Titles and Abstracts

A quantitative Koukoulopoulos-Maynard theorem in metric Diophantine approximation

Christoph AISTLEITNER
Graz University of Technology

The Duffin-Schaeffer conjecture was one of the central open problems in metric Diophantine approximation for a period of almost 80 years, before finally being resolved by Koukoulopoulos and Maynard in 2019. Roughly speaking, the conjecture (now theorem) asserts that almost all real numbers admit infinitely many co-prime rational approximations of a certain quality, as long as the function measuring the quality of these approximations satisfies a natural divergence condition. In this talk, we prove a quantitative version of the theorem, by calculating for almost all reals the asymptotic order of the number of such co-prime rational approximations of prescribed quality, up to a given threshold for the size of the denominator

Discretized rotation and Salem numbers

Shigeki AKIYAMA
University of Tsukuba.

Discretized rotation acting on $\mathbf{Z}^d$ serves interesting number theoretical problems, but they are usually pretty difficult. I review several results and open problems on the case $d = 2$.
Lately, with H. Hichri, we found a strong connection between this discretized rotation and the Markovian property of beta expansion in the Salem number base. By this bridge, we guess that for a given Salem number β of degree 6 , almost all β^n ($n = 1, 2, \ldots$) give rise to the Markovian system. Without any hypothesis, we can show that half of $n \in \mathbf{N}$ satisfies this property, w.r.t.natural density of $\mathbf{N}$.

Simultaneous Diophantine approximation on the Veronese curve

Dzmitry BADZIAHIN
University of Sydney

Measuring the set of simultaneously well approximable points on manifolds is one of the most intricate problems in metric theory of Diophantine approximation. Unlike the dual case of well approximable linear forms, the results here are known to depend on a manifold. For example, some of the manifolds do not contain simultaneously very well approximable points at all, while for the others the set of such points always has positive Hausdorff dimension. In this talk, we will closely look at the Veronese curve$\{x, x^2, x^3, \ldots, x^n\}$, discuss what is known about the sets of simultaneously well approximable points on it and provide several new results. In particular, for $\mathrm{n} = 3$ we provide the Hausdorff dimension of the set of x such that $\lambda_3(x) \leq \lambda$ where $\lambda \leq \frac{2}{5}$ or $\lambda \geq \frac{7}{9}$.

Some recent developments on Brjuno functions

Ayreena BAKHTAWAR
Scuola Normale Superiore di Pisa

An irrational number is called a Brjuno number if the sum of the series of $\log\left(q_{n+1}\right)/q_n$ converges, where q_n is the denominator of the n-th principal convergent of the regular continued fraction. Brjuno numbers play an important role in the study of small divisors problems in dynamical systems. In 1988, J.-C. Yoccoz proved the optimality of the Brjuno condition for the linearization of quadratic polynomials, introducing a version of the Brjuno function well suited for the estimate the size of Siegel disks. Motivated by the work of Balazard-Martin 2020, we study the scaling properties of the Brjuno function around its global and local minima. We give results both for the Brjuno function associated to the usual regular continued fraction expansion as well as for the generalized Brjuno function associated to α continued fractions where $\alpha \in [1/2, 1)$. This is based on joint work with Carlo Carminati and Stefano Marmi.

On Korobov's problem on the discrepancy associated to normal numbers

Verónica BECHER
Facultad de Ciencias Exactas y Naturales Universidad de Buenos Aires

A real number x is normal to base 2 if the sequence$(2^n x)_{n>0}$ is uniformly distributed modulo 1. That is, the discrepancy $D_N\left(\left(2^n x(\operatorname{mod} 1)\right)_{n>0}\right)$ goes to 0 as N goes to infinity. In 1955 Korobov posed the problem of finding a function f with maximum decay such that there is a number x for which $D_N\left(\left(2^n x(\operatorname{mod} 1)\right)_{n>0}\right) < f(N)$, for $N = 1, 2, 3, \ldots$. We show work in progress on the study of the discrepancy of sequence $\left(2^n x(\operatorname{mod} 1)\right)$ restricted to dyadic intervals of fixed size: we construct normal numbers x for which the discrepancy of the first N terms of $\left(2^n x(\operatorname{mod} 1)\right)$ at the dyadic intervals $[0, 1/2)$ and $[1/2, 1)$ is $O(\log N)/N$.

