

# Title and Abstract for List of Boundaries of Topological Groups, $C^*$ -algebras, Symmetric Spaces, and Lyapounov Exponents

1. Principle Speaker: Hillel Furstenberg, The Hebrew University in Jerusalem, Israel

**Lecture 1:**

**Abstract:**

**Lecture 2:**

**Abstract:**

2. Uri Bader, Weizmann Institute, Rehovot, Israel

**Title:** Algebraic Representations of Ergodic Actions

**Abstract:** I will define the notion of AREA and survey its properties and uses, in particular its applications to Lattices Rigidity. I will also state and explain some results that will be used later in Furman's talk. (Based on a long joint work with Alex Furman).

3. Tim de Laat, University of Munster, Germany

**Title:** Fixed Point Properties for Group Actions on Banach Spaces

**Abstract:** Group actions on Banach spaces (in particular on  $L_p$ -spaces) have seen a growing interest in the last decades. After an introduction to this topic, I will explain a joint work with Mikael de la Salle, in which we established a spectral criterion for fixed point properties of group actions on large classes of Banach spaces (including  $L_p$ -spaces). This criterion can be applied to random groups in certain models. Our work also lead to new estimates on the conformal dimension of the boundary of random groups.

4. Alex Furman, University of Illinois at Chicago, USA

**Title:** Boundary Theory and Lyapunov Spectrum

**Abstract:** Multiplicative Ergodic Theorem of Oseledets describes the asymptotic behavior of products of an integrable matrix valued function over an invertible, ergodic, pmp system in terms of the Lyapunov spectrum, and a measurable family of pairs of flags. In the talk we shall describe a class of situations where we prove that the spectrum is simple and is stable under certain perturbations of the matrix valued function. Our approach uses boundary theory of groups. (Joint work with Uri Bader).

5. Yair Hartman, Ben Gurion University, Beer Sheva, Israel

**Title:** Stationary  $C^*$ -Dynamical Systems

**Abstract:** We introduce the notion of stationary actions in the context of  $C^*$ -algebras, and prove a new characterization of  $C^*$ -simplicity in terms of unique stationarity. This ergodic theoretical characterization provides an intrinsic understanding for the relation between  $C^*$ -simplicity and

the unique trace property, and provides a framework in which  $C^*$ -simplicity and random walks interact. (Joint work with Mehrdad Kalantar).

6. Lizhen Ji, University of Michigan, USA

**Title:** Compactifications and Metrics of Symmetric Spaces viewed as Teichmuller Spaces

**Abstract:** In this talk, I will describe a joint work with Mark Greenfield on defining several metrics on the symmetric space  $SL(n, \mathbf{R})/SO(n)$  when it is viewed as the Teichmuller space of flat tori of volume 1. In particular, we define the analogue of the Thurston metric on the usual Teichmuller space, the resulting horofunction compactification of the symmetric space, and its relation with the Furstenberg–Satake compactification.

7. Vadim Kaimanovich, University of Ottawa, Canada

**Title:** Arboreal Structures on Groups and the Associated Boundaries

**Abstract:** For any countable group with infinite conjugacy classes we construct a family of forests on the group. For each of them there is a random walk on the group with the property that its sample paths almost surely converge to the geometric boundary of the forest in a way that resembles the simple random walks on trees. It allows us to identify the Poisson boundary of the random walk with the boundary of the forest and to show that the group action on the Poisson boundary is free (which, in particular, implies non-triviality of the Poisson boundary). As a consequence we obtain that any countable group carries a random walk such that the stabilizer of almost every point of the Poisson boundary coincides with the hyper-FC-centre of the group. Our work is a development of a recent result of Frisch–Hartman–Tamuz–Vahidi–Ferdowsi who proved that any ICC group admits a measure with a non-trivial Poisson boundary. (Joint work with Anna Erschler).

8. Mehrdad Kalantar, University of Houston, USA

**Title:** Stationary States and Their Applications to  $C^*$ -simplicity/Rigidity Problems

**Abstract:** We give applications of stationary states in some  $C^*$ -simplicity/rigidity problems. In particular, we discuss simplicity of crossed product  $C^*$ -algebras and their intermediate subalgebras, as well as certain rigidity properties of groups relative to their subgroups. This is based on joint works with Yair Hartman, and Tattwamasi Amrutam.

9. Anders Karlsson, University of Geneva, Switzerland

**Title:** Horofunction Compactifications, Non-linear Lyapunov Exponents and Operator Ergodic Theorems

**Abstract:** Metric functionals (a variant of horofunctions) play a role for metric spaces akin to continuous linear functionals for topological vector spaces. In particular one has weak compactness. This notion is used for a metric spectral (ergodic) theorem, which in turn can help to define genuinely non-linear Lyapunov exponents for random products of maps. When applied to symmetric spaces it reproves Oseledets' theorem. The most perfect non-linear analogy so far is a

generalization of Thurston's spectral theorem for surface homeomorphisms. It also extends in particular the Wolff–Denjoy theorem in complex dynamics and the Carleman–von Neumann–Riesz mean ergodic theorem.

