Dynamic Programming and Splitting up Method

Alain Bensoussan, Binh Tran, Phillip Yam

Abstract

We develop methods of Machine Learning to solve Bellman equation in large dimensions. Among the techniques, we make use of splitting up, which is efficient to deal with the curse of dimensionality. We prove some convergence results and provide numerical results.

A splitting algorithm for the numerical approximation of rigid maps in paper folding theory

Alexandre Caboussat
Geneva School of Business Administration
University of Applied Sciences and Arts Western Switzerland (HES-SO),
1227 Carouge, Switzerland, Email : alexandre.caboussat@hesge.ch

Abstract

Orthogonal maps are used in paper folding theory to model so-called origami problems. The model consists of a system of first order implicitly nonlinear equations. We thus consider the Dirichlet problem for orthogonal maps that reads: find $\mathbf{u} : \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$\begin{cases} \nabla \mathbf{u} \in \mathcal{O}(2) & \text{in } \Omega, \\ \mathbf{u} = \mathbf{g} & \text{on } \partial\Omega. \end{cases}$$

where $\mathbf{g}$ is a given, sufficiently smooth, boundary data, and $\mathcal{O}(2)$ the space of orthonormal 2×2 matrix-valued functions. We advocate a variational approach for the numerical approximation of this problem. We introduce a suitable objective function to enforce the uniqueness of the solution. We look for the stationary solutions of the flow problem associated to the corresponding first-order conditions. The complexity of the system comes from the non-uniqueness and the low-regularity of the solutions, and the implicit character of the equations.

We rely on a strategy based on operator splitting in order to decouple the time-dependent problem into a sequence of local nonlinear problems and a global variational problem at each time step. Numerical experiments validate the accuracy and the efficiency of the method for various computational domains and meshes. Adaptive space discretization methods are introduced to better track singularities.

This is a joint work with D. Gourzoulidis, M. Picasso (EPFL, Switzerland), and R. Glowinski (Univ. of Houston, USA).

Subsampled trust-region methods for finite-sum minimization

Benedetta Morini
University of Firenze

Abstract

Convex and non-convex finite-sum minimization arises in many scientific computing and machine learning applications. Recently, second-order methods have received great attention due to their distinguishing features with respect to first-order methods: resilience to problem ill-conditioning and low sensitivity to parameter tuning.

We propose a new trust-region method which is more efficient than the standard scheme in terms of cost-per iteration as it can employ suitable approximations of the objective function, gradient and Hessian matrix built via random subsampling techniques. We discuss the local and global properties for finding approximate first and second-order optimal points and show results from the numerical experience.

This is a joint work with Stefania Bellavia (University of Firenze) and Nataša Krejić (University of Novi Sad).

Subspace acceleration of split Bregman methods for constrained fused lasso problems in portfolio optimization

Daniela di Serafino
Department of Mathematics and Physics
University of Campania “Luigi Vanvitelli”, Caserta, Italy

Joint work with
Valentina De Simone, University of Campania “Luigi Vanvitelli”, Italy
and Marco Viola, Sapienza University of Rome, Italy

Abstract

Regularization by fused lasso has been successfully applied in minimization problems modelling a variety of applications, to promote sparsity and smoothness in the solution. In this talk, we focus on constrained fused lasso problems of the following form, which arise, e.g., in multi-period portfolio optimization:

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}\mathbf{w}^T C\mathbf{w} + \tau_1\|\mathbf{w}\|_1 + \tau_2 \sum_{i=1}^{m-1} \|\mathbf{w}_{i+1} - \mathbf{w}_i\|_1, \\ \text{s.t.} & A\mathbf{w} = \mathbf{b}, \end{array}$$

where $\mathbf{w}_i \in \mathbb{R}^n$ for $i = 1, \ldots, m$, $\mathbf{w} = (\mathbf{w}_1^T, \ldots, \mathbf{w}_m^T)^T \in \mathbb{R}^{nm}$, $C \in \mathbb{R}^{nm\times nm}$ is symmetric positive definite, $A \in \mathbb{R}^{s\times nm}$ with $s < nm$, $\mathbf{b} \in \mathbb{R}^s$, $\tau_1 > 0$ and $\tau_2 > 0$. We propose an acceleration technique for split Bergman methods, based on second-order subspace minimization steps, where the subspaces are orthant faces identified by the zero entries of the current iterate. A condition based on suitable measures of optimality is used to decide when the acceleration is needed. Numerical experiments on multi-period portfolio selection problems using real data sets show the effectiveness of the proposed method.

On the Equivalence of Inexact Proximal ALM and ADMM for a Class of Convex Composite Programming

Defeng Sun
Department of Applied Mathematics, The Hong Kong Polytechnic University

Abstract

We show that for a class of linearly constrained convex composite optimization problems, an (inexact) symmetric Gauss-Seidel based majorized multi-block proximal alternating direction method of multipliers (ADMM) is equivalent to an *inexact* proximal augmented Lagrangian method (ALM). This equivalence not only provides new perspectives for understanding some ADMM-type algorithms but also supplies meaningful guidelines on implementing them to achieve better computational efficiency. Even for the two-block case, a by-product of this equivalence is the convergence of the whole sequence generated by the classic ADMM with a step-length that exceeds the conventional upper bound of $(1+\sqrt{5})/2$, if one part of the objective is linear. This is exactly the problem setting in which the very first convergence analysis of ADMM was conducted by Gabay and Mercier in 1976, but, even under notably stronger assumptions, only the convergence of the primal sequence was known. A collection of illustrative examples are provided to demonstrate the breadth of applications for which our results can be used. Numerical experiments on solving a large number of semidefinite programming problems are conducted to illustrate how the theoretical results established here can lead to improvements on the corresponding practical implementations. [This is a joint work with Liang Chen, Xudong Li, and Kim-Chuan Toh]

Nonlinear Preconditioning and the Acceleration of Nonlinear Fixed Point Iterations

Felix Kwok
Hong Kong Baptist University

Abstract

In multiphysics problems modelled by systems of nonlinear partial differential equations, the solution of the global discretized problem can be challenging for the nonlinear solver, because of the complex interaction between different dynamics and time scales caused by the various physical processes. For such problems, the use of generic nonlinear solvers, such as Newton's method with line search, may lead to convergence difficulties. Fixed point methods derived based on operator splitting and/or physical properties of the system can lead to more robust convergence; however, the resulting method typically exhibit convergence that is linear, if not slower.