On the Polynomial Szemerédi Theorem over Finite Commutative Rings

Andrew BEST
BIMSA

The polynomial Szemerédi theorem implies that, for any $\delta \in (0, 1)$, any family $\{P_1, \ldots, P_m\} \subset \mathbb{Z}[y]$ of nonconstant polynomials with constant term zero, and any sufficiently large $N \in \mathbb{N}$, every subset of $\{1, \ldots, N\}$ of cardinality at least δN contains a nontrivial configuration of the form $\{x, x + P_1(y), \ldots, x + P_m(y)\}$. When the polynomials are assumed independent, one can expect a sharper result to hold over finite fields, special cases of which were proven recently in various articles by Bourgain and others, culminating with a 2018 result of Peluse, which deals with the general case of independent polynomials. In this talk we discuss, over general finite commutative rings, a version of the polynomial Szemerédi theorem for multivariable independent polynomials $\{P_1, \ldots, P_m\} \subset \mathbb{Z}\left[y_1, \ldots, y_n\right]$, deriving new combinatorial consequences, such as the following. Let $\mathcal{R}$ be a collection of finite commutative rings satisfying a technical condition which limits the amount of torsion. There exists $\gamma \in (0, 1)$ such that, for every $R \in \mathcal{R}$, every subset $A \subset R$ of cardinality at least $|R|^{1-\gamma}$ contains a nontrivial configuration $\left\{x, x + P_1\left(y_1, \ldots, y_n\right), \ldots, x + P_m\left(y_1, \ldots, y_n\right)\right\}$ for some

$x, y_1, \ldots, y_n \in R$. The move from finite fields to finite commutative rings introduces many unexpected obstacles, which we will try to give a flavor of, and depends heavily on the idea of asymptotic total ergodicity.

On angles between linear subspaces in $\mathbb{R}^4$ and singularity

Artem CHEBOTARENKO
Moscow State University

The singularity phenomenon in the theory of Diophantine approximations was discovered by Khintchine. He proved that for any function $\varphi(t)$ that decreases to zero as t approaches infinity, there exist two real numbers α_1 and α_2 such that for the corresponding irrationality measure function

$$\psi_{\alpha_1,\alpha_2}(t) = \min_{x_0,x_1,x_2\in\mathbb{Z}:1\leq\max(|x_1|,|x_2|)\leq t} |x_0 + x_1\alpha_1 + x_2\alpha_2|$$

inequality

$$0 < \psi_{\alpha_1,\alpha_2}(t) \leq \varphi(t), \quad \forall t > 1.$$

It should be noted that according to Minkowski's theorem on convex bodies, for any two numbers α_1 and α_2, the inequality $\psi_{\alpha_1,\alpha_2}(t) < t^{-2}$ always holds. A pair of numbers α_1 and α_2 is called singular if for any $\varepsilon > 0$, there exists t_0 such that for all $t > t_0$, the inequality $\psi_{\alpha_1,\alpha_2}(t) < \varepsilon t^{-2}$ holds. The general result for arbitrary systems of linear forms was obtained by Jarnik. Currently, there are many studies dedicated to the existence of singular vectors and their generalizations, as well as applications. We generalize the singularity phenomenon for the problem of finding rational subspaces that form the smallest angle with a given irrational linear subspace. Here, we consider only the simplest non-trivial case, which involves approximating a two-dimensional irrational subspace in $\mathbb{R}^4$ with two-dimensional rational subspaces in $\mathbb{R}^4$.

Large Weyl sums and Hausdorff dimension

Changhao CHEN(陈昌昊)
Anhui University

I will introduce Weyl sums and some metric results about these sums. In particular, I show the exact value of the Hausdorff dimension of the set of coefficients of large Gauss sums. Diophantine approximation plays a key role in our proof. Joint work with Roger Baker and Igor Shparlinski.

Dynamical view of Tijdeman's solution of the Chairman Assignment problem

Nicolas CHEVALLIER
University of Haute Alsace

Let $T : \mathbb{T}^d \to \mathbb{T}^d$ be an ergodic translation. Given a partition $P_1, \ldots, P_k$, each point x of the torus can be encoded by the sequence (u_n) defined by $T^n(x) \in P_{u_n}$. We will explain how to construct a partition of the torus based on the Tijdeman solution of the chairman assignment problem. This partition has an interesting discrepancy property. This is a joint work with V. Berthé, O. Carton, W. Steiner and R. Yassawi.

