

# The 2023 Workshop on

# **Lie Theory and Representation**



Sanya, China November 17-19, 2023

Hosted by

Yau Mathematical Sciences Center, Tsinghua University

Organized by

**Tsinghua Sanya International Mathematics** 

### Forum (TSIMF)

### The 2023 Workshop on Lie Theory and Representation

### (WLTR 2023)

### Aim:

Lie theory is an important and popular research direction in modern mathematics. Lie theory is closely related to all branches of mathematics and has important applications in fields such as physics and chemistry. This workshop will bring experts to exchange the latest research progress and trends in the field of Lie theory and representations, and promote academic cooperation among experts in the field of Lie theory and representations.

### **Conference Time:**

November 17-19, 2023;

Full-day registration on November 17th, and departure on November 19th.

### **Conference Venue:**

Tsinghua Sanya Internation Mathematical Forum

No.100, Tsinghua Road, Tianya District, Sanya, Hainan, P. R. China.

### Accommodation

Tsinghua Sanya Internation Mathematical Forum

No.100, Tsinghua Road, Tianya District, Sanya, Hainan, P. R. China.

### **Contact:**

Honglian Zhang, Email: hlzhangmath@shu.edu.cn, Phone number: 13636353711

Jiancai Sun, Email: jcsun@shu.edu.cn, Phone number: 13681808277

### **Transportation**

### Getting from the Airport to TSIMF:

TSIMF provids pick-up and drop-off services for participants. Detailed directions can be found on the official TSIMF website. In case you missed the pick-up, there are two ways you can go from the Sanya Phoenix International Airport to Sanya International Mathematics Forum:

- By a shuttle car provided by the Forum. In this case we will need your flight information in advance.
- By taxi. One way to the Forum should cost from CNY15 to CNY25.

## **Conference Agenda**

Time	Proceeding	Place	
November 17	Registration		
	Opening		
November 18	Ceremony, Group	A-110	
	Photo & Lectures		
November 19	Lectures	A 110	
Morning	Lectures	A-110	
November 19	Deporturo		
Afternoon		Departure	

## Program (November 18, 2023, Saturday)

Opening Ceremony		Room: A-110			
8:00-8:10	<ol> <li>Opening remarks by Naihuan Jing</li> <li>Opening remarks by Honglian Zhang</li> </ol>				
8:10-8:30	Group Photo				
Chair: Naihuan Jing					
8:30-9:00	Chongying Dong	Coset theory and Kac-Wakimoto hypothesis			
9:00-9:30	Alexander Molev	Representations of the orthosymplectic Yangian			
9:30-10:00	Vyacheslav Futorny	Representations of Affine Lie superalgebras			
10:00-10:20	Coffee Break				
Chair: Chongying Dong					
10:20-10:50	Jun Hu	On the centers and cocenters of the cyclotomic Hecke and KLR algebras			
10:50-11:20	Chengming Bai	Parity duality of super r-matrices via O-operators and pre-Lie superalgebras			
11:20-11:50	Jinkui Wan	Some multiplication formulas in queer q-schur superalgebras			
11:50-13:30	Lunch				
	Chair: Alex	kander Molev			
13:30-14:00	Naihong Hu	Some new isoclasses of one-parameter exotic small quantum groups arising from the two-parameter setting			
14:00-14:30	Bin Shu	Basic quasi-reductive root data and supergroups			
14:30-15:00	Zhaobing Fan	Global bases, canonical bases and perfect bases for quantum Borcherds-Bozec algebras			
15:00-15:20	Coffee Break				
Chair: Vyacheslav Futorny					
15:20-15:50	Qing Wang	Representations of the orbifold of the parafermion vertex operator algebra K(osp(1 2),k)			
15:50-16:20	Li Ren	Pointed modular tensor category			
16:20-16:50	Jian Zhang	General Capelli-type identities			
16:50-18:30	Dinner				

Chair: Naihong Hu				
8:00-8:30	Xiangmao Ding	Bethe/Gauge Duality for 3d Gauge Theory		
8:30-9:00	Jinwei Yang	Tensor category arising from N=1 super Virasoro algebra		
9:00-9:30	Fei Xu	关于自同态平凡模的上同调刻画		
9: 30-9: 50	Coffee Break			
Chair: Bin Shu				
9:50-10:10	Heguo Liu	从无挠幂零群导出的几个矩阵结果		
10:10-10:40	Lizhong Wang	Modular version of Frobenius theory		
10:40-11:10	Yucong Du	The Blockwise Alperin Weight Conjecture for Finite Groups of Lie Type D		
10:10-11:50	Pengcheng Liao	Frobenius' arithmetic theory and its application to supercharacter theory.		
11:50-13:30	Lunch			
Goodbye				