In this talk, we explain how the same fixed point method can be used as a preconditioner to the underlying nonlinear problem; this allows the preconditioned problem to be solved using methods such as Newton or Anderson acceleration. Moreover, we show how the action of the Jacobian can be computed cheaply by reusing computations already performed during the fixed point step. We illustrate the effectiveness of our approach on two problems: the first one is based on the restricted additive Schwarz method, and the other from the simulation of porous media flow. These examples show how the nonlinearly preconditioned method achieves the quadratic convergence of Newton, while retaining the robustness of the underlying fixed point method.

Whiteness Constraints in a Unified Variational Framework for Image Restoration

Fiorella Sgallari
University of Bologna
Department of Mathematics, Via Saragozza 8, I-40123 Bologna -Italy
E-mail: fiorella.sgallari@unibo.it

Abstract

A real captured image may be distorted by many expected or unexpected factors among which blur and random noise are typical and often unavoidable examples. Hence, image deblurring and denoising are fundamental tasks in the field of image processing. Over the years, one of the most studied class of noises is that of additive, independent identically distributed (in short i.i.d.) noises, which affect all the pixels by independent random corruptions coming from the same distribution. This class includes important noises such as those characterized by Gaussian, Uniform, Laplacian and Cauchy distributions, which can be found in many applications, such as e.g. medical and astronomic imaging. For any of these noise distributions, ad hoc variational models have been devised in the past for image restoration.

However, in many practical applications it is difficult to know a priori the noise distribution and, in any case, the noise might be the outcome of several sources thus giving raise to mixed noise models with very specific/complex distributions.

To overcome these inherent difficulties, in this talk we discuss a robust variational model aimed at restoring images corrupted by blur and by the generic wide class of additive white - or uncorrelated - noises, which include i.i.d noises. The key idea behind our proposal relies on a novel hard constraint imposed on the residual of the restoration, namely we characterize a residual whiteness set to which the restored image must belong. As the feasible set is unbounded, solution existence results for the proposed variational model are given. Moreover, based on theoretical derivations as well as on Monte Carlo simulations, we provide well-founded guidelines for setting the whiteness constraint limits. The solution of the nontrivial optimization problem, due to the non-smooth non-convex proposed model, is efficiently obtained by an alternating directions method of multipliers, which in particular reduces the solution to a sequence of convex optimization subproblems. Numerical results show the potentiality of the proposed model for restoring blurred images corrupted by several kinds of additive white noises.

Joint work with Alessandro Lanza, Serena Morigi and Federica Schiacchitano.

Discrete total variation based cone-beam reconstruction for axially symmetric object tomography

Haibo Xu
Institute of Applied Physics and Computational Mathematics, Beijing, China

Abstract

In this paper, we consider 2D tomographic reconstruction for axially symmetric objects from a single radiograph formed by cone-beam X-rays. All contemporary density inversion methods in high-energy flash radiography are based on the assumption that the cone beam can be treated as fan beam located at parallel planes perpendicular to the symmetric axis, so that the density of the whole object can be recovered layer by layer. Considering the relationship between the upper and lower layers, we construct the cone-beam global reconstruction to solve the ambiguity effect at material interfaces of the inversion results. In view of the anisotropy of classical discrete total variations, a new discretization of total variation which yields sharp edges and has better isotropy is introduced in our reconstruction model. The optimization problem is solved using the alternating proximal gradient method which has been recently applied in image processing. Density reconstruction results are presented for MCNP simulated radiographs, which shows that the proposed method has improved the preservation of edge location and the accuracy of the density level when compared with the well-known isotropic total variation regularization method.

A Two Stage Method for Color Image Segmentation Based on SVM and Variation Models

Haixia Liang
Xi'an Jiaotong-Liverpool University

Abstract

In this talk, I will present our work on color image segmentation. The work is based on Prof. Raymond Chan' s two papers. One is on color image segmentation and another one is on hyperspectral imaging. We combine them together and propose a two stage method for color image segmentation based on SVM and Variaition models.

CT image reconstruction on incomplete data

Hongwei Li
Capital Normal University, Beijing

Abstract

Limited-angle computed tomography is a very challenging problem in applications. Due to high degree of ill-posedness, conventional reconstruction algorithms will introduce blurring along the directions perpendicular to the missing projection lines, as well as streak artifacts when applied on limited-angle data. Various models and algorithms have been proposed to improve the reconstruction quality by incorporating priors, among which the total variation, i.e. l_1 norm of gradient, and l_0 norm of gradient are the most popular ones. These models and algorithms partially solve the blurring problem under certain situations. However, the fundamental difficulty remains. We propose a reconstruction model for limited-angle computed tomography, which incorporates two regularization terms that play the role of edge-preserving diffusion and smoothing, along the x-direction and y-direction respectively. Then, an alternating minimization algorithm, which is motivated physically rather than mathematically, is proposed to solve the model approximately. The proposed model and algorithm are based on the theory of visible and invisible singularities of limited-angle data, developed by Quinto et al. By incorporating visible singularities as priors into an iterative procedure, the proposed algorithm could produce promising results and outperforms state-of-the-art algorithms for certain limited-angle computed tomography applications. Extensive experiments on both simulated data and real data are provided to validate our model and algorithm.

