Title and Abstract

1. Tsuyoshi Ando, Hokkaido University (Emeritus), Japan
   Title: M-norms and L-norms
   Abstract: A subset $\mathbb{U}$ of M_n is said to be C^*-convex if

$$\Sigma_j C_j^* \mathbb{U} C_j \subset \mathbb{U} \text{ whenever } \Sigma_j C_j^* C_j = I.$$

A norm $||\cdot||$ will be called an M-norm if its unit ball is C^*-convex, or equivalently

$$||\Sigma_j C_j^* X_j C_j|| \leq \max_j ||X_j|| \text{ whenever } \Sigma_j C_j^* C_j = I.$$

Its dual norm $||\cdot||^*$, which is characterized by the property

$$\Sigma_j ||C_j Y C_j^*||^* \leq ||Y||^* \text{ whenever } \Sigma_j C_j^* C_j = I$$

will be called an L-norm. Familiar examples of M-norms $||\cdot||_\infty$ (spectral norm) and $w(\cdot)$(numerical radius) while those of L-norms are $||\cdot||_1$ (trace norm) and $w^*(\cdot)$ (the dual norm of $w(\cdot)$).

The p-radius $w_p(\cdot)$ $(1 \leq p \leq 2)$, whose unit ball consists of all matrices admitting unitary p-dilations, forms a scale of M-norms interpolating $w_1(\cdot) := ||\cdot||_\infty$ and $w_2(\cdot) := w(\cdot)$.We discuss also another scale of M-norms interpolating $||\cdot||_\infty$ and $w(\cdot)$.

We will show that the identity $/\!/ X /\!/_1 = w_1(X)$ and the inequality of Marcus and Sandy

$$||X||_\infty \leq n \cdot w(X) = n \cdot w_2(X)$$

can be interpolated as

$$||X||_p \leq n^{1/p} \cdot w_{2^{1/p}}(X),$$

where $||\cdot||_p$ is the Schatten p-norm for $1 \leq p \leq \infty$:

2. Jor-Ting Chan, University of Hong Kong
   Title: Numerical Ranges of Weighted Composition Operators on ℓ^2
   Abstract: Let ℓ^2 be the Hilbert space of all square-summable sequences under the usual inner product. A weighted composition operator on ℓ^2 is an operator of the form $(x_n) \mapsto (u_n \cdot x_{\varphi(n)})$, where (u_n) is a sequence and φ is a mapping on N. In this talk, we will look at the numerical range of this type of operators.

3. Chi-Tung Chang, Feng Chia University, Taiwan
   Title: Compactness of $f(T)$
   Abstract: Let H be a separable Hilbert space and T be a essentially normal C_0-contraction on H. We give necessary and sufficient conditions for the compactness of $f(T)$, where f is a bounded holomorphic function on the unit disc D. We also obtain related results when f is a holomorphic function on D and

continuous on $\bar{D}$.

4. Wai-Shun Cheung, University of Hong Kong
   Title: On the common boundary of generalized numerical ranges
   Abstract: For any n×n matrix A, we define a new genearlized numerical range W (A; c) where $c = (c_1, \ldots, c_n)^t \in IR^n$. We will show that, given $A \in M_n, B \in M_m$, $c \in IR^n$ and $d \in IR^m$, if W(A; c) and W(B; d) shared a common curved boundary then there exist $\sigma \in S_n$ and $\mu \in S_m$ such that
   $$c_1\lambda_{\sigma(1)}(A) + \ldots + c_n\lambda_{\sigma(n)}(A) = d_1\lambda_{\mu(1)}(B) + \ldots + d_m\lambda_{\mu(m)}(B).$$
   Some applications will be discussed.

5. Mao-Ting Chien, Soochow University, Taiwan
   Title: Determinantal representations of hyperbolic forms via weighted shift matrices.
   Abstract: This is a joint work, still in progress, with Hiroshi Nakazato. It aims to characterize hyperbolic ternary forms which admit determinantal representations via cyclic weighted shift matrices.

6. Man-Duen Choi, University of Toronto
   Title: The Panorama of Numerical Ranges in Modern Times
   Abstract: Suddenly, there arrives the new era of REAL quantum computer with all sorts of mathematical applications by means of non-commutative analysis. We seek new meanings of old value, as well as to realize new values of old meaning of numerical ranges, in terms of quantum information.

