler metrics as introduced by Apostolov, Calderbank, Gauduchon, Legendre, and Maschler and developed further by Lahdili. These metrics are generalizing conformally Kähler, Einstein–Maxwell metrics or (conformally) Einstein-Maxwell Kähler metrics, as defined by LeBrun, Apostolov-Calderbank-Gauduchon, Apostolov-Maschler, and Futaki-Ono.

We work with admissible projective bundles, as defined in the joint work with Apostolov-Gauduchon-Calderbank. Much like the well-known story for Calabi's extremal Kähler metrics, we obtain a general existence result for weighted extremal metrics on such admissible manifolds, which yields, for example, many new examples of conformally Kähler, Einstein–Maxwell metrics, adding to the collection of explicit examples already obtained by Lebrun, Apostolov-Maschler, Futaki-Ono, and Koca-T-F.

For each admissible Kähler class on an admissible projective bundle, we associate an explicit function of one variable and show that if it is positive on the interval $(-1, 1)$, then there exists a weighted extremal Kähler metric in the given class. On the other hand, if it is strictly negative somewhere in $(-1, 1)$, there is no Kähler metric of constant weighted scalar curvature in that class. To see the last part, we use the relative Mabuchi energy, introduced by Lahdili, in the admissible setting.

Time permitting, we will also discuss a notion of weighted K-stability, thus establishing a Yau–Tian–Donaldson type correspondence for the existence of Kähler metrics of constant weighted scalar curvature in the rational admissible Kähler classes on an admissible projective bundle.

2. Nobuhiro Honda, TIT University

Title: Twistors, Quartics, and del Pezzo Fibrations

Abstract: I will talk about a recent result on algebraic description of a wide class of twistor spaces associated to anti-self-dual metrics on compact 4-manifolds. Each of these

twistor spaces is birational to the total space of a del Pezzo fibration over $\mathbb{CP}^1$, and may be described by a single quartic polynomial of a particular form. Generic fibers of the fibration are (possibly singular) del Pezzo surfaces of degree two. We also show that any Moishezon twistor space has at least one of the following structures, provided that the fundamental system (i.e. the half-anti-canonical system) of the twistor space is of positive dimen- sion:

- the above type (i.e. birational to a del Pezzo fibration over $\mathbb{CP}^1$),
- birational to conic bundles over minitwistor spaces associated to the twistor spaces of Joyce metrics,
- birational to conic bundles over $\mathbb{CP}^2$, investigated by Campana-Kreussler.

The second ones can be regarded as a generalization of the famous LeBrun twistor spaces.

3. Hisashi Kasuya, Osaka University

Title: DGA-Models of Variations of Mixed Hodge Structures

Abstract: A variation of mixed Hodge structures (VMHS) over a complex manifold M is a vary important object for complex algebraic or analytic geometry. The purpose of this paper is to give an algebraic model of variations of mixed Hodge structures over compact Kähler manifolds or their complements of normal crossing divisors.

In [5], extending Deligne's theory in [1] to a theory on differential graded algebra (DGA), Morgan tried to understand the mixed Hodge structures on the fundamental groups and homotopy groups of smooth algebraic varieties. More precisely Morgan studied DGA-models of de Rham complex (resp. logarithmic de Rham complex) of compact Kähler manifolds (resp. complements of normal crossing divisors), called *m*ixed Hodge diagrams. In this talk, we define VMHS-like objects $VMHS^u(\mathcal{D})$ over Morgan's mixed Hodge diagrams $\mathcal{D}$. The main purpose of this talk is to give a "fiber-taking"

$$VMHS^u(\mathcal{D}) \to MHS$$

like the usual fiber-taking $(\mathbf{E}, \mathbf{W}_*, \mathbf{F}^*) \mapsto (E_x, W_{*x}, F_x^*)$ of a VMHS $(\mathbf{E}, \mathbf{W}_*, \mathbf{F}^*)$ over a compact Kähler manifold M at a point $x \in M$. For this, we prove an equivalence between the category of our VMHS-like objects and the category of mixed Hodge representations of the dual Lie algebra of Sullivan's minimal model with Morgan's mixed Hodge structure an analogue of Hain–Zucker's equivalence ([2]) between unipotent variations of mixed Hodge structures and mixed Hodge representations of the fundamental group with Hain's mixed hodge structure.

References

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[2] R. M. Hain, S. Zucker, Unipotent variations of mixed Hodge structure. Invent. Math. **88** (1987), no. 1, 83–124.

[3] H. Kasuya, Techniques of constructions of variations of mixed Hodge structures. Geom. Funct. Anal. **28** (2018), no. 2, 393–442.
[4] H. Kasuya, DGA-Models of variations of mixed Hodge structures. preprint arXiv:1809.03716
[5] J. W. Morgan, The algebraic topology of smooth algebraic varieties. *Inst. Hautes Études Sci. Publ. Math.* No. **48** (1978), 137–204.