Sparse Regularization via Convex Analysis

Ivan Selesnick
New York University

Abstract

Sparse approximate solutions to linear equations are often obtained via L1 norm regularization, but the L1 norm tends to underestimate the non-zero components of sparse solutions. We introduce a non-convex alternative to the L1 norm. Unlike other non-convex regularizers, the proposed regularizer maintains the convexity of the objective function to be minimized. This leads to more accurate estimates of the non-zero components of sparse solutions without sacrificing the convexity of the optimization problem. We also consider the problem of total variation denoising using non-convex regularization that maintains the convexity of the objective function. Although the new regularizer is non-convex, it is defined using tools of convex analysis. In particular, it is defined using a generalization of the Moreau envelope. The resulting optimization problem can be performed by proximal algorithms.

The SAV approach for complex nonlinear systems

Jie Shen
Purdue University and Xiamen University

Abstract

We present in this talk the scalar auxiliary variable (SAV) approach to deal with complex nonlinear dissipative or conservative systems. The technique leads to linear and unconditionally energy stable schemes which only require solving decoupled linear equations with constant coefficients. Hence, these schemes are extremely efficient as well as accurate, and rigorous convergence and error analyses are also established.

In particular, we shall combine this approach and a operator-splitting approach (in the form of pressure-correction) for Navier-Stokes equations to develop decoupled, linear and unconditionally energy stable schemes for the phase-field models of incompressible -multiphase flows.

Solving systems of quadratic equations: Manifold algorithms and global geometry

Ke Wei
Fudan University

Abstract

A Riemannian gradient descent algorithm and a truncated variant will be presented for solving systems of phaseless equations $|Ax|^2 = y$. The algorithms are developed by exploiting the inherent low rank structure of the problem based on the embedded manifold of rank-1 positive semidefinite matrices. Theoretical recovery guarantee has been established for the truncated variant, showing that the algorithm is able to achieve successful recovery when the number of equations is proportional to the number of unknowns. In addition, we will present a loss function without spurious local minima when the sampling complexity is optimal.

ReProducible Kernel Hilbert Space (RKHS) and Heaviside Functions in Image Fusion

Liangjian Deng
University of Electronic S&T of China

Abstract

In this talk, we present an iterative scheme to address the problem of pansharpening that is to fuse a high-resolution panchromatic image and a low-resolution multispectral image to get a high-resolution multispectral image. We solve the problem from image intensity function estimation perspective and assume the image contains smooth and edge components. We model the smooth components of an image using a thin-plate reproducing kernel Hilbert space (RKHS) and the edges using approximated Heaviside functions. The proposed method is applied to image patches, aiming to reduce computation and storage. Visual and quantitative comparisons with some competitive approaches show the effectiveness of the proposed framework.

A Localized Higher-Order Regularization Model for Simultaneous Image Reconstruction and Bias Field Correction

LingHai kong
Institute of Applied Physics and Computational Mathematics

Abstract

A preprocessing method of simultaneously treating intensity inhomogeneity and mixed Laplace-Gaussian noise is proposed.

Under some assumptions on the bias field, an inhomogeneous image model

$$f(x) = b(x)(Hu(x) + n(x)), x \in \Omega \tag{1}$$

is considered, where $f, b, u, n : \Omega \to \mathbb{R}$ represent the observation, the bias field, the original image and the mixed Laplace-Gaussian noise with PDF

$$p(z; \Theta_0) = \sum_{k=1}^{2} \gamma_k p_k(s; \sigma_k^2), \tag{2}$$

respectively,

$$p_1(s; \sigma_1^2) = \frac{1}{2\sigma_1^2} \exp\left(\frac{-|s|}{\sigma_1^2}\right), \tag{3}$$

$$p_2(s; \sigma_2^2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(\frac{-|s|^2}{2\sigma_2^2}\right), \tag{4}$$

and H denotes a degeneration operator related with the imaging process. An iterative higher order regularization model

$$\min_{\theta,u} \Psi(\theta, u, \varphi) + \Phi(\nabla u, \nabla^2 u) \tag{5}$$

is proposed, where $\varphi = (\varphi_1, \varphi_2)$ belongs to

$$\Delta_+ = \left\{ \varphi(x) = (\varphi_1(x), \varphi_2(x)) | 0 < \varphi_i(x) < 1, \sum_{i=1}^{2} \varphi_i(x) = 1, x \in \Omega \right\},$$

$$\begin{aligned} \Psi(\theta, u, \varphi) \quad = \quad & \int_\Omega \int_\Omega G_\sigma(s-x) \left(\frac{\varphi_1 |f(x)/b(s) - Hu(x)|}{\sigma_1^2} + \frac{\varphi_2 |f(x)/b(s) - Hu(x)|}{2\sigma_2^2} \right) dsdx - \\ & \int_\Omega \int_\Omega G_\sigma(s-x) \left(\varphi_1 \ln \frac{\gamma_1}{2\sigma_1^2 b(s)} + \varphi_2 \ln \frac{\gamma_2}{b(s)\sqrt{2\pi\sigma_2^2}} - \sum_{i=1}^{2} \varphi_i \ln \varphi_i \right) dsdx, \end{aligned}$$

and

$$\Phi(\nabla u, \nabla^2 u) = \int_\Omega \left(\lambda_1 |\nabla u| + \lambda_2 |\nabla^2 u| \right) dx.$$

G_σ is regarded as the localized weighting function, $\lambda_i, i = 1, 2$ are spacially adaptive parameters.

The algorithm to be analyzed for the proposed model is based on the variable splitting method and the alternating direction method of multipliers. An adaptive soft-shrinking formulation is advanced, by which an alternating minimization algorithm is further established with some modification to derive anisotropic diffusion.