7. Jianlian Cui, Tsinghua University
   Title: Numerical range preservers of skew Lie products
   Abstract: Let H and K be complex separable Hilbert spaces with dimensions at least three, and $\mathrm{B}(H)$ the Banach algebra of all bounded linear operators on H. Let $A \in \mathrm{B}(H)$. Denote by $W(A)$ the numerical range of A. It is shown that a surjective map (no algebraic structure assumed) $\Phi : \mathrm{B}(H) \to \mathrm{B}(K)$ satisfies that $W(AB - BA^*) = W(\Phi(A)\Phi(B) - \Phi(B)\Phi(A)^*)$ for all $A, B \in \mathrm{B}(H)$ if and only if there exists a unitary operator $U \in \mathrm{B}(H, K)$ such that $\Phi(A) = \mu UAU^*$ for all $A \in \mathrm{B}(H)$, where $\mu \in \{-1, 1\}$. We discuss also additive maps preserving numerical radius of skew Lie products on factor von Neumann algebras.

8. Hongke Du, Shaanxi Normal University
   Title: Spectra and numerical ranges of operators
   Abstract: In this talking, some closed connection between spectra and numerical ranges of operators.

9. Runyao Duan, Centre for Quantum Computation and Intelligent Systems, UTS, Australia

Title: Perfect Distinguishability of Quantum Operations and Numerical Range

Abstract: A fundamental problem in quantum information is to characterize the perfect distinguishability of quantum operations (also known as Completely Positive Trace-Preserving maps, or Quantum Channels), which formalize all physically realizable processes allowed by Quantum Mechanics. Since the seminal work by Childs, Preskill, and Renes [Journal of Modern Optics 47, 155 (2000)], this problem have been studied extensively and many interesting partial results have been reported. However, it remained unknown for a long time when two general quantum operations are perfectly distinguishable.

In 2009 we obtained a feasible necessary and sufficient condition for the perfect distinguishability of a given finite set of quantum operations, thus successfully solved this open problem. An optimal protocol requiring minimal resources was also found for the case of distinguishing two quantum operations. As by-products, we found that the notion of numerical range and many of its variants such as local numerical range and q-numerical range, which have been extensively studied in Matrix Analysis, play crucial roles in characterizing the perfect distinguishability of quantum operations. Most notably, we re-discovered the notions of local numerical range and q-numerical range of a linear operator.

In this talk I will first provide a brief review of basic notions in quantum information, and show how basic properties of numerical ranges can be used to settle the perfect discrimination of unitary and isometries. I will then generalize these techniques to provide a complete solution to the perfect distinguishability of quantum operations. Hopefully the nontrivial connection between numerical range and the distinguishability of quantum operations will stimulate research interest in both topics.

This talk is based on the following joint works with Yuan Feng and Mingsheng Ying:

1. Runyao Duan, Yuan Feng and Mingsheng Ying, Perfect distinguishability of quantum operations,
Physical Review Letters 103 (2009), 210501.

2. Runyao Duan, Yuan Feng and Mingsheng Ying, Local Distinguishability of Multipartite Unitary
Operations, Physical Review Letters 100 (2008), 020503.

3. Runyao Duan, Yuan Feng and Mingsheng Ying, Entanglement Is Not Necessary for Perfect Discrimination between Unitary Operations, Physical Review Letters 98 (2007), 100503.

10. Ajda Fosner, University of Primorskem

Title: Linear maps preserving numerical radius of tensor products of matrices

Abstract: Let $m,n \geq 2$ be integers and let us denote by M_m the set of $m \times m$ complex matrices and by $w(X)$ the numerical radius of a square matrix X. Motivated by the study of operations on bipartite systems of quantum states and the linear preserving problem, we show that a linear map $\phi : M_{mn} \to M_{mn}$

satisfies

$$w(\phi(A \otimes B)) = w(A \otimes B), \qquad A \in M_m, B \in M_n,$$

if and only if there is a unitary matrix $U \in M_{mn}$ and a complex unit ξ such that

$$\phi(A \otimes B) = \xi U(\varphi_1(A) \otimes \varphi_2(B))U^*, \qquad A \in M_m, B \in M_n,$$

where φ_k is the identity map or the transposition map $X \mapsto X^t$ for $k = 1, 2$. More precisely, if $m, n \geq 3$, the maps φ_1 and φ_2 must be of the same type. In particular, if $m, n \geq 3$, the map ϕ corresponds to an evolution of a closed quantum system (under a fixed unitary operator), possibly followed by a transposition. The results are extended to multipartite systems.