## Program (November 19, 2023, Sunday)

### Abstracts

**Title:** Parity duality of super \$r\$-matrices via \$\mathcal O\$-operators and pre-Lie superalgebras

Speaker: Chengming Bai (Nankai University)

Abstract: We interpret the homogeneous solutions of the super classical Yang-Baxter equation, also called super \$r\$-matrices, in terms of \$\mathcal O\$-operators by a unified treatment. Furthermore, by a parity reversion of Lie superalgebra representations, a duality is established between the even and odd \$\mathcal O\$-operators. This leads to a parity duality of the super \$r\$-matrices induced by the \$\mathcal O\$-operators in semi-direct product Lie superalgebras. Therefore a pre-Lie superalgebra naturally defines an even \$\mathcal O\$-operator, and hence an odd \$\mathcal O\$-operator by the duality, thereby giving rise to a parity pair of super \$r\$-matrices. This is a joint work with Li Guo and Runxuan Zhang.

Title: Bethe/Gauge Duality for 3d Gauge Theory

**Speaker:** Xiangmao Ding (Academy of Mathematics and Systems Science Chinese Academy of Sciences)

**Abstract:** We give a new effective superpotential for 3d N=2 gauge theories with matter, it ensures a clear Bethe/Gauge correspondence between 3d (A)BCDEFG gauge theories and (closed) open XXZ spin chains with diagonal boundary conditions, respectively, and it also works in the case of 3d BC\_N-type gauge theories which is not previously discussed in the literature. For an A\_2 quiver gauge theory, which is the simplest non-trivial quiver gauge theory, and sl3 open XXZ spin chain with diagonal boundary condition. We demonstrate the correspondence between the vacuum equations of different gauge groups and Bethe Ansatz equations with different boundary parameters. The 2d gauge theory correspondence with XXX spin chain is obviously as a degeneration.

Title: Coset theory and Kac-Wakimoto hypothesis

Speaker: Chongying Dong (University of California)

**Abstract:** Kac-Wakimoto hypothesis is about the positivity of S-matrices and was proposed by Kac-Wakimoto in 1988 for studying cost constructions associated to affine Kac-Moody algebra. We prove recently that Kac-Wakimoto hypothesis holds in categorical coset construction setting. This is a joint work with Li Ren and Feng Xu.

Title: The Blockwise Alperin Weight Conjecture for Finite Groups of Lie Type D

Speaker: Yucong Du (Southern University of Science and Technology)

**Abstract:** The Alperin weight conjecture is one of the most important conjectures in modular representation theory for finite groups. It has been reduced to several inductive conditions for finite simple groups, i.e., to prove the Alperin weight conjecture, it suffices to verify the inductive Alperin weight conditions for finite simple groups. In this talk, we will establish a graph-field automorphism equivariant bijection between weights and a basic set for conformal special orthogonal groups of even dimension, which is a regular embedding of the special orthogonal groups of even dimension. In particular, we can prove that the blockwise Alperin weight conjecture holds for conformal special orthogonal groups of even dimension.

**Title:** Global bases, canonical bases and perfect bases for quantum Borcherds-Bozec algebras

Speaker: Zhaobing Fan (Harbin Engineering University)

**Abstract:** By using new Kashiwara operators, we construct global bases for quantum Borcherds-Bozec algebra. Moreover, we introduce the primitive canonical bases, and prove it coincides with global bases. This is a joint work with Han, Kang and Kim.

**Title:** Representations of Affine Lie superalgebras

Speaker: Vyacheslav Futorny (Southern University of Science and Technology)

**Abstract :** We will discuss representation theory of Affine Lie algebras and superalgebras.

Title: On the centers and cocenters of the cyclotomic Hecke and KLR algebras

Speaker: Jun Hu (Beijing Institute of Technology)

**Abstract:** In this talk, I shall report our recent work on the centers and cocenters of the cyclotomic Hecke and KLR algebras (including type A). We shall characterize minimal length elements in each conjugacy class of the complex reflection group G(r,1,n). The talk are based on some joint work with Shi Lei, and with Shi Xiaolin and Shi Lei.