4. Claude LeBrun, Stony Brook University

Title: Mass, Kähler Manifolds, and Symplectic Geometry

Abstract: In the author's previous joint work with Hans-Joachim Hein, a mass formula for asymptotically locally Euclidean (ALE) Kähler manifolds was proved, assuming only relatively weak fall-off conditions on the metric. However, the case of real dimension 4 presented technical difficulties that led us to require fall-off conditions in this special dimension that are stronger than the Chruściel fall-off conditions that sufficed in higher dimensions. In this talk, I will explain how a new proof of the 4-dimensional case, using ideas from symplectic geometry, shows that Chruściel fall-off suffices to imply all our main results in any dimension. In particular, I will explain why our Penrose-type inequality for the mass of an asymptotically Euclidean Kähler manifold always still holds, given only this very weak metric fall-off hypothesis.

5. Eveline Legendre, Toulouse University

Title: The Einstein-Hilbert Functional and the Transversal Futaki Invariant

Abstract: We apply an equivariant localisation formula to the volume, the total scalar curvature and the Einstein-Hilbert functionals defined on the Reeb cone of a Sasaki manifold and prove that they are all proper. This implies that there exists a Reeb vector field with (transversal) Futaki invariant in any the Reeb cone.

6. Haizhong Li, Tsinghua University

Title: Inverse Mean Curvature Flow and Some Geometric Applications

Abstract: In this talk, we give some important properties of inverse mean curvature flow for hypersurfaces in space forms or some warped product Riemannian manifolds. By use of the properties of inverse mean curvature flow, we prove some geometric inequalities for such hypersurfaces.

7. Xiang-Dong Li, Academy of Mathematics and Systems Science, Chinese Academy of Sciences

Title: W-entropy Formulas on Super Ricci Flows and Langevin Deformation on Wasserstein Space over Riemannian Manifolds

Abstract: In this talk, we give an overview of some recent works on the study of the W-entropy for the heat equation of the Witten Laplacian on super-Ricci flows and the Langevin deformation on the Wasserstein space over Riemannian manifolds. Inspired by Perelman's seminal work on the Ricci flow, we establish the W-entropy formula for the heat equation of the Witten Laplacian on complete Riemannian manifolds with the $CD(K, m)$-condition and for the heat equation of the time dependent Witten Laplacian on compact manifolds equipped with (K, m)-super Ricci flows, where $m \in [n, \infty]$ and $K \in \mathbb{R}$. Furthermore, we prove an analogue of the W-entropy formula for the Wasserstein geodesic flow which corresponds to the Monge-Kantorovich optimal transportation problem with quadratic distance cost function on Riemannian manifolds. Our result improves a previous result due to Lott and Villani on the displacement convexity of the Boltzmann-Shannon type entropy on Riemannian manifolds with non-negative Ricci curvature. To better understand the similarity between above two W-entropy formulas, we introduce the Langevin deformation of geometric flows, which interpolate the geodesic flow and the gradient flows on the Wasserstein space over Riemannian manifolds, and derive the W-entropy formula for the Langevin deformation. Finally, we present some recent results on the study of Shannon entropy power inequalities for the heat equation and the porous medium equation on manifolds with the $CD(K, m)$ condition and with (K, m)-super Ricci flows. Joint work with Songzi Li.

8. Toshiki Mabuchi, Osaka University

Title: Stabilities in the Existence Problem of Extremal Metrics

Abstract: In this talk, we consider various stabilities on polarized algebraic manifolds from the viewpoints of the existence problem of extremal metrics. For Kähler-Einstein metrics, the Yau-Tian-Donaldson Conjecture was recently solved affirmatively by Chen-Donaldson-Sun and Tian. However, the conjecture is still open for extremal Kähler cases.

To study the conjecture for extremal Kähler cases, we often consider stronger versions of relative K-stability. Typical examples of such stabilities are uniform relative K-stability and strong relative K-stability. In this talk, by using such stabilities, we discuss extremal Kähler versions of the conjecture with emphasis on how to apply the peak section method to the problem.

9. Yuji Sano, Fukuoka University

Title: A Polar Dual to the Momentum of Toric Fano Manifolds

Abstract: On a toric Fano manifold, Futaki invariant gives the necessary and sufficient condition to the existence of Kähler-Einstein metrics (Wang-Zhu). On the other hand, the invariant is characterized as the momentum of the Delzant polytope P^* of the toric manifold (Mabuchi). In this talk, I introduce an invariant on the dual polytope P as a polar dual to the momentum of P^*.

10. Xujia Wang, Australian National University

Title: Locally Convex Surfaces and Related Topics

Abstract: In this talk we will develop some properties of locally convex surfaces, such as the uniform cone condition, and use them to study related problems, such as the existence of locally convex surfaces with constant Gauss curvature and the Euclidean completeness of locally convex surfaces. In particular we will consider the four vertex theorem for space curves. The classical four vertex theorem states that a planar Jordan curve contains at least four vertices. We show that a closed space curve contains four torsion zero points if it spans a locally convex surface which is a topological disc.