11. Hwa-Long Gau, National Central University, Taiwan
Title: Numerical ranges and numerical radii for tensor products of matrices.
Abstract: For n-by-n and m-by-m complex matrices A and B, it is known that the inequality w(A⊗B) ≤ ||A||w(B) holds, where w(.) and||. || denote, respectively, the numerical radius and the operator norm of a matrix. In this talk, we consider when this becomes an equality. We give necessary and suffcient conditions for w(A⊗B) = ||A||w(B) to hold. In this case, we show that the numerical range W(A⊗B) must be a circular disccentered at the origin. Among other things, for some classes of matrices A, we also show that W(A⊗A) is a circular disc centered at the origin if and only if W(A) is a circular disc centered at the origin.

12. Kan He, Taiyuan University of Technology
Title: Maps on quantum states preserving numerical radius of convex combinations
Abstract: In quantum control theory, unitary revolution of a quantum state is controllable by the maximum expectation value of observables. One of well-known observables may be the rank-one projective operator, and its maximum expectation value is the numerical radius of the quantum state. So the concept of the numerical radius plays an important pole in quantum control theory. In the note, we claim that two states ρ, σ with the same numerical radius w under perturbation of every pure state just be the same one, i.e., $\rho=\sigma$ if and only if there exists some $\lambda_0 \in (0,1)$ such that $w(\lambda_0\rho+(1-\lambda_0)\gamma)=w(\lambda_0\sigma+(1-\lambda_0)\gamma)$ for every pure state γ. Applying the above result, we also give a characterization of maps preserving numerical radius of the convex combinations of quantum states.

13. Jinchuan Hou,
Title: Maps preserving numerical range of Lie-products of operators
Abstract: Let H,K be complex separable Hilbert spaces of dimension >2 and let ${\mathcal B}(H)$ be the von Neumann algebra of all bounded linear operators acting on H. We discuss the question of characterizing the maps $\Phi:{\mathcal B}(H)\to {\mathcal B}(K)$ satisfying $W(AB-BA)=W(\Phi(A)\Phi(B)-\Phi(B)\Phi(A))$ for any

$A,B\in{\mathcal B}(H)$,where $W(T)$ stands for the numerical range of operator T.

14. Zejun Huang, Hunan University
Title: Linear maps preserving the higher numerical ranges of tensor products of matrices.
Abstract: For a positive integer k, let M_k be the set of $k\times k$ complex matrices. Suppose m, $n\geq 2$ are positive integers. In this talk, we will present the characterization of linear maps ϕ on M_{mn} leaving invariant, the higher numerical ranges of matrices in tensor form $A\otimes B$ with $A \in M_m$ and $B \in M_n$.
This talk is based on joint work with Ajda Fosner, Chi-Kwong Li, Yiu-Tung Poon, and Nung-Sing Sze.

15. Seung-Hyeok Kye, Seoul National University
Title: Permanents of matrices arising from the quantum information theory
Abstract: One of the most important problems in quantum information theory is to determine if a given state is separable or entangled. The PPT criterion by Choi in 1980 is quite strong for this purpose, and was rediscovered by Peres in the nineties. But, it is very difficult in general to determine if a given PPT state is separable or not. The range criterion is quite useful for this purpose. In order to apply the range criterion, we have to consider algebraic equations in terms of complex variables and their conjugates. Existences and numbers of solutions naturally depend on the numbers of equations and variables, and it is very subtle to determine the existence in the critical case when two numbers coincide. In the multi-qubit cases, we show that the existence of a solution depends on the permanent of the associated matrix, which has been studied from the era of Cauchy. This talk will be based on a co-work with Y.-H. Kiem and J. Na [arXiv 1401.3181]

16. Chi-Kwong Li, College of William and Mary, USA
Title: Numerical ranges of operator products
Abstract: We show that a matrix A in M_n is a multiple of a unitary matrix if and only if W(AB) = W(BA) for all (rank one) matrices B. We then extend the results in several directions.
a. We replace the numerical range by the numerical radius.
b. We extend the result to bounded linear operators.
c. We extend the result to other kinds of generalized numerical ranges.

17. Yuan Li, Shaanxi Normal University
Title: The structure of quantum operations
Abstract: We firstly show that for any quantum states p on H and σ on K there exists a quantum channel ϕ such that $\phi(p) = \sigma$, where H and K are finite or infinite dimensional Hilbert spaces. Then we consider some conclusions for the quantum

channel ϕ such that ϕ (p) = σ and ϕ (I_H) exists or ϕ (I_H) = I_K. Also, we give a generalization of a theorem due to Uhlmann theorem, extending it to infinite dimensional Hilbert spaces. Finally we show that for any quantum channelϕ, one has S(ϕ (p)) = S(σ) for all quantum states p if and only if there exists an isometric operator V such that ϕ (p) = V pV $*$.