A dynamical application of Ostrowki's algorithm (joint work with Pascal Hubert)

Sébastien FERENCZI
Aix-Marseille Université

We look at d-point extensions of a rotation of angle α with r marked points, which are also interval exchange transformations and generalize the famous examples of Veech 1969 and Sataev 1975. The Ostrowski expansions of the marked points by α allow us to study the dynamical properties of minimality, unique ergodicity and rigidity for these examples, as well as the word-combinatorial property of linear recurrence for their natural codings. In particular, this allows us to build the first examples of non linearly recurrent and non rigid interval exchange| transformations.

On a new version of Siegel's lemma

Lenny FUKSHANSKY
Claremont McKenna College

We establish a new version of Siegel's lemma over a number field K, providing a bound on the maximum of heights of basis vectors of a subspace of K^n. In addition to the small-height property, the basis vectors we obtain satisfy certain sparsity condition. Further, we produce a nontrivial bound on the heights of all the possible subspaces generated by subcollections of these basis vectors. Our bounds are absolute in the sense that they do not depend on the field of definition. The main novelty of our method is that it uses only linear algebra and does not rely on the geometry of numbers or the Dirichlet box principle employed in the previous works on this subject. This is joint work with Max Forst.

Uniformly distributed periodic orbits of Arnold's cat map

Shaobo GAN(甘少波)
Peking University

Several years ago, Yi Shi asked if Arnold's cat map has a sequence of periodic orbits O_n uniformly distributed in the following sense:

$$\liminf_{n\to\infty} \delta\left(O_n\right)^2 \pi\left(O_n\right) > 0?$$

where $\delta\left(O_n\right)$ is the minimal distance of two points in O_n and $\pi\left(O_n\right)$ is the period of O_n. In this talk, I will give an affirmative answer to Shi's question. In fact, I will show that such uniformly distributed periodic orbits exists for any ergodic endomorphisms on tori. This is a joint work with Daohua Yu.

Fourier analysis of fractal measures and Diophantine approximation

Xiang GAO(高翔)
Hubei University

It is well known that Lebegues almost everywhere points are normal numbers. Firstly, we will survey some recent results on normal numbers on fractals. And also discuss how to use some powerful tools from probability theory, harmonic analysis, measure rigidity in ergodic theorem, to study the problems of normality on fractals. The second part is devoted to investigate the Fourier transform of fractals measure, including the Fourier decay estimate of some self-similar measure with special algebraic contractive ratios, and of projected measures of product of Bernoulli convolutions under different directions parameters. We study various kinds of conditions which are imposed on fractal measures to guarantee that generic points are absolutely normal. Our main goal is to show how we use Diophantine approximation tools, such as Thue-Siegel-Roth theorem and Linear forms in logarithms to deal with the Fourier analytic behaviors of some fractal measures.

Hausdorff dimension estimates for Sudler products with positive lower bound

Dmitrii GAYFULIN
Institute for Information Transmission Problems of RAS

For $\alpha \in \mathbb{R}$ and $N \in \mathbb{N}$, the Sudler product at stage N is defined as

$$P_N(\alpha) := \prod_{r=1}^{N} 2|\sin \pi r\alpha|.$$

It is known that $\liminf_{N\to\infty} P_N(\alpha) = 0$ whenever the sequence of partial quotients in the continued fraction expansion of α contains infinitely many digits greater than 6. In fact, it was conjectured by Lubinsky that liminf equals zero for all real numbers. However, it was shown by Verschueren and independently by Grepstad, Kaltenböck and Neumüller that for α equal to the golden ratio $[0; 1, 1, 1, \ldots]$ one has $\liminf_{N\to\infty} P_N(\alpha) > 0$. Later, some other counterexamples were found, all of them also were quadratic irrationals. In a joint paper with Manuel Hauke, we show that $\liminf_{N\to\infty} P_N(\alpha) > 0$ whenever the sequence of partial quotients in the continued fraction expansion of α exceeds 3 only finitely often. Furthermore we deduce a nontrivial lower bound of the HD of the set of α satisfying $\liminf_{N\to\infty} P_N(\alpha) > 0$. The research was funded by the Russian Science Foundation (project No. 22-4105001).