**Title:** Some new isoclasses of one-parameter exotic small quantum groups arising from the two-parameter setting

Speaker: Naihong Hu (East China Normal University)

**Abstract:** Using the isomorphism theorem of two-parameter small quantum groups of types \$A, B, C, D, F\_4, G\_2\$, when parameters are of odd prime orders, we reduce and obtain a complete classification of one-parameter exotic small quantum groups with the Drinfeld double structures.

For even order  $\left|\left|\right| \left(4, 6, 8\right)\right)$ , we acquire partial classification lists including some undistinguishable representatives by the aforementioned method. In particular, for some representatives of type  $A_2$ , we make a fine distinction via calculating dimension distributions of their simple modules.

Accordingly, to the \$42\$ choosen one-parameter standard small quantum groups, we get the corresponding \$201\$ new isoclasses of one-parameter exotic (i.e.,

non-standard) small quantum groups arising from the two-parameter setting. These form new finite dimensional pointed Hopf algebras with the Drinfeld doubles without the assumption \$(ell, 210)=1\$ in the classification work of Andruskiewitsch-Schneider's Annals of Math. This is a joint work with Xiao Xu.

Title: Frobenius' arithmetic theory and its application to supercharacter theory.

Speaker: Pengcheng Liao (Southern University of Science and Technology)

**Abstract:** In order to study prime decomposition of group-determinant on the field of complex numbers , Frobenius used arithmetic method to develop the theory of characters, which is considered the origin of representation theory for finite groups.

In 2008, P. Diaconis and I. Isaacs first gave the definition of supercharacter theory. We applied the arithmetic method of Frobenius to supercharacter theory and obtained some results. In addition, we will introduce the computer algorithm and the classification of supercharacter theory for some finite groups.

#### Title:从无挠幂零群导出的几个矩阵结果

Speaker: Heguo Liu (Hubei University)

Abstract:从无挠幂零群出发,研究代数闭域上矩阵的根。运用中国剩余定理, 得到矩阵的根能够表示为该矩阵的多项式的充要条件。

Title: Representations of the orthosymplectic Yangian

Speaker: Alexander Molev (The University of Sydney)

**Abstract:** The Yangians form a remarkable family of quantum groups with a deep and substantive representation theory and numerous connections in mathematical physics. The Yangians admit at least three different presentations, including the R-matrix presentation going back to the work of Faddeev's school in the 1980s. It is the R-matrix approach which turned out to be more suitable for the introduction of the super-versions of the Yangians as given by Nazarov (for the general linear Lie superalgebras, 1991) and Arnaudon et al. (for the orthosymplectic Lie superalgebras, 2003). The classification problem for simple finite-dimensional modules over the Yangians associated with the orthosymplectic Lie superalgebras osp(1|2n) and osp(2|2n) has been solved recently (2022). We will discuss the solution which describes the representations in terms of their highest weights. Key arguments rely on an explicit construction of a family of elementary modules of the Yangian for osp(1|2).

#### Title: Pointed modular tensor category

#### Speaker: Li Ren (Sichuan University)

**Abstract:** A modular tensor category is pointed if every simple object is a simple current. We show that any pointed modular tensor category is equivalent to the module category of a lattice vertex operator algebra. Moreover. if the pointed modular tensor category C is the module category of a twisted Drinfeld double associated to a finite abelian group G and a 3-cocycle with coefficients in U(1), then there exists a self dual positive definite even lattice L such that G can be realized an automorphism group of lattice vertex operator algebra V\_L, V\_L^G is also a lattice vertex operator algebra and C is equivalent to the module category of V\_L^G. This is a joint work with C. Dong and S. Ng.

#### Title: Basic quasi-reductive root data and supergroups

#### **Speaker:** Bin Shu (East China Normal University)

**Abstract:** In this talk, we will talk about the root data for basic classical algebraic Lie supergroups which is an algebraical generalization of root-data theory for connected reductive groups. We show an existence and uniqueness theorem, and explain a

classification of connected quasi-reductive algebraic supergroups of monodromy type under the requirement (i) the root system does not contain 0; (ii) g := Lie(G) admits a non-degenerate even symmetric bilinear form. This is a joint work with Rita Fioresi.