18. Minghua Lin, University of Victoria
Title: Singular value inequalities for matrices with numerical ranges in a sector
Abstract: By a sector, I mean the set $S_\alpha = \{z \in \mathbb{C}: \Re z > 0, |\Im z| \le (\Re z)\tan\alpha\}$ on the complex plane. Motivated by a conjecture of N. Highamon the growth factor in Gaussian elimination, several recent papers are devoted to studying a new class of matrices, matrices whose numerical ranges are in a sector. In this talk, I survey some results on this topic, from determinantal inequalities to singular value inequalities. The talk is based on joint work with S. Drury.

19. Hiroshi Nakazato, Hirosaki University, Japan
Title: The genus of the boundary generating curves of numerical ranges
Abstract: Helton and Vinnikov proved the validity of the Fiedler-Lax conjecture in 2005. Recently Plaumann and Vinzant gave a rather elementary proof of the Helton-Vinnikov theorem. We are interested in concrete construction of herbolic ternary forms F (t; x; y) for which the algebraic curve F (t; x; y) =0 has a low genus g = 0 or g = 1. Typical examples of g = 0 curves are given by using some Toeplitz matrices or trigonometric polynomials. Many examples of g = 1 curves are given by using cyclic shift matrices.

20. Abbas Salemi Parizi, Shahid Bahonar University of Kerman
Title: Joint higher rank numerical range of Pauli group
Abstract: A quantum channel is called a Pauli channel if each of its error operators are scalar multiple of elements in N-qubit Pauli group P_N. The Pauli channels are a central class of quantum channels in quantum computing.
In this lecture, we are looking to find the largest k such that the joint rank-2^k numerical range $\Lambda_{2^k}(\mathbf{A}) \neq \varnothing$, for $\mathbf{A} = (A_1, \ldots, A_m) \in \mathrm{P}_N^m$. In the context of quantum error correction, this means that there exists an N-qubit encoding which accommodates a k-qubit data states. Note that in this sense, the rate of the quantum error correction code is maximal. So our purpose is to study the joint rank-2^{N-1} numerical range of $\mathbf{A} \in \mathrm{P}_N^m$. Understanding the properties of $\Lambda_{2^{N-1}}(\mathbf{A})$ is useful to constructing quantum error correction codes. But if $\Lambda_{2^{N-1}}(\mathbf{A}) = \varnothing$, then we focus on $\Lambda_{2^{N-2}}(\mathbf{A})$. Also, we analyze the geometric structure of the set

$$\Omega_k(\mathbf{A}) := \{\mathbf{a} \in \square^{1\times m} : \mathbf{c}\cdot\mathbf{a} \le \lambda_k(\mathbf{c}\cdot\mathbf{A}) \text{ for all } \mathbf{c} \in \square^{1\times m} \text{ with } \|\mathbf{c}\| = 1\},$$

for a generic Hermitian m-tuple $\mathbf{A} \in \mathrm{H}_n^m$.
Moreover, the rank-k numerical range of an independent triple of Hermitian matrices in the N-qubit Pauli group are studied. We show that

$$\operatorname{conv}(\Lambda_k(\mathbf{A})) = \Omega_k(\mathbf{A}) \quad \text{for all } \mathbf{A} \in \mathrm{P}_N^3.$$

Finally, we obtained the largest, for which the joint rank-2^k numerical range associated with error operators of the arbitrary (not necessarily abelian) Pauli subgroup of N-qubit Pauli group is nonempty.

21. Rajesh Jose Pereira, University of Guelph

Title: Applications of and Open Questions about Compressions of Matrices

Abstract: Let M be an operator in $B(PC^n)$. A compression of M is an operator in $B(PC^n)$ of the form $PMP|_{P\mathbb{C}^n}$ where P is an n by n orthogonal projection. We give some applications of compressions to the geometry of polynomials and to numerical ranges and higher-rank numerical ranges. We explore some open questions: some old such as Sendov's conjecture, and some new such as a conjectured representation of the higher rank numerical ranges of M in terms of the star centres of the normal compressions of M.