On uniform Diophantine exponents of lattices

Oleg GERMAN
Moscow State University

We are going to discuss uniform analogues of Diophantine exponents of lattices. There are at least two natural ways to define them. We call the respective exponents weak and strong. It is well known that the uniform Diophantine exponent of a real number is trivial. We are going to show that the strong Diophantine exponents of lattices are also trivial, whereas the weak ones turn out to be nontrivial.

The research was funded by the Russian Science Foundation (project No. 22-41-05001).

Density of minimal points and Bergelson-Hindman question

Wen HUANG(黄文)
University of Science and Technology of China

Furstenberg's multiply recurrent theorem states that any dynamical system has multiply recurrent points, and points out that this result is equivalent to the van der Waerden theorem. An equivalent form of van der Waerden's theorem is that any piecewise syndetic subset of a natural number contains any arbitrarily long arithmetic progressions. In this talk, we discuss the correlation between multiple recurrence and piecewise syndetic set, and provide some applications in combinatorial number theory. In 1998 Furstenberg and Glasner proved that the set composed of the first term and common difference of all arithmetic progressions of length k appearing in the piecewise syndetic subset of natural numbers is also piecewise syndetic subsets in $\mathbb{Z}^2$. In 2001 Bergelson and Hindman raised the question of whether the polynomial version of this result holds. We will answer the Bergelson-Hindman question by showing the density of minimal points of a dynamical system of $\mathbb{Z}^2$ action associated with the piecewise syndetic set and the polynomials. This based on joint works with Prof. Shao and Ye.

Local distribution of rational points on flag varieties

Zhizhong HUANG(黄治中)
AMSS

The Manin–Peyre principle predicts that rational points of bounded height on Fano varieties are equidistributed in the adelic space. A local version of this principle is concerned with how rational points are distributed around a fixed point, whose Diophantine approximation property plays a key role. We shall present various results for quadrics based on the circle method (joint work with D. Schindler and A. Shute), and for flag varieties based on homogeneous dynamics (joint work in progress with N. de Saxc´e).

Sums of squares in short intervals and Jacobi-type modular forms

Alexander KALMYNIN
National Research University Higher School of Economics

The problem of estimating intervals between consecutive sums of two squares is among the most well-known questions in analytic number theory. It turns out that one can connect this problem with behaviour of certain two-variable function possessing properties similar to the Jacobi theta-function. In this talk, we are going to construct of this function and show that its Taylor expansion produces a sequence of rational functions with remarkable properties. These results can be generalised to arbitrary modular forms with respect to certain Hecke subgroup of full modular group.
The research was funded by the Russian Science Foundation (project No. 22-41- 05001).

Distribution of discriminants of integer polynomials

Nikolai KALOSHA
Mathematics of the National Academy of Sciences of Belarus

This is a joint work with V.I.Bernik.
In Diophantine approximation, it is very natural to ask various questions about the distribution of discriminants of integer polynomials. The first results in this direction were obtained by Davenport [1], and extended by Kaliada [2] for the degrees $n = 2$, $n = 3$.
In 2015, Beresnevich, Bernik and Götze obtained the following result [3].

Let a natural n be fixed, let $Q > 1$ be a sufficiently large real number, and let $\mathcal{P}_n(Q)$ denote the set of polynomials with integer coefficients that have the degree n and heights bounded from above by Q. Further, let $D(P)$ denote the discriminant of a polynomial P :

$$D(P) = a_n^{2n-2} \prod_{i<j} (\alpha_i - \alpha_j)^2 ,$$

where a_n is the leading coefficient of P, and $\alpha_1, \ldots, \alpha_n$ are its complex roots. Now let the set $\mathcal{P}_n(Q, v)$ consist of polynomials from $\mathcal{P}_n(Q)$ such that

$$0 < D(P) \leq Q^{2n-2-2v}.$$

Then for $0 \leq v \leq n-1$ we have

$$\#\mathcal{P}_n(Q, v) \gg Q^{n+1-\frac{n+2}{n}v}.$$

However, obtaining similarly tight upper bounds proved itself to be a difficult problem. In our talk, we are going to discuss this family of problems in more detail and present several recent results related to bounds for $\#\mathcal{P}_n(Q, v)$.