11. Xiaohua Zhu, Peking University

Title: Tian's $\alpha_{m,k}^{\hat{K}}$-invariants on Group Compactifications

Abstract: Tian's α-invariant plays an important role in the study of Kaehler-Einstein metrics on Fano manifolds. However, there is not so much work on $\alpha_{m,k}$-invariant. In this talk, we talk about how to compute $\alpha_{m,k}^{K\times K}$-invariant for any positive integer k on a polarized G-group compactification, where K denotes a connected maximal compact subgroup of G. We prove that Tian's conjecture for $\alpha_{m,1}^{K\times K}$-invariant is still true on such manifolds, but it is not true in general for $k \geq 2$ by showing encounter-examples.

30 Mininutes' Report

1. Yoshinori Hashimoto, Firenze-TIT University

Title: Variational Approach to the Kobayashi–Hitchin Correspondence and the Quot-scheme Limits

Abstract: The Kobayashi–Hitchin correspondence states that the existence of Hermitian–Einstein metrics on a holomorphic vector bundle is equivalent to an algebro-geometric stability condition, and was proved by Donaldson and Uhlenbeck–Yau. Their original proofs were based on technical and sophisticated applications of nonlinear PDE theory. We present

some results that clarify variational aspects of the Kobayashi–Hitchin correspondence for smooth projective varieties; they are based on the theory of Quot-schemes in algebraic geometry and rely much less on analysis. Joint work with Julien Keller.

2. Satoshi Nakamura, Tohoku University

Title: A Generalization of Kähler Einstein Metrics for Fano Manifolds with Non-vanishing Futaki Invariant

Abstract: The existence problem of Kähler Einstein metrics for Fano manifolds was one of the central problems in Kähler Geometry. The vanishing of the Futaki invariant is known as an obstruction to the existence of Kähler Einstein metrics. Generalized Kähler Einstein metrics (GKE for short), introduced by Mabuchi in 2000, is a generalization of Kähler Einstein metrics for Fano manifolds with non-vanishing Futaki invariant. In 2017, GKE metrics were rediscovered by Yao in the story of infinite dimensional GIT picture for Fano manifolds. In this talk, we give recent development on GKE metrics, in particular deformation theory of complex structures for Fano manifolds admitting GKE metrics.

3. Shunsuke Saito, Tohoku University

Title: Equivalence of Strengthenings of Relative K-stability for Polarized Toric Manifolds

Abstract: This talk will concern relations between algebro-geometric stabilities for polarized toric manifolds. I will first introduce several strengthenings of relative K-stability such as uniform stabilities and K-stability tested via general convex functions. Then I will prove the equivalence of all of them. This is based on an ongoing joint work with Yasufumi Nitta.

4. Ryosuke Takahashi, Tohoku University

Title: The Inverse Monge-Ampère Flow and Applications to Kähler-Einstein Metrics

Abstract: In this talk, we introduce the "inverse Monge-Ampère Flow", a new parabolic flow which is designed to deform a given Kähler metric to a Kähler-Einstein one, and fits Donaldson's new GIT picture. We provide some convergence results for the flow. This talk is based on a joint work with T. C. Collins (Harvard Univ.) and T. Hisamoto (Nagoya Univ.).

5. Hikaru Yamamoto, Tokyo University of Science

Title: ε-regularity Theorem for Line Bundle Mean Curvature Flows

Abstract: The line bundle mean curvature flow was recently defined by Jacob and Yau. It is a kind of nonlinear parabolic flows to obtain deformed Hermitian Yang–Mills metrics of a holomorphic line bundle over a Kähler manifold. In this talk I will give an ε-regularity theorem for the line bundle mean curvature flow which ensure the regularity of the flow when Gaussian density is sufficiently close to 1. To establish the theorem, I will provide a scale invariant monotone quantity and as the critical point of this quantity I will give a definition of self-shrinker solutions of line bundle mean curvature flows. A Liouville type theorem for self-shrinkers will be also given. It plays an important role in the proof of the ε-regularity theorem.

6. Jun Zhang, University of Michigan-Ann Arbor

Title: Interaction of Statistical Structure with Almost (Para-)Complex Structures

Abstract: A statistical manifold is one prescribed with a Riemannian metric g and a pair of torsion-free g-conjugate connections; the connection and metric is Codazzi-coupled. Statistical manifolds arise out of Information Geometry, i.e., geometric characterization of statistical (probability) models, machine learning algorithms, and other topics of information science. Assuming that a statistical manifold (of even dimension) further admits an almost complex or almost para-complex structure. We consider conjugate connections that admit torsion in the (para-)Hermitian setting, and demonstrate a quadruple of torsion-and curvature-carrying connections that are compatible with (para-)Kahler and (para-)Hermitian structures. (Collaborative work with Teng Fei @Columbia University and with Sergey Gregorian @University of Texas Grande Rio Valley)

7. Yingying Zhang, Tsinghua University

Title: Complex Deformation of Kähler-Einstein Manifolds

Abstract: In this talk, we will discuss the existence of the Kähler-Einstein metrics on a Fano manifold under the deformation of complex structures, this leads to the understanding of Weil-Petersson metric on the moduli space of Fano Kähler-Einstein manifolds. We will also talk about a plurisubharmonic function the Teichmullër space of Kähler-Einstein manifolds of general types. This is based on joint work with H.-D. Cao, X. Sun and S.-T. Yau.