22. Sarah Joelle Plosker, Brandon University

Title: Quantum expectations: a matricial range perspective

Abstract: The matricial range is the matrix-valued form of the numerical range. In this lecture, I will explain how the expectation of a quantum random variable can be understood and studied from the point of view of matricial ranges and spectra.

23. Edward Poon, Embry-Riddle Aeronautical University

Title: Preserving the relative C-numerical range

Abstract: The relative C-numerical range of an $n \times n$ matrix A relative to a subgroup G of the unitary group U_n is the set of complex numbers

$$W_G(C, A) = \{\operatorname{tr}(C^* UAU^*) : U \in G\}.$$

We consider the problem of characterizing linear preservers of relative C-numerical range in certain special cases.

24. Yiu-Tung Poon, Iowa State University

Title: Joint numerical range and states of maximum entropy.

Abstract: Let $W(F_1, \ldots, F_k)$ be the joint numerical range of n×n Hermitian matrices $F_1, \ldots, F_k$. Then the convex hull of $W(F_1, \ldots, F_k)$ is given by

$$L(F_1, \ldots, F_k) = \{(\operatorname{tr}(pF_1, \ldots, \operatorname{tr}(pF_k)) : p \in Dn\}$$

where Dn is the set of n×n positive semidefinite matrices with trace 1. Given $\mathbf{a} \in L(F_1, \ldots, F_k)$, let

$$L(\mathbf{a}) = \{p \in Dn : (\operatorname{tr}(pF_1), \ldots, \operatorname{tr}(pF_k)) = \mathbf{a}\}$$

and $p^*(\mathbf{a})$ be the state in $L(\mathbf{a})$ with the maximum entropy. We study the continuity of the maps $\mathbf{a} \to L(\mathbf{a})$ and $\mathbf{a} \to \mathbf{p^*}(\mathbf{a})$.

25. Nung-Sing Sze, Hong Kong Polytechnic University

Title: Determinantal and eigenvalue inequalities for matrices with numerical ranges in a sector

Abstract: Let $A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \in M_n$, where $A_{11} \in M_m$ with $m \le n/2$, be such that the numerical range of A lies in the set $\{e^{i\varphi} z \in \mathbb{C} : |\Im z| \le (\Re z)\tan\alpha\}$, for some $\varphi \in [0, 2\pi)$ and $\alpha \in [0, \pi/2)$. We obtain the optimal containment region for the generalized eigenvalue λ satisfying

$$\lambda \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} x = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix} x \quad \text{for some nonzero } x \in \mathbb{C}^n,$$

and the optimal eigenvalue containment region of the matrix $I_m - A_{11}^{-1}A_{12}A_{22}^{-1}A_{21}$ in case A_{11} and A_{22} are invertible. From this result, one can show $|\det(A)| \le \sec^{2m}(\alpha)|\det(A_{11})\det(A_{22})|$ .In particular, if A is an accretive-dissipative matrix, then $|\det(A)| \le 2^m |\det(A_{11})\det(A_{22})|$. These affirm some conjectures of Drury and Lin.

This talk is based on a joint work with and C.K. Li (College of William \& Mary) and has been published in J. Math. Anal. Appl., 410:487-491, 2014.

26. Tin-Yau Tam, Auburn University
Title: Generalized Numerical Range, Schur-Horn's Projection, and Iwasawa Projection
Abstract: We discuss the generalized numerical range, the Schur-Horn's projection and Iwasawa projection. They are related to the classical numerical range and its beautiful convexity result. We will mention the average of the numerical range as well as the projections. This is a joint work with Ming Liao.

27. Ming-Cheng Tsai, National Sun Yat-sen University
Title: Completely positive interpolations of compact, trace-class and Schatten-p class operators.
Abstract: Extending Li and Poon's results on interpolation problems for matrices, we give characterizations of the existence of a completely positive linear map between compact (or Schatten-p class) operators sending a particular operator A to another B. It is shown that such a completely positive map exists if and only if a positive one does. Moreover, we show that such a completely positive map exists as above if and only if a multiple of the numerical range of A contains the numerical range of B. Similar results are also established for families of operators.