References

(a) H. Davenport. A note on binary cubic forms, Mathematika 8 (1961), 58-62.
(b) D. Kaliada, F. Götze, O. Kukso. The asymptotic number of integral cubic polynomials with bounded heights and discriminants, Lith. Math. J. 54 (2014), 150-165.
(c) V. Beresnevich, V. Bernik, F. Götze. Integral polynomials with small discriminants and resultants, Adv. Math. 298 (2016), 393-412.
Noncommutative Hamiltonian structures and quantizations on preprojective algebras

On a Kurzweil type theorem via ubiquity

Taehyeong KIM
Tel Aviv University

Kurzweil's theorem ('55) is concerned with zero-one laws for well approximable targets in inhomogeneous Diophantine approximation under the badly approximable assumption. In this talk, we discuss the divergent part of a Kurzweil type theorem via ubiquity when the badly approximable assumption is relaxed. We also discuss some counterparts of Kurzweil's theorem.

On the distribution of sequences of the form $(q_n\alpha)$

Simon KRISTENSEN
Aarhus University

Abstract: The distribution of sequences of the form $(q_n\alpha)$ with (q_n) a sequence of integers and α a real number have attracted quite a bit of attention, for instance due to their relation to inhomogeneous Littlewood type problems. In this talk, we will provide some results on the Lebesgue measure and Hausdorff dimension on the set of points in the unit interval approximated to a certain rate by points from such a sequence. A feature of our approach is that we obtain estimates even in the case when the sequence (q_n) grows rather slowly. This is joint work with Tomas Persson.

The shrinking target problem for matrix transformations of tori: revisiting the standard problem

Bing LI(李兵)
South China University of Technology

We study the shrinking target problem for matrix transformations of tori. First we prove a quantitative form of this zero-one criteria that describes the asymptotic behavior of the counting function. The counting result makes use of a general quantitative statement that holds for a large class measure-preserving dynamical systems (namely, those satisfying the so called summable-mixing property).
We next turn our attention to the Hausdorff dimension of the shrinking target set. In the case the targets are balls, rectangles or hyperboloids we obtain precise formulae for the dimension. These shapes correspond, respectively to the simultaneous, weighted and multiplicative theories of classical Diophantine approximation. The dimension results for balls generalize those obtained in Hill and Velani (1999) for integer matrices to real matrices. This is joint work with Lingmin Liao, Sanju Velani and Evgeniy Zorin.

From BCZ map to RH

Yiming LI(李一鸣)
Tsinghua University

In 1924, J´erˆome Franel and Edmund Landau characterized Riemann Hypothesis in terms of Farey sequence which attracts many mathematicians to study Farey sequence including Boca, Cobeli and Zaharescu. They give the definition of BCZ map which leads to the possibility of studying RH in a dynamical system point of view. In this talk, we'll first introduce BCZ map and then reformulate RH in terms of estimates of L1-averages of BCZ cocycle along periodic orbits of the BCZ map, replace the cocycle with a discrete approximation and establish that the discretized analog of RH holds in a stronger sense. We'll also show the relationship between BCZ map and diophantine approximation.

Simultaneous shrinking target problem of the dynamical systems $\times 2$ and $\times 3$

Lingmin LIAO(廖灵敏)
Wuhan University

We study the size of the sets of points in the unit interval whose orbits under the dynamical systems $\times 2$ and $\times 3$ simultaneously fall into a given sequence of shrinking balls (shrinking targets). We prove a zero-one law for the Lebesgue measure of such sets. We also obtain the Hausdorff dimension formula when the radii of the balls decrease exponentially. One part of the dimensional formula is established under the famous abc conjecture. This is a joint work with Bing Li, Sanju Velani and Evgeniy Zorin.

Lacunary sequences, diophantine approximation and Glasner sets

Yuval PERES
BIMSA

In this expository talk, I will first describe the connection of diophantine approximation to chromatic number of Cayley graphs on the integers, leading to the Katznelson conjecture on Bohr recurrence. Work with Schlag gave almost sharp results for the lacunary case, improving previous bounds by Khinchin and Katznelson but leaving a logarithmic gap. This method was developed further by Moschevitin. I will also discuss quantitative versions (due to Berend, Alon and myself) of the Glasner lemma, that states: For any infinite set A in $[0,1)$ and any $\mathrm{r}_{\llcorner}0$, there exists an integer n so that nA is r-dense $\mathrm{mod}1$, i.e. the maximal gap between elements of nA in the reals $\mathrm{mod}1$ is less than r.