A flexible space-variant directional regularisation for image restoration problems

Luca Calatroni
CMAP, École Polytechnique, Palaiseau, France
joint work with: A. Lanza, M. Pragliola, F. Sgallari (University of Bologna, Italy)

Abstract

We consider a space-variant and directional variational regularisation term for image restoration problems based on the statistical assumption that the gradient magnitude of the objective image is distributed at any pixel according to a bivariate generalised Gaussian distribution. Compared to standard TV regularisation, this new regulariser is highly flexible as it is defined in terms of several space-variant parameters describing local image geometry and orientation preferences via a combination of an anisotropic and a variable-exponent modelling possibly making the model non-convex . For an automatic estimation of such parameters, we propose a robust maximum likelihood approach and verify empirically its reliability on synthetic and natural imaging data. The numerical solution of the corresponding image restoration model by means of the Alternating Direction Method of Multipliers (ADMM) for which the efficient calculation of a non-convex proximal step is required. The proposed model outperforms related competing methods in terms of image quality and, due to its directional behaviour, strongly improves reconstruction qualities in terms of texture and detail preservation.

Structure preserving preconditioning for frame-based image deblurring

Marco Donatelli
University of Insubria, Como, Italy.

Abstract

Regularizing preconditioners for accelerating the convergence of iterative regularization methods for inverse problems have been extensively investigated in the literature. For inverse problems, a correct choice of the preconditioner is crucial for accelerating the convergence of iterative methods without to spoil the quality of the computed approximation. Classical regularizing preconditioners for image deblurring are usually based on the circular convolution, which is equivalent to impose periodic boundary conditions.

We show that a preconditioner preserving the same structure of the convolution operator can be more effective than the circulant approach both for the quality of the restoration and the robustness of the regularization parameter, in particular when the choice of appropriate boundary conditions is crucial. We explore the use of a structure preserving preconditioner for a modification of the linearized Bregman iteration applied to the image deblurring problem with a ℓ_1-norm regularization term in the wavelet domain.

Unfortunately, the theoretical results proved for the circulant preconditioning can not be easily carry on to the structured preconditioning, but a large number of numerical experiments shows the effectiveness of our preconditioning strategy.

Subjective parameter choice for total variation denoising of real noisy images

Markus Juvonen
University of Helsinki

Abstract

Variational methods for image denoising require the choice of a regularization parameter that balances the regularization and data-fidelity terms. The correct selection of this parameter is crucial to obtain a good result that preserves image details while removing unwanted noise. We compare a subjective choice for regularization parameters with an objective methods such as the S-curve method in the case of total variation denoising. As our data we use natural images with different levels of real noise.

Controlling Backward Diffusion Using Asynchronous Time-Stepping

Martin Welk
Private University of Health Sciences, Medical Informatics and Technology, Hall/Tyrol, Austria

Abstract

Nonlinear diffusion is a well-established tool for structure-preserving image denoising. Directional and local splitting schemes are well-established tools for the numerical evaluation of nonlinear diffusion.

Traditionally, diffusivities are assumed to be nonnegative. Nevertheless, some of these processes achieve edge enhancement which can be associated with a backward diffusion component in the overall evolution. Transcending the established framework, some recent studies have therefore turned to consider nonlinear diffusion models in which diffusivities can take negative values in some regions.

The talk addresses the stable numerical evaluation of these processes. A time-explicit scheme with a nonstandard spatial discretisation can be used for this purpose. Unfortunately, it requires prohibitively small time step sizes. A way out is offered by spatially adaptive time stepping. The resulting asynchronous updates at different grid locations can be understood as a refined spatio-temporal splitting. Two realisations of such schemes are discussed, demonstrating impressive speed-up factors over conventional synchronous time-stepping.

This talk is based on joint work with Joachim Weickert, Saarland University, Saarbrücken, Germany, and Guy Gilboa, Technion, Haifa, Israel.

(Pre)Dualization, Dense Embeddings of Convex Sets, and Applications in Image Processing

Michael Hintermüller
Weierstrass Institute for Applied Analysis and Stochastics Berlin

Abstract

Variable splitting schemes for the function space version of the image reconstruction problem with total variation regularization (TV-problem) in its primal and pre-dual formulations are considered. For the primal splitting formulation, while existence of a solution cannot be guaranteed, it is shown that quasi-minimizers of the penalized problem are asymptotically related to the solution of the original TV-problem. On the other hand, for the predual formulation, which depends on dense embeddings of convex intersections, a family of parametrized problems is introduced and a parameter dependent contraction of an associated fixed point iteration is established. Moreover, the theory is validated by numerical tests. Additionally, the augmented Lagrangian approach is studied, details on an implementation on a staggered grid are provided and numerical tests are shown.

A Peaceman-Rachford Splitting Method with Monotone plus skew-symmetric splitting for Nonlinear Saddle Point Problems

Michael K. Ng
Hong Kong Baptist University

Abstract

This talk is devoted to solving the linearly constrained convex optimization problems by the Peaceman-Rachford splitting method with monotone plus skew-symmetric splitting on KKT operators. This approach generalizes the Hermitian and skew-Hermitian splitting method (HSS), an unconditionally convergent algorithm for non-Hermitian positive semi-definite linear systems, to the nonlinear scenario. Theoretical and numerical results are presented to demonstrate the usefulness of the proposed approach.

On Glowinski's Open Question of Alternating Direction Method of Multipliers

Min Tao
Department of Mathematics, Nanjing University

Abstract

The alternating direction method of multipliers (ADMM) was proposed by Glowinski and Marrocco in 1975; and it has been widely used in a broad spectrum of areas, especially in some sparsity-driven application domains. In 1982, Fortin and Glowinski suggested to enlarge the range of the step size for updating the dual variable in ADMM from 1 to $(0, \frac{1+\sqrt{5}}{2})$; and this strategy immediately accelerates the convergence of ADMM for most of its applications. Meanwhile, Glowinski raised the question of whether or not the range can be further enlarged to $(0, 2)$; this question remains open with nearly no progress in the past decades. In this paper, we answer this question affirmatively for the case where both the functions in the objective are quadratic. Glowinski's open question is thus partially answered. We further establish the global linear convergence of the ADMM with the step size range $(0, 2)$ for the quadratic programming case under a condition that turns out to be tight.