28. Kuo-Zhong Wang, National Chiao Tung University
Title: Diagonals and numerical ranges of weighted shift matrices
Abstract: For any n-by-n matrix A, we consider the maximum number $k = k(A)$ for which there is a k-by-k compression of A with all its diagonal

entries in the boundary $\partial W(A)$ of the numerical range $W(A)$ of A. For any such compression, we give a standard model under unitary equivalence for A. When A is a matrix of the form

$$\begin{pmatrix} 0 & w_1 & & \\ & 0 & \ddots & \\ & & \ddots & w_{n-1} \\ w_n & & & 0 \end{pmatrix},$$

we show that $k(A)=n$ if and only if either $|w_1|=\cdots=|w_n|$ or n is even and $|w_1|=|w_3|=\cdots=|w_{n-1}|$ and $|w_2|=|w_2|=\cdots=|w_n|$. For such matrices A with exactly one of the w_j's zero, we show that any k, $2\le k\le n-1$, can be realized as the value of $k(A)$, and determine exactly when the equality $k(A)=n-1$ holds.

29. Ngai-Ching Wong, National Sun Yat-sen University
Title: Maps Preserving Schatten p-Norms of Convex Combinations
Abstract: In this talk, we study maps ϕ of positive operators of Schatten p-classes ($1 < p < +\infty$), which preserve the p-norms of convex combinations, that is,

$$\|tp + (1 - t)\sigma\|_p = \|t\phi(p) + (1 - t)\phi(\sigma)\|_p, \ \forall p, \sigma \in S_p^+(H)_1, \quad t \in [0,1].$$

They are exactly those carrying the form $\phi(p) = UpU^*$ for a unitary or antiunitary U. In the case p = 2, we have the same conclusion whenever it just holds

$$\|p + \sigma\|_2 = \|\phi(p) + \phi(\sigma)\|_2$$

for all positive Hilbert-Schmidt class operators p, σ of norm 1. Some examples are demonstrated.

This paper is included in "Preserver Problems on Function Spaces, Operator Algebras, and Related Topics", as a special issue of Abstr. Appl. Anal. (Co-edited by Peralta, Antonio M.; Ng, Chi-Keung; Wong, Ngai-Ching; Yao, Jen-Chih), Volume 2014 (2014), Article ID 520795.

30. Pei Yuan Wu, National Chiao Tung University
Title: Circular Numerical Range of a Partial Isometry
Abstract: Let A be an n-by-n partial isometry whose numerical range W(A) is a circular disc $\{x \in C : |z-c| \le r\}$. In this talk, we consider the possible values of the center c and radius r. We show that c must be 0 for $n\le 4$, and suspect that this is the case for all n. On the other hand, for a general n and for $W(A) = \{z \in C: |z| \le r\}$, all the possible values of *r* are $\{0,\cos(\pi/(n + 1))\} \cup [\cos(\pi/3), \cos(\pi/n)]$.

This is a joint work with H.-L. Gau and K.-Z. Wang.

31. Fuzhen Zhang, Nova Southeastern University, Florida 33314, USA
Title: A Matrix Decomposition and Its Applications
Abstract: We discuss a matrix decomposition, show the uniqueness and

construction (of the Z matrix in our main result) of the matrix decomposition, and give an affirmative answer to a question proposed in [J. Math. Anal. Appl. 407 (2013) 436-442]. The theorem is stated as Sectoral Decomposition:
Let A be an $n\times n$ complex matrix such that its numerical range is contained in a sector in the 1st and 4th quadrants, i.e., $W(A)\subseteq S_{\alpha}$ for some $\alpha \in [0, \frac{\pi}{2})$. Then there exist an invertible matrix X and a unitary diagonal matrix $Z={\rm diag} (e^{i\theta_1}, \dots, e^{i\theta_n})$ with all $|\theta_j|\leq \alpha$ such that $A=XZX^*$. Moreover, such a matrix Z is unique up to permutation.

32. Karol Zyczkowski, Jagiellonian University
Title: On restricted numerical range and numerical shadow
Abstract: Numerical shadow $P_X(z)$ of an operator X is the probability measure on the complex plane supported by the numerical range $W(X)$, defined as the probability that the inner product (Xu,u) is equal to z, where u denotes a normalized N-dimensional random complex vector.

Restricting vectors u to certain subset of the set of all states (e.g. real/product/entangled states) one arrives at the notion of the restricted numerical range, which in general needs not to be convex. Analyzing numerical shadow of hermitian matrices with respect to real states we show that they form a generalization of the standard B-spline.

We study the standard numerical range of a non-hermitian random Ginibre matrix G and show that $W(G)$ converges asymptotically to the disk of radius larger by factor $\sqrt{2}$ than the radius of the support of the spectrum of G on the complex plane. The with of the belt of the numercial range without eigenvalues can be related to the degree of non-normality of the matrix analyzed.

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