Main references:

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2. Alon, Noga, and Yuval Peres. "Uniform dilations." Geometric and Functional Analysis GAFA 2 (1992): 1-28.
3. Moshchevitin, N. "Density modulo 1 of lacunary and sublacunary sequences: application of Peres-Schlag's construction." Journal of Mathematical Sciences 180, no. 5 (2012).

The Duffin-Schaeffer conjecture for systems of linear forms

Felipe RAMIREZ
Wesleyan University

The Duffin-Schaeffer conjecture, posed in 1941 and proved by Koukoulopoulos and Maynard in 2020, gives a complete characterization of when almost every real number can be approximated by infinitely many reduced rationals at a given rate of approximation. The characterization is given by the divergence or convergence of a measure sum associated to the desired rate of approximation. Between 1941 and 2020, many related results were proved, including a higher dimensional version of the conjecture by Pollington and Vaughan in 1990. The work of Koukoulopoulos-Maynard and Pollington-Vaughan hinges on controlling the measures of pairwise intersections of certain sets that arise naturally in the problem. I will discuss how the control that they achieve can also be used to establish a version of the Duffin-Schaeffer conjecture for systems of linear forms that was conjectured in 2009 by Bersnevich, Bernik, Dodson, and Velani.

Bounded orbits with prescribed limit points

Anurag RAO
Peking University

Abstract: Consider a homogeneous space X - such as the space of lattices or the space of grids in Euclidean space - and the action of a diagonal one-parameter subgroup. In 1996 Kleinbock-Margulis proved that the set of points in X with precompact forward orbit has full Hausdorff dimension. We study a slight refinement of this problem: given some x in X, does the same dimension result hold for points with precompact forward orbit accumulating on x ? We prove that, barring a certain topological condition on x, the codimension of this set is less than the

dimension of the centralizer of the subgroup. All of this is inspired by a 1969 result of Davenport-Schmidt regarding badly approximable matrices and we explain this along with some new applications to Diophantine approximation. Joint work with Manfred Einsiedler and Dmitry Kleinbock.

Recent progress on Schmidt's problem of rational approximations to subspaces

Nicolas de SAXCÉ
Institut Galilée, LAGA, Université Sorbonne Paris Nord

The goal of the talk will be to present Schmidt's questions from his 1967 paper "On heights of algebraic subspaces and diophantine approximation" and recent progress by various authors on different aspects of the subject. I will also try to highlight the remaining open questions in Schmidt's program that I find most intriguing.

Integrability of extremal bundles for symplectic Anosov diffeomorphisms on $\mathbb{T}^4$

Yi SHI(史逸)
Sichuan University

We show that every symplectic Anosov diffeomorphism f on $\mathbb{T}^4$ which is C^1-close to an irreducible non-conformal automorphism $A \in \mathrm{Sp}(4, \mathbb{Z})$, the extremal symplecitc subbundle of f is integrable if and only if f is smoothly conjugate to A.

Dense Forests Constructed from Grids

Victor SHIRANDAMI
The University of Manchester

A dense forest is a subset F of $\mathbb{R}^n$ with the property that for all $\epsilon > 0$ there exists a number $V(\epsilon) > 0$ such that all line segments of length $V(\epsilon)$ are ϵ-close to a point in F. The function V is called a visibility function of F. In the work of Adiceam, Solomon, and Weiss (2022) it was shown that, given an $\eta > 0$, there exists dense forests constructed from a finite union of grids which admit a visibility function of order $\epsilon^{-(n-1)-\eta}$. This is arbitrarily close to optimal in the sense that a finite union of grids admits only visibility functions bounded below at order $\epsilon^{-(n-1)}$. In this talk we will first provide a necessary and sufficient condition for a finite union of grids to be a dense forest in terms of the irrationality properties of the matrices defining them. This result, however, does not provide an explicit visibility function. We provide a result to complement this, namely, given an $\eta > 0$ there exists a $k \in \mathbb{N}$ such that almost all unions of k grids are dense forests admitting a visibility function of order $\epsilon^{-(n-1)-\eta}$. That is, the visibility achieved in the construction of Adiceam, Solomon, and Weiss almost always occurs. The notion of 'almost all' is considered with respect to several underlying measures which are defined according to the Iwasawa decomposition of the matrices used to define the grids.