This is a joint work with Xiaoming Yuan from The University of Hong Kong.

Self Functional Maps

Oshri Halimi, Ron Kimmel
Technion - Israel Institute of Technology

Abstract

A classical approach for surface classification is to find a compact algebraic representation for each surface that would be similar for objects within the same class and preserve dissimilarities between classes. We introduce Self Functional Maps as a novel surface representation that satisfies these properties, translating the geometric problem of surface classification into an algebraic form of classifying matrices. The proposed map transforms a given surface into a universal isometry invariant form defined by a unique matrix. The suggested representation is realized by applying the functional maps framework to map the surface into itself. The key idea is to use two different metric spaces of the same surface for which the functional map serves as a signature. Specifically, in this paper, we use the regular and the scale invariant surface laplacian operators to construct two families of eigenfunctions. The result is a matrix that encodes the interaction between the eigenfunctions resulted from two different Riemannian manifolds of the same surface. Using this representation, geometric shape similarity is converted into algebraic distances between matrices.

A Novel Optimization Approach to Fictitious Domain Methods

Daniel Agress and Patrick Guidotti
University of California, Irvine

Abstract

A new approach to the solution of boundary value problems within the so-called *fictitious domain methods* philosophy is proposed which avoids well known shortcomings of other fictitious domain methods, including the need to generate extensions of the data. The salient feature of the novel method, which we refer to as SSEM (Smooth Selection Embedding Method), is that it reduces the whole boundary value problem to a linear constraint for an appropriate optimization problem formulated in a larger simpler set containing the domain on which the boundary value problem is posed and which allows for the use of straightforward discretizations. The proposed method in essence computes a (discrete) extension of the solution to the boundary value problem by selecting it as a smooth element of the complete affine family of solutions of the extended, yet unmodified, under-determined problem. The actual regularity of this extension is determined by that of the analytic solution and the choice of objective functional. Numerical experiments will demonstrate that it can be stably used to efficiently deal with non-constant coefficients, general geometries, and different boundary conditions in dimensions $d = 1, 2, 3$ and that it produces solutions of tunable (and high) accuracy.

ON THE SOLUTION OF NONLINEAR EIGENVALUE PROBLEMS BY OPERATOR-SPLITTING METHODS

Roland Glowinski

University of Houston & Hong-Kong Baptist University

Abstract. The main goal of this lecture is to discuss the numerical solution of ***nonlinear eigenvalue problems*** associated with elliptic, possibly multivalued, operators. A simple example of such problems is given by

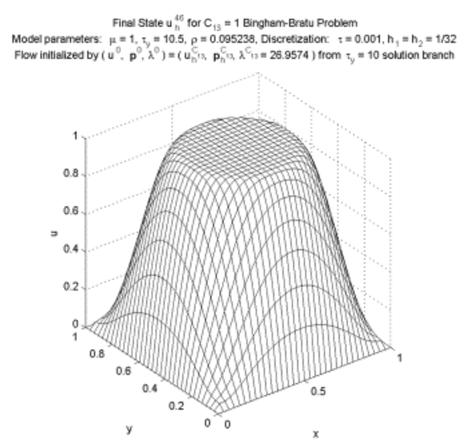
(NLEVP)
$$\begin{cases} \begin{cases} -\nabla^2 u + \partial j(u) \ni \lambda u \text{ in } \Omega, \\ u = g \text{ on } \partial\Omega, \end{cases} \\ \int_\Omega |u|^2 dx = U^2 |\Omega|, \end{cases}$$

where: (i) $\partial j(u)$ denotes the sub-differential at u of j a functional that we suppose convex, proper and lower semi-continuous, and (ii) $|\Omega|$ = measure of Ω.

The solution of these nonlinear eigenvalue problems is made (almost) trivial by applying an operator-splitting scheme to the time-discretization of an initial value problem associated with the eigenvalue problem under consideration. The results of numerical experiments validating the above approach will be presented. They include results for *Monge-Ampère* related eigenvalue problems such that

(MAEV)
$$\begin{cases} \begin{cases} \det \mathbf{D}^2 u = \lambda \varphi(u) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \\ \int_\Omega \Phi(u) = C, \end{cases}$$

where φ is a continuous increasing function from $\mathbf{R}$ into $\mathbf{R}$, $\Phi' = \varphi$, and C is a constant.



Visualization of a solution of $-\mu\nabla^2 u + \tau_y \partial j(u) \ni \lambda e^u$ in $\Omega, u = 0$ on $\partial\Omega$ ($j(v) = \int_\Omega |\nabla v| dx$).

Shape matching via quasiconformal maps

Ronald Lok Ming Lui
Department of Mathematics
The Chinese University of Hong Kong

Abstract

Shape matching is a process of finding meaningful dense correspondences between 3D shapes. It has important applications in imaging, computer graphics and visions. Quasiconformal theories provide a useful tool to compute shape matching with high accuracy, even if two shapes differ by a large deformation. In this talk, we will explore how quasiconformal maps can be used to tackle different shape matching problems, including the computation of large deformation shape registration, inconsistent shape matching and folded deformation. This work is support by HKRGC GRF (Project ID: 14304715).

Regularizer split for the splitting of scalar fields over surfaces

Serena Morigi
Department of Mathematics
University of Bologna
Italy

Abstract

The decomposition of images as well as signals is a widely used tool in data processing applications such as data coding, analysis and synthesis. In such problems it is typically assumed that the given signal is a weighted combination of different source signals with specific homogeneous characteristics. We focus on the decomposition of noisy scalar functions defined over an arbitrary topology 2-manifold embedded in R^3, where the underlying data may contain jump discontinuities that separate smoothly varying regions. We present a Convex-NonConvex variational approach where the split of the regularizer in the objective function and the optimization splitting strategy adopted lead to efficient and efficacy solutions, without renouncing to convexity of the total energy functional.