Title: Some multiplication formulas in queer q-schur superalgebras

**Speaker:** Jinkui Wan (Beijing Institute of Technology)

Abstract: Building on an earlier work by Du and Wan, where some standard basis for the queer q-Schur superalgebra  $Q_q(n, r; R)$  is defined by a labelling set of matrices and their associated double coset representatives, we investigate the matrix representation of the regular module of  $Q_q(n, r; R)$  with respect to this basis. More precisely, we derive explicitly (resp., partial explicitly) the multiplication formulas of the basis elements by certain even (resp., odd) generators of a queer q-Schur superalgebra. These multiplication formulas are highly technical to derive, especially in the odd case. It requires to discover many multiplication (or commutation) formulas in the Hecke–Clifford algebra Hc r,R associated with the labelling matrices. For example, for a given such a labelling matrix  $A^*$ , there are several matrices w(A),  $\sigma(A)$ ,  $tilde{A}$ , and  $hat{A}$  associated with the base matrix A of A<sup>\*</sup>, where w(A) is used to compute a reduced expression of the distinguished double coset representatives d A, and the other matrices are used to describe the permutation d A and theSDP (commutation) condition between T\_{d\_A} nd generators of the Clifford subsuperalgebra. With these multiplication formulas, we will construct a new realisation of the quantum queer supergroup in a forthcoming paper, and to give new applications to the integral Schur-Olshanski duality and its associated representation theory at roots of unity. This is a joint work with Jie Du, Haixia Gu and Zhenhua Li.

Title: Modular version of Frobenius theory

Speaker: Lizhong Wang (Peking University)

**Abstract:** In this talk, we will introduce the factorization and construction of the modular group determination.

**Title:** Representations of the orbifold of the parafermion vertex operator algebra K(osp(1|2),k)

Speaker: Qing Wang (Xiamen University)

Abstract: In this talk, we present our recent progress about the orbifold theory of parafermion vertex operator algebras K(osp(1|2),k) associated to the affine vertex operator superalgebra  $L_{\langle videhat \{ osp(1|2) \} }(k,0)$  with any positive integer k. In particular, we classify the irreducible modules for the orbifold of the parafermion vertex operator algebra K(osp(1|2),k). This is a joint work with Cuipo Jiang.

#### Title:关于自同态平凡模的上同调刻画

Speaker: Fei Xu (Shantou University)

Abstract:设 G 为有限群。自同态平凡模是模表示的重要研究对象之一,它们的 同构类构成所谓的自同态平凡模群 T(G)。Balmer 和 Grodal 采用 Cech 上同调 和范畴上同调,分别刻画了 T(G)。通过拓扑斯上同调理论,我们说明他们的刻 画本质上相同。在此基础上,我们得到一个新的刻画。

Title: Tensor category arising from N=1 super Virasoro algebra

Speaker: Jinwei Yang (Shanghai Jiao Tong University)

**Abstract:** N=1 super Virasoro algebra plays an important role in the study of superstring theory. We construct tensor structures on the representation categories of N=1 super Virasoro algebra at all central charges, and then prove rigidity of these tensor categories. We will also relate these categories with those of the affine Lie superalgebra osp(1|2). This talk is based on joint work with T. Creutzig, R. McRae

and F. Orosz.

Title: General Capelli-type identities

**Speaker:** Jian Zhang (Central China Normal University)

**Abstract:** The Capelli identity is one of the most important results of classical invariant theory. It played the central role in Weyl's famous book, "The Classical Groups". In this talk, we will discuss the Capelli identities from different points of view. A new generalization of Capelli identity for immanant will be introduced. We will also present generalized Turnbull's identities for both symmetric and antisymmetric matrices, as well as the generalized Howe-Umeda-Kostant-Sahi identities for antisymmetric matrices which confirm the conjecture of Caracciolo, Sokal and Sportiello.