Covering Radii in Positive Characteristic

Noy SOFFER-ARANOV
Israel Institute of Technology

Abstract: A fascinating question in geometry of number pertains to the covering radius of lattice with respect to an interesting function. For example, given a convex body C and a lattice L in $\mathbb{R}^d$, it is interesting to ask what is the infimal $r \geq 0$ such that $L + rC = \mathbb{R}^d$. Another interesting covering radius is the multiplicative covering radius, which connects to dynamics due to its invariance under the diagonal group. It was conjectured by Minkowski that the multiplicative covering radius is bounded above by 2^{-d} and that this upper bound is obtained only on $A\mathbb{Z}^d$. In this talk I will discuss surprising results pertaining to covering radii in the positive characteristic setting and discover several surprising results. Some of my results include explicitly connecting between the covering radii with respect to convex bodies and successive minima and proving a positive characteristic analogue of Minkowski's function.

Non-dense orbits: an approach via specification-like properties

Peng SUN(孙鹏)
Central University of Finance and Economics

Weak variations of Bowen’s specification property hold for a broad class of dynamical systems, including certain compact homogeneous spaces. Meanwhile, some elegant results can be obtained via such specification-like properties. In particular, we show that certain sets of points with non-dense orbits, which can avoid a rather large set, carry full topological pressure.

Nontrivial time-changes of unipotent flows on quotients of Lorentz groups

Siyuan TANG(唐思远)
Peking University

The theory of unipotent flows plays a central role in homogeneous dynamics. Time-changes are a simple perturbation of a given flow. In this talk, we shall discuss the rigidity of time-changes of unipotent flows. More precisely, we shall see how to utilize the branching theory of the complementary series, combining it with the works of Ratner and Flaminio-Forni to get to our purpose.

On the periodicity of Somos sequences

Alexey USTINOV
Karlsruhe Institute of Technology

For integer $k \geq 4$ Somos $-k$ sequence is a sequence generated by quadratic recurrence relation of the form

$$s_{n+k}s_n = \sum_{j=1}^{[k/2]} \alpha_j s_{n+k-j} s_{n+j},$$

where α_j are constants and $s_0, \ldots, s_{k-1}$ are initial data. Among them exist a class of sequences

with many properties.
They are finite rank sequences. The sequence $\{s_n\}_{n=-\infty}^{\infty}$ has a (finite) rank r if maximal rank of two infinite matices $(s_{m+n}s_{m-n})|_{m,n=-\infty}^{\infty}$, $(s_{m+n+1}s_{m-n})|_{m,n=-\infty}^{\infty}$
is r. If $r = 2$ then general term of Somos sequence can be expressed in terms of elliptic function. One can consider a general finite rank sequence as a sequence admitting more complicated addition theorem.
Presumably the following properties are more or less equivalent: finitness of the rank, Laurent phenomenon, periodicity $(\mathrm{mod} N)$, solvability in theta-functions. The talk will be mostly devoted to periodicity $(\mathrm{mod} N)$ of general finite rank sequences. The research was funded by the Russian Science Foundation (project No. 22-4105001).

Representation of integers by binary forms

Michel WALDSCHMIDT
Sorbonne Université

We consider some families of binary binomial forms $aX^d + bY^d$, with a and b integers. Under suitable assumptions, we prove that every rational integer m with $|m| \geq 2$ is only represented by a finite number of the forms of this family (with varying d, a, b). Furthermore the number of such forms of degree $\geq d_0$ representing m is bounded by $O\left(|m|^{(1/d_0)+\epsilon}\right)$ uniformly for $|m| \geq 2$. We also prove that the integers in the interval $[-N, N]$ represented by one of the forms of the family with degree $d \geq d_0$ are almost all represented by some form of the family with degree $d = d_0$.
In a previous paper we investigated the particular case where the binary binomial forms are positive definite. We now treat the general case by using a lower bound for linear forms of logarithms.
This is a joint work with Etienne Fouvry.