Operator-Splitting Based Methods for Hamilton-Jacobi Type Equations

Shingyu Leung
The Hong Kong University of Science and Technology

Abstract

We present two applications of the operator-splitting method. In the first part of the talk, we consider wave propagation in an isotropic acoustic medium in a moving fluid governed by an anisotropic eikonal equation. Since this anisotropic eikonal equation is associated with an inhomogeneous Hamiltonian, most of existing anisotropic eikonal solvers are either inapplicable or of unpredictable behavior in convergence. Realizing that this anisotropic eikonal equation is defined by a sum of two well-understood first-order differential operators, we propose novel operator-splitting based fast sweeping methods to solve this generalized eikonal equation. After applying the operator-splitting strategy, each splitting step corresponds to a much simpler Hamilton-Jacobi (HJ) equation so that we can apply the Lax-Friedrichs sweeping method to solve these splitted equations efficiently and easily. In the second part, we try to understand effective Hamiltonians quantitatively for the homogenization of HJ equations. We develop a simple efficient operator-splitting method for computing effective Hamiltonians when the Hamiltonian is either convex or nonconvex in the gradient variable. To speed up our Lie scheme-based operator-splitting method, we further propose a cascadic initialization strategy so that the steady state of the underlying time-dependent HJ equation can be reached more rapidly. This is a joint work with Prof. Roland Glowinski (UH) and Prof. Jianliang Qian (MSU).

First Order Methods Beyond Convexity and Lipschitz Gradient Continuity with Applications to Quadratic Inverse Problems

Shoham Sabach
Technion - Israel Institute of Technology

Abstract

We focus on nonconvex and nonsmooth minimization problems with a composite objective, where the differentiable part of the objective is freed from the usual and restrictive global Lipschitz gradient continuity assumption. This long-standing smoothness restriction is pervasive in first order methods, and was recently circumvented for convex composite optimization by Bauschke, Bolte and Teboulle, through a simple and elegant framework which captures, all at once, the geometry of the function and of the feasible set. Building on this work, we tackle genuine nonconvex problems. We first complement an extend their approach to derive a full extended descent lemma by introducing the notion of smooth adaptable functions. We then consider a Bregman-based proximal gradient method for the nonconvex composite model with smooth adaptable functions, which is proven to globally converge to a critical point under natural assumptions on the problem's data, and, in particular, for semi-algebraic problems. To illustrate the power and potential of our general framework and results, we consider a broad class of quadratic inverse problems with sparsity constraints, which arises in many fundamental applications, and we apply our approach to derive new globally convergent schemes for this class.

The talk is based on joint work with Jerome Bolte (Toulouse), Marc Teboulle (TAU) and Yakov Vaisbroud (TAU).

Convex shape priori with level set representation

Shousheng Luo
Henan University and CSRC

Abstract

For many applications, we need to use techniques to represent convex shapes and objects. In this work, we use level set method to represent shapes and find a necessary and sufficient condition on the level set function to guarantee the convexity of the represented shapes. We take image segmentation as an example to apply our techniques. Efficient numerical algorithm is developed to solve the variational model. In order to improve the performance of segmentation for complex images, we also incorporate landmarks into the model.

One option is to specify points that the object boundary must contain. Another option is to specify points that the foreground (the object) and the background must contain. Numerical experiments on different images validate the efficiency of the proposed models and algorithms. We want to emphasis that the proposed technique could be used for general shape optimization with convex shapes. For other applications, the numerical algorithms need to be extended and modified.

This work is complete under the supervison of Prof. Xuecheng Tai and Prof. Yang Wang.

Besov semi-norm in image processing

Stacey Levine
Duquesne University, Pittsburgh, PA, USA

Abstract

Besov space semi-norms have a natural use as a regularizer in variational problems used in signal processing given both their 'closeness' to the bounded variation semi-norm as well as their connection with wavelets. Furthermore, the $B^1_\infty(L_1(I))$ Besov semi-norm prefers planar regions to staircases, potentially making them a desirable regularizer for recovering piecewise affine data, and allows for functions containing 'fractal-like' rough regions. Equivalent Besov space semi-norms can be defined in terms of moduli of smoothness or sequence norms of coefficients in appropriate wavelet expansions. Existing algorithms for wavelet-based semi-norms in variational problems for processing image data do not preserve edges and many result in blocky artifacts. Here we present algorithms that utilize operator splitting using moduli of smoothness for the $B^1_\infty(L_1(I))$ semi-norm. The novelty in this work is a new algorithm that incorporates a translation invariant Besov regularizer that does not depend on wavelets. The algorithm naturally exposes a range of scales that depend on the image data, noise level, and the smoothing parameter. Numerical results demonstrate properties of solutions obtained from this moduli of smoothness based regularizer. We also analyze the norms of smooth, textured, and random Gaussian noise data in $B^1_\infty(L_1(I))$, $B^1_1(L_1(I))$, $BV(I)$ and $L^2(I)$ and their dual spaces, motivating alternative and complementary approaches that incorporate geometry and learning based algorithms.

The Inverse Problem Approach for x-ray Radiograph Tomography

Suhua Wei
Institute of Applied Physics and Computational Mathematics, Beijing 100094, China

Abstract

X-ray is an excellent tool to pear into the interior of an object. Image reconstruction means to inverse the object physical parameters from x-ray radiographs. This is an inverse problem. In this talk, the ill-posedness of the inverse problem will be analyzed and the mathematical model will be given based on variational method. The mathematical model deals with Abel transform inversion and the determination of the indirectly measured data. Two terms of regularizers are introduced in our model. Numerical tests show the efficiency of the proposed model.

