Titles and Abstracts

Elliptic integrable lattice equations: an overview and open problems.

Frank Nijhoff
University of Leeds

By integrable lattice equations are meant integrable partial difference equations on a space-time lattice. There exist a number of interesting examples, such as Adler's discretization of the Krichever-Novikov equation (a.k.a. Q4), lattice versions of Landau-Lifschitz, etc., for which very few results exist to date, in particular w.r.t. explicit solutions. The talk will provide a brief overview, mentions open problems and possible suggestions for progress.

Elliptic discrete Painlevé equations with constraints and their symmetry groups

Anton Dzhamay
University of Northern Colorado

We consider an elliptic discrete Painlevé equation obtained from a geometric deautonomization of a certain QRT map along the elliptic fiber. We emphasize that there are different types of orbits of discrete Painlevé dynamics — generic ones, whose symmetry group is the affine E8, and constrained ones, whose symmetry groups do not appear in the generic Sakai classification scheme. This talk is based on a joint work with S. Carstea and T. Takenawa.

On the lattice Heisenberg magnet with Sklyanin's discretization

Xiaoxue Xu(许晓雪)
Zhengzhou University

By using the stereographic variables, we give a new Lax pair for the lattice isotropic Heisenberg magnet model. Through nonlinearization of the spectral problem, we construct an integrable Hamiltonian system and an integrable symplectic map. Then the finite genus potentials are calculated. Finally we obtain the solutions to the lattice isotropic Heisenberg magnet equation in terms of Riemann theta functions.

Fredholm determinant as isomonodromic τ-function of higher rank Painlev\'e system

Yuancheng Xie(谢远成)
Peking University

Painlev\'e transcendents as nonlinear special functions are widely used in mathematical physics such as random matrices, conformal field theory etc. A fruitful way to study them is through the isomonodromic deformations of some auxiliary linear systems, and a key notion in this method is the Jimbo-Miwa-Ueno τ-function. In this talk I will outline a procedure to represent isomonodromic τ-functions of a linear system with irregular singularity as Fredholm determiant, and the simplest case of which is equivalent to Painlev\'e VI. This talk is based on a joint work with Xinxing Tang (BIMSA) and Xiaomeng Xu (PKU).

Rogue waves arising on the standing periodic waves in the Ablowitz-Ladik equation

Jinbing Chen(陈金兵)
Southeast University

We study the standing periodic waves in the semi-discrete integrable system modelled by the Ablowitz-Ladik equation. We have related the stability spectrum to the Lax spectrum by separating the variables and by finding the characteristic polynomial for the standing periodic waves. We have also obtained rogue waves on the background of the modulationally unstable standing periodic waves by using the end points of spectral bands and the corresponding eigenfunctions. The magnification factors for the rogue waves have been computed analytically and compared with their continuous counterparts.

Oscillatory exact global solutions to integrable systems

Ruomeng Li(李若梦)
Zhengzhou University

Resorting to the Baker-Akhiezer functions in the study of algbraic-geometric solutions, I shall demonstrate the construction of a class of oscillatory exact solutions to some long-wave-short-wave equations. The procedure consists of (1) finding oscillatory seed solutions, (2) solving the spectral problems on oscillatory backgrounds exactly via a so-called inverse-algebraic-geometric- solution method, (3) solving inhomogeneous spectral problems for the derivatives of the spectral problems, and (4) piecing together the spectral functions and the seed solutions to yield novel, singularity-free solutions on oscillatory backgrounds. I shall also present some examples of oscillatory breathers, rogue waves, and their interactions.

Title: TBA

Alexander Mikhailov
University of Leeds

Abstract: TBA

Discrete integrable systems related to elliptic curves

Xingbiao Hu(胡星标)
Chinese Academy of Sciences

In this talk, I will present some known and new results for discrete integrable systems related to elliptic curves. The talk is based on joint work with Wu Jingrui.