### List of Participants

No	Name	Institute	Email
1	Chengming Bai 白承铭	Nankai University	baicm@nankai.edu.cn
2	Xiangmao Ding 丁祥茂	Academy of Mathematics and Systems Science Chinese Academy of Sciences	xmding@amss.ac.cn
3	Chongying Dong 董崇英	University of California	dong@ucsc.edu
4	Yucong Du 杜予聪	Southern University of Science and Technology	duke@pku.edu.cn
5	Zhaobing Fan 樊赵兵	Harbin Engineering University	fanzhaobing@hrbeu.edu.cn
6	Vyacheslav Futorny	Southern University of Science and Technology	futorny@sustech.edu.cn
7	Weiqiang He 何伟强	Sun Yat-sen University	hewq25@mail.sysu.edu.cn
8	Jun Hu 胡峻	Beijing Institute of Technology	junhu404@bit.edu.cn
9	Naihong Hu 胡乃红	East China Normal University	nhhu@math.ecnu.edu.cn
10	Pengcheng Liao 廖鹏程	Southern University of Science and Technology	liaopc@sustech.edu.cn
11	Aming Liu 刘阿明	Hainan University	amliu@hainanu.edu.cn
12	Heguo Liu 刘合国	Hubei University	ghliu@hubu.edu.cn
13	Ming Liu 刘明	South China University of Technology	mamliu@scut.edu.cn
14	Alexander Molev	The University of Sydney	alexander.molev@sydney.edu.au
15	Li Ren 任丽	Sichuan University	renl@scu.edu.cn
16	Bin Shu 舒斌	East China Normal University	bshu@math.ecnu.edu.cn
17	Jinkui Wan 万金奎	Beijing Institute of Technology	wjk302@bit.edu.cn
18	Lizhong Wang 王立中	Peking University	lwang@math.pku.edu.cn

19	Qing Wang 王清	Xiamen University	qingwang@xmu.edu.cn
20	Yongjie Wang 汪永杰	Hefei University of Technology	wyjie@mail.ustc.edu.cn
21	Fei Xu 徐斐	Shantou University	fxu@stu.edu.cn
22	Jinwei Yang 杨进伟	Shanghai Jiao Tong University	jinwei2@sjtu.edu.cn
23	Jian Zhang 张健	Central China Normal University	jzhang@ccnu.edu.cn
24	Jing Zhao 赵静	Hainan University	jzhao0@163.com
25	Naihuan Jing 景乃桓	North Carolina State University	nhjing@shu.edu.cn
26	Honglian Zhang 张红莲	Shanghai University	hlzhangmath@shu.edu.cn
27	Jiancai Sun 小建才	Shanghai University	jcsun@shu.edu.cn
28	Chiheng Zhang 张弛恒	Shanghai University	zchmath@shu.edu.cn

### Profile of The Department of Mathematics

The Department of Mathematics is the home of 109 well qualified individuals, 93 of who are full-time faculty members. The faculty team has 29 professors, 36 doctoral advisors, 28 associate professors, 1 Double hired academician, 3 national leading talents including Distinguished Young Scholars, 9 Shanghai municipal talents. Every member under 50 has a doctoral degree, 95% of who are overseas graduates or have overseas research experiences. The Department has over 500 undergraduates, 200 graduates, and 60 doctoral candidates. The Department of Mathematics offers first-level doctoral program in mathematics, first-level master's program in mathematics and, master's program in statistics. It also has a mathematical postdoctoral research station. The discipline of Mathematics is a Shanghai municipal education commission key discipline, a Shanghai key discipline, a Shanghai first-class discipline, and a Shanghai plateau discipline. In addition, Shanghai Institute of Applied Mathematics and Systems Science, Institute of Core Mathematical Research of Shanghai University, Shanghai University Open Laboratory for Operations Research & Optimization, International Research Center for Tensor and Matrix Theory of Shanghai University, and Institute of Systems Science of Shanghai University are all affiliated to the Department of Mathematics.

In 2022, the Department of Mathematics of SHU is ranked 103th in the best global Mathematics Programs by US News Rankings of Universities in the World. According to the latest data from Essential Science Indicators database, 333 mathematical research institutions are recognized as world's top 1%, among which, Shanghai University ranks 154, entering the top 4.62‰ in the world. In recent years, The Department of Mathematics hosts nearly 300 academic talks every year for extensive academic exchanges and research collaborations. Among the speakers, there are more than 30 domestic and foreign academicians, including Fields Medalists – Zelmanov and Birkar. Besides, the Department of Mathematics has hosted or undertook more than 50 grand international academic conferences. In the past five years, the faculty members of Department of Mathematics have published or accepted nearly 600 academic papers on top-tier international academic journals, such as Comm. Pur. Appl. Math., Adv. Math., Arch. Ration. Mech. An., SIAM Review, Trans. Amer. Math. Soc., Automatica, J. Algebra, J. Comput. Phys., SIAM J. Numer. Anal., SIAM. J. Appl. Math., SIAM J. Optim., and Math. Program. Moreover, we have received 55 NSFC grants in the past five years.