10. Sanghyun Kim, Seoul National University, Korea

**Title:** Diffeomorphism Groups of Critical Regularity

**Abstract:** For each real number  $r = k + a \geq 1$  where  $k = [r]$ , we let  $\text{Diff}^r(S^1)$  denote the group of orientation-preserving  $C^k$ -diffeomorphisms on  $S^1$  whose  $k$ -th derivatives are Hölder continuous. For each  $r \geq 1$ , we construct a finitely generated group  $G_r$  inside  $\text{Diff}^r(S^1)$  such that  $G_r$  admits no injective homomorphisms into  $\cup_{s>r} \text{Diff}^s(S^1)$ . We also have a dual result: there exists a finitely generated group  $H_r$  inside  $\cap_{s<r} \text{Diff}^s(S^1)$  such that  $H_r$  admits no injective homomorphisms into  $\text{Diff}^r(S^1)$ . This result was previously known for  $r = 1$ , and partially for  $r = 1.5$  and  $r = 2$ . (Joint work with Thomas Koberda).

11. Zhiqiang Li, Chongqing University, China

**Title:** Joint Topological Entropy for Dynamical Systems

**Abstract:** We study a version of topological entropy inspired by S. Friedland for dynamical systems. This entropy is an useful invariant up to generalized flip conjugacy. We exhibit concrete examples to demonstrate its feature of jointness, and apply it to distinguish (or classify) group actions in certain cases, especially for natural  $\mathbf{Z}^k$  actions, for which this joint entropy will not vanish.

12. Shahar Mozes, Hebrew University, Jerusalem, Israel

**Title:** Surface Subgroups in Uniform Lattices of Some Semisimple Lie Groups

**Abstract:** In a joint work with Jeremy Kahn and Francois Labourie we prove that any uniform lattice in a simple complex Lie group  $G$  contains a surface group (i.e. the the fundamental group of an orientable surface of genus at least 2). This theorem is a generalization of the celebrated Kahn–Markovic Theorem which deals with the case of  $G = \text{PSL}(2, \mathbf{C})$  and its proof follows a similar scheme.

13. Amos Nevo, The Technion, Haifa, Israel

**Title:** Boundary Theory and Ergodic Theory

**Abstract:** We will give an exposition of a recent approach to several aspects of ergodic theory, which treats amenable groups and non-amenable groups on an equal footing. The key to this approach is the use of an auxiliary space, which carries an amenable Borel equivalence relation with finite invariant measure. This space can often be chosen as the Poisson boundary of the acting group. We will demonstrate how this approach can be used to prove pointwise ergodic theorems for the group action, and to define a notion of entropy satisfying a Shannon–McMillan–Breiman theorem, both for general non-amenable groups. (Based on joint work with L. Bowen and on joint work with F. Pogorzelski).

14. Ebrahim Samei, University of Saskatchewan, Canada

**Title:** Amenability of Quasi-Hermitian Groups

**Abstract:** A locally compact group  $G$  is *Hermitian* if the spectrum  $\text{Sp}_{L^1(G)}(f) \subseteq \mathbb{R}$  for every  $f \in L^1(G)$  satisfying  $f = f^*$ , and *quasi-Hermitian* if  $\text{Sp}_{L^1(G)}(f) \subseteq \mathbb{R}$  for every  $f \in C_c(G)$  satisfying  $f = f^*$ . We show that every quasi-Hermitian locally compact group is amenable. This provides an affirmative answer to the question of whether every Hermitian locally compact group is amenable, a problem which has remained open since 1960's. Our approach involves introducing a theory of "spectral interpolation of triple Banach  $*$ -algebras" and applying this theory to a family  $\text{PF}_p^*(G)$  ( $1 < p < \infty$ ) of Banach  $*$ -algebras related to convolution operators. We also give an alternative proof to Jenkin's result that a discrete group containing a free sub-semigroup on two generators is not quasi-Hermitian. This, in particular, provides a dichotomy on discrete elementary amenable groups: either they are non quasi-Hermitian or they have subexponential growth.

(Joint work with Matthew Wiersma, University of Alberta, Canada).

15. Matan Tal, The Hebrew University, Israel

**Title:** Conic Representations of Topological Groups

**Abstract:** Affine representations, namely continuous group actions by affine transformations on compact convex sets, play a central role in Furstenberg's Boundary theory. In analogy to that, a conic representation is a group action on a cone – a set closed under linear combinations of positive coefficients – by appropriate transformations. I shall present the basic theory of such representations. (The work was done as a master's thesis under the guidance of Hillel Furstenberg).

16. Omer Tamuz, California Institute of Technology, USA

**Title:** Strong amenability and the infinite conjugacy class property

**Abstract:** A group is said to be strongly amenable if each of its proximal topological actions has a fixed point. We show that a countable discrete group is strongly amenable if and only if none of its quotients have the infinite conjugacy class property. It follows that a finitely generated group is strongly amenable if and only if it is virtually nilpotent. Another consequence is a connection between proximal actions and the group von Neumann algebra, which mirrors the known relation between strongly proximal actions and the reduced  $C^*$ -algebra. (joint with Joshua Frisch and Pooya Vahidi Ferdowsi).