A generalized sine-Gordon equation: reductions and integrable discretizations

Guofu Yu(虞国富)
Shanghai Jiaotong University

In this talk, we propose fully discrete analogues of a generalized sine-Gordon (gsG) equation $u_{t x}=\left(1+\nu \partial_x^2\right) \sin u$. The key points of the construction are based on the the bilinear discrete KP hierarchy and appropriate definition of discrete reciprocal transformations. We derive semi-discrete analogues of the gsG equation from the fully discrete gsG equation by taking the temporal parameter limit $b\rightarrow0$. In particular, one fully discrete gsG equation is reduced to a semi-discrete gsG equation in the case of $\nu=-1$ (Feng et al. Numer. Algorithms. 2023). Furthermore, N-soliton solutions to the semi- and fully discrete analogues of the gsG equation in the determinant form are presented. Dynamics of one- and two-soliton solutions for the discrete gsG equations are analyzed. We also investigate the reductions from the gsG to the sine-Gordon (sG) equation and the short pulse (SP) equation, respectively. By introducing a parameter c, we demonstrate that the gsG equation can reduce to the sG equation and the SP equation, and the discrete gsG equation can reduce to the discrete sG equation and the discrete SP equation, respectively, in the appropriate scaling limits. The limiting forms of the N-soliton solutions to the gsG equation also correspond to those of the sG equation and the SP equation. This is a joint work with Bao-Feng Feng and Han-Han Sheng.

Complex crystallographic groups and Seiberg--Witten integrable systems

Oleg Chalykh
University of Leeds

For any smooth complex variety Y with an action of a finite group W, Etingof defines the global Cherednik algebra H_c and its spherical subalgebra B_c as certain sheaves of algebras over Y/W. When Y is an n-dimensional abelian variety, the algebra of global sections of B_c is a polynomial algebra on n generators, as shown by Etingof, Felder, Ma, and Veselov. This defines an integrable system on Y. In the case of Y being a product of n copies of an elliptic curve E and W=S_n, this reproduces the usual elliptic Calogero----Moser system. Recently, together with P. Argyres and Y. Lü, we proposed that many of these integrable systems at the classical level can be interpreted as Seiberg----Witten integrable systems of certain super-symmetric quantum field theories. I will describe our progress in understanding this connection for groups W=G(m, 1, n), corresponding to the case Y=E^n where E is an elliptic curves with Z_m symmetry, m=2,3,4,6.

Masur--Veech volumes of moduli spaces of quadratic differentials and their asymptotics

Di Yang(杨迪)
University of Science and Technology of China

Based on the Chen--Moeller--Sauvaget formula, we apply the theory of integrable systems to derive three Painle\'e-type equations for the generating series of the Masur--Veech (MV) volumes associated with the principal strata of the moduli spaces of quadratic differentials, and propose refinements of the conjectural formulas given by Delecroix et al. and by Aggarwal et al. on the large genus asymptotics of the MV volumes and of the associated area Siegel--Veech constants. The talk is based on a joint work with Don Zagier and Youjin Zhang.

Elliptic functions and their applications in complex differential equation

Chengfa Wu(吴成发)
Shenzhen University

This talk focuses on applications of elliptic functions in the study of complex differential equations. In the first part, we discuss meromorpohic solutions of Briot-Bouquet differential equations and classifications of meromorphic solutions of certain autonomous complex differential equations. In the second part, we will present our recent works on meromorphic solutions of two types of Schwarzian differential equations.

On hybrid integrable systems and low dimensional Liouville tori

Nicolai Reshetikhin
BIMSA

A hybrid integrable systems has a classical integrable system as a subsystem and a quantum integrable system which is "driven" by the classical one. The talk starts with an overview of Hybrid integrable systems and then will be focused on spin Calogero-Moser models.

Miura transformations and geometric curve flows

Zhiwei Wu(吴志伟)
Sun Yat-sen University

In this talk, we will discuss curve flows whose invariants are solutions to the KdV and mKdV type equations. The generalized Miura transformations will be induced from the correspondence between different sets of geometric invariants. This is a joint work with Qu, Changzheng.

Elliptic curves, continued fractions and Somos recurrences

Xiangke Chang(常向科)
Chinese Academy of Sciences

Somos-4 and Somos-5 are bilinear recurrence relations that can obtained as reductions of the discrete KP equation. They exhibit interesting integrality, behind which it is the Laurent phenomenon appearing as a key property of cluster variables in Fomin and Zelevinsky's cluster algebras. This talk is devoted to presenting how to derive explicit solutions of Somos-4 and Somos-5 in terms of Hankel determinants based on elliptic curves and continued fractions. The talk is mainly based on the following works and our recent results.

References:

[1]. X.K. Chang, X.B. Hu and G. Xin. Hankel determinant solutions to several discrete integrable systems and the Laurent property. SIAM. J. Discrete Math., 2015.

[2]. A.N.W. Hone. Continued fractions and Hankel determinants from hyperelliptic curves. Comm. Pure Appl. Math., 2020.

The Volterra map: a discrete integrable map in dimension 3g+1 (g=1,2,3...)

John Roberts
University of New South Wales

We study [1,2] the Stieltjes continued fraction expansion of a certain rational function of the plane on a hyperelliptic curve of genus g=1,2,3,.. . We show how it gives rise to a birational map