Hausdorff dimension of Cartesian product of limsup sets in Diophantine approximation

Baowei WANG(王保伟)
Huazhong University of Science and Technology

In 1962, Erdos observed that every real number can be represented as the sum of two Liouville numbers. Thus it follows that the Cartesian product of two sets of Liouville numbers is of dimension 1. More or less, this is a surprising result since the dimension of the set of Liouville numbers is 0. Motivated by this, we consider the dimension of the Cartesian product of limsup sets in Diophantine approximation. It will be seen that for many limsup sets, the dimension of their Cartesian product can be as large as possible.

Time change rigidity for unipotent flows

Daren WEI(魏达仁)
National University Singapore

Two flows are said to be Kakutani equivalent if one is isomorphic to the other after time change, or equivalently if there are Poincare sections for the flows so that the respective induced maps are isomorphic to each other. Ratner showed that if $G = \mathrm{SL}(2,\mathbb{R})$ and Γ is a lattice in G, and if U is a one parameter unipotent subgroup in G then the U action on G equipped with Haar measure is loosely Bernoulli, i.e. Kakutani equivalent to a circle rotation. Thus any two such systems $\mathrm{SL}(2,\mathbb{R})/\Gamma$ are Kakutani equivalent to each other. On the other hand, Ratner showed that if $G' = \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$and Γ' is a reducible lattice, and U' is the diagonally embedded one parameter unipotent subgroup in G', then $(G'/\Gamma', U', m)$ is not loosely Bernoulli. We show that in fact in this case and many other situations one cannot have Kakutani equivalence between such systems unless they are actually isomorphic. This is a joint work with Elon Lindenstrauss.

An upper bound of the Hausdorff dimension of singular vectors on affine subspaces

Pengyu YANG(杨鹏宇)
Chinese Academy Sciences

We give an upper bound of the Hausdorff dimension of singular vectors on affine subspaces of the Euclidean space. This upper bound is expressed in terms of the Diophantine exponent of the parameter of the affine subspaces. The proof involves our construction of a new Margulis function on homogeneous spaces. Joint work with Shah.

Large gaps between directions in certain planar quasicrystals

Shucheng YU(于树澄)
University of Science and Technology of China

Given a planar point set with a constant density, one can study the limiting distribution of gaps for directions (i.e. projection to the unit sphere) of points from this point set in larger and larger disks. For primitive integer points an explicit formula for the limiting gap distribution function was obtained by Boca, Cobeli, and Zaharescu using analytic number theory. This formula was later extended by Marklof and Str¨ombergsson for any lattices using homogeneous dynamics. More recently, they showed the limiting gap distribution function exists for point sets of cut-and-project type. In this talk I will describe a tail asymptotic formula for this limiting gap distribution function for a certain family of cut-and-project sets of arithmetic origin, which, in particular includes some classical quasicrystals. This is work in progress with Gustav Hammarhjelm and Andreas Str¨ombergsson.

On connectedness in parametric geometry of numbers

Han ZHANG(张涵)
Suzhou University

Parametric geometry of numbers is the study of Minkowski’s successive minima of a one-parameter family of unimodular lattices in a n-dimensional Euclidean space. Connectedness of these successive minima is a property measuring the rationality of lattices. We reformulate the connectedness criterion of Schmidt and Summerer in the language of multi-linear algebra and sharpen their results concerning connectedness arising from simultaneous Diophantine approximation and approximation by linear forms. We also discuss the relation between connectedness and pencils, which is a concept describing extremity in Diophantine approximation. Joint with Yuming Wei.

Asymptotics of integer points on homogeneous spaces

Runlin ZHANG(张润林)
Chongqing University

Assume that a homogeneous variety has infinitely many integer points, can one count them with the help of a height function? We will explain that in some cases, this problem can be solved for some good heights using equidistribution results on homogeneous spaces. As for the remaining heights, this seems to be a problem in algebraic geometry.

Jarn´ık-Besicovitch type theorems for semisimple algebraic groups

Cheng ZHENG(郑骋)
Shanghai Jiao Tong University

The Jarn´ık-Besicovitch theorem in the metric theory of Diophantine approximation states that the set of real numbers which are not Diophantine of type γ has Hausdorff dimension $2/(1+\gamma)$. In this talk, we discuss a recent result about Jarn´ık- Besicovitch type theorems for semisimple algebraic groups from the representationtheoretic point of view.