IMEX HDG-DG: A coupled implicit hybridized discontinuous Galerkin and explicit discontinuous Galerkin approach for shallow water systems

Tan Bui-Thanh
The University of Texas at Austin

Abstract

We propose IMEX HDG-DG schemes for planar and spherical shallow water systems. Of interest is subcritical flow, where the speed of the gravity wave is faster than that of nonlinear advection. In order to simulate these flows efficiently, we split the governing system into a stiff part describing the gravity wave and a non-stiff part associated with nonlinear advection. The former is discretized implicitly with the HDG method while an explicit Runge-Kutta DG discretization is employed for the latter. The proposed IMEX HDG-DG framework: 1) facilitates high-order solutions both in time and space; 2) avoids overly small time-step sizes; 3) requires only one linear system solve per time stage; and 4) relative to DG generates smaller and sparser linear systems while enabling further parallelism. Numerical results for various test cases demonstrate that our methods are comparable to explicit Runge-Kutta DG schemes in terms of accuracy, while allowing for much larger time step sizes.

Convex and Non-convex Optimization in Image Recovery and Segmentation

Tieyong Zeng
The Chinese University of Hong Kong

Abstract

In this talk, we present some recent progress on variational approaches for image recovery and segmentation. First, a new convex variational model for restoring images degraded by blur and Rician noise is proposed. Based on the statistical property of the noise, a quadratic penalty function technique is utilized to obtain a strictly convex model under mild condition, which ensures the uniqueness of the solution and the stabilization of the algorithm. Numerical results are presented to demonstrate the good performance of our approach. The idea of convex relaxation is then extended to other image recovery and segmentation tasks. Finally, we also discuss the image recovery issue in the framework of dictionary learning if time permitted.

An efficient Peaceman-Rachford splitting method for constrained TGV-shearlet-based MRI reconstruction

Tingting Wu
Nanjing University of Posts and Telecommunications, Nanjing, 210023, China

Abstract

As a fundamental application of compressive sensing, magnetic resonance imaging (MRI) can be efficiently achievable by exploiting fewer k-space measurements. In this paper, we propose a constrained total generalized variation and shearlet transform based model for MRI reconstruction, which is usually more undemanding and practical to identify appropriate trade-offs than its unconstrained counterpart. The proposed model can be facilely and efficiently solved by the strictly contractive Peaceman-Rachford splitting method, which generally outperforms some state-of-the-art algorithms when solving separable convex programming. Numerical simulations demonstrate that the sharp edges and grainy details in magnetic resonance images can be well reconstructed from the under-sampling data.

Towards Sparse Extreme Minimal Learning Machine

Tommi Kärkkäinen
University of Jyväskylä
Faculty of Information Technology

Abstract

In [1], Kärkkäinen and Glowinski proposed a new operator-splitting, Douglas-Rachford method for solving convex and nonconvex, nonsmooth optimization problems resulting from supervised machine learning with random basis. The purpose was to advocate sparsity of the models. Then, in [2], Kärkkäinen proposed a novel machine learning method referred as Extreme Minimal Learning Machine (EMLM), whose construction uses distance-based basis with random selection of the so-called reference points. In this talk, I consider combination of these two methodological premises and describe preliminary results from attempts to enforce sparsity in the EMLM models.

References

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Non-Convex Optimization Problem in Multi-Spectral Computed Tomography

Xiaochuan Pan
The University of Chicago

Abstract

X-ray computed tomography (CT) refers to a broad class of tomographic imaging techniques that has impact profoundly medicine and other disciplines. It is perhaps fair to say that it is the success of CT that has inspired at least in part the development of other advanced tomographic imaging techniques such as magnetic resonance imaging and molecular imaging techniques. Numerous mathematical problems derived in CT imaging remain topics of interest in the community of applied mathematics. While the current CT technology can yield 3D images of the imaged subject, it uses no spectral information of X-ray source. The appropriate use of the spectral information can minimize numerous types of artifacts in current CT images and, more importantly, engender additional contrast information of high value to medical and other applications. In the presentation, I will discuss recent advances in the development of a non-convex optimization-based image reconstruction (OBIR) method for accurate image reconstruction in multi-spectral CT. Following the discussion of the method design, the effectiveness of the non-convex OBIR method will be demonstrated for image reconstruction from data collected with current multi-spectral CT, and reveal the potential of the non-convex OBIR method for enabling innovative workflows and applicatoins of multi-spectral CT.

An Inexact Uzawa Algorithmic Framework for Nonlinear Saddle Point Problems

Xiaoming Yuan
The University of Hong Kong

Abstract

Saddle point problems are fundamental in many areas; their particular applications arise in various mathematical fields such as optimization, scientific computing, data science, control and game theory. While a rich set of literature exists for linear saddle point problems, the research for nonlinear saddle point problems is still in its infancy. In this talk, we will discuss an algorithmic framework derived from an inexact Uzawa method for a class of application-driven nonlinear saddle point problems. We uniformly establish its convergence and linear convergence rate, and show how it can be specified as some highly implementable splitting algorithms for special convex optimization problems. Some numerical results for solving elliptic optimal control problems will be shown. This is a joint work with Yongcun Song and Hangrui Yue.

An efficient threshold dynamics method for topology optimization for fluids

Xiaoping Wang
Hong Kong University of Science and Technology

Abstract

We propose an efficient threshold dynamics method for topology optimization for fluids modeled with the Stokes equation. The proposed algorithm is based on minimization of an objective energy function that consists of the dissipation power in the fluid and the perimeter approximated by nonlocal energy, subject to a fluid volume constraint and an incompressibility condition. We show that the minimization problem can be solved with an iterative scheme in which the Stokes problem is approximated by a Brinkman equation and solved with the mixed finite-element method. The indicator functions of the fluid-solid regions are then updated according to simple convolutions followed by a thresholding step. We demonstrate mathematically that the iterative algorithm has the total energy decaying property. The proposed algorithm is simple and easy to implement. Extensive numerical experiments in both two and three dimensions show that the proposed iteration scheme is robust, efficient and insensitive to the initial guess and the parameters in the model.

Proximal iterative hard thresholding for sparse optimization

Xiaoqun Zhang
Shanghai Jiaotong University

Abstract

The iterative thresholding algorithms, both soft and hard thresholding, are widely adopted for sparsity promoting in imaging and data science. The analysis of iterative soft thresholding algorithms has been well studied under the framework of convexity and operator splitting methods and inspired many algorithms for related minimization problems. However, iterative hard thresholding methods are less understood due to its non-convexity and discontinuity, although there are some studies related to sparse signal recovery and image restorations. We considered the minimization of the sum of L0-norm and a smooth function arising from image restoration and sparse learning. We first proposed an extrapolated proximal iterative hard thresholding (EPIHT) algorithm and its variant with line-search. Under the theoretical framework of Kurdyka-Lojasiewicz (KL) property, we show that the sequences generated by the two algorithms converge to a local minimizer with linear convergence rate. Moreover, extensive numerical experiments on sparse signal reconstruction and wavelet frame based image restoration problems demonstrate the improvement of L0-norm based regularization models over some prevailing ones. We further considered the minimization of the sum of L0-norm and a convex smooth function under box constraint and designed an accelerated extrapolated proximal algorithm. The global convergence to a local minimizer of this algorithm is also established based on the convexity of the first function and the property of L0 function, while without using the tool of KL property. Numerical experiments on compressive sensing and sparse logistic regression showed the effectiveness of the latter one compared to other multi-steps algorithms for L0 minimization.

Uncertainty estimation of density reconstruction using Markov chain Monte Carlo method

Xinge Li
Institute of Applied Physics and Computational Mathematics, Beijing, China

Abstract

High-energy X-ray radiography is a measuring technique for quantitative measurement and diagnosis to the object and its internal structure. This technique utilizes the strong penetrability of X-ray and its interaction with the material, and determines the geometric and physical properties of the object according to the energy distribution on the imaging plane. For the axisymmetric single image photographic system, we cast this inversion problem within a statistical framework in order to compute volumetric object densities from X-ray radiographs and to quantify uncertainties in the reconstruction. A hierarchical Bayesian model is developed with a likelihood based on a Gaussian noise model and with priors placed on the unknown nonnegative density profile, the prior precision matrix, and two scale parameters. This results in a joint posterior distribution which can be readily sampled using the Markov chain Monte Carlo method. To study the rule of hyperparameters and their sensitivity analysis, many tests are done and some results are concluded. Results of the density reconstructions and pointwise uncertainty estimates are presented for MCNP simulated signals with various physical factors in the imaging process included.

An Adaptive Directional Haar Framelet-based Reconstruction Algorithm for Parallel Magnetic Resonance Imaging

Yan-Ran Li
College of Computer Science and Software Engineering, Shenzhen University, China.

Abstract

Parallel magnetic resonance imaging (pMRI) is a technique to accelerate the magnetic resonance imaging process. The problem of reconstructing an image from the collected pMRI data is ill-posed. Regularization is needed to make the problem well-posed. We first construct a 2-dimensional tight framelet system whose filters have the same support as the orthogonal Haar filters and are able to detect edges of an image in the horizontal, vertical, and $\pm 45^o$ directions. This system is referred to as directional Haar framelet (DHF). We then propose a pMRI reconstruction model whose regularization term is formed by the DHF. This model is solved by a fast proximal algorithm with low computational complexity. The regularization parameters are updated adaptively and determined automatically during the iteration of the algorithm. Numerical experiments for in-silico and in-vivo data sets are provided to demonstrate the superiority of the DHF-based model and the efficiency of our proposed algorithm for pMRI reconstruction.

A Efficient Algorithm for Image Restoration under Poisson Noise

You-Wei Wen
School of Mathematics and Statistics, Hunan Normal University
Changsha, Hunan 410081, P. R. China

Abstract

We consider the problem of restoring images corrupted by Poisson noise. Under the framework of maximum a posteriori estimator, the problem can be converted into a minimization problem where the objective function is composed of a Kullback-Leibler (KL)-divergence term for the Poisson noise and a regularization term. Due to the logarithm function in the KL-divergence term, the non-differentiability of regularization term, it is not easy to design stable and efficiency algorithm for the problem. In this talk, we use the splitting and regularization approach for solving Poisson noise removal problem. Numerical results show that our proposed algorithms are efficient.

A Linear Reaction-Diffusion System with Interior Degeneration for Color Image Compression

Zhichang Guo
Department of Mathematics, Harbin Institute of Technology

Joint work with Boying Wu, Dazhi Zhang, Jiebao Sun and Kehan Shi

Abstract

We considers colorization-based image compression in RGB color space. In compression, we store only the compressed luminance component of the original color image and a few representative pixels extracted from the original color image. In decompression, by explicitly introducing the relation between the luminance component and the original color image into diffusion equations, a linear reaction-diffusion system with Perona-Malik type diffusion coefficient is proposed to reconstruct R, G and B channels simultaneously. The Perona-Malik type diffusion coefficient is a function of the luminance component and leads to interior degenerations, in general. It yields anisotropic smoothing in the restored color image and constrains the geometry of the restored image to follow the geometry of the luminance component. The existence and uniqueness of solutions for the proposed system with a specific class of diffusion coefficients are proved in a weighted Sobolev space. The selection of representative pixels has a big impact on reconstruction results. We also propose a local-optimal strategy that splits the original color image into a series of different size subimages and searches the optimal representative pixel in each subimage. Comparisons with recent colorization-based image compression methods, as well as transform-based JPEG and JPEG2000 standards are performed to show the potential for successful compression applications of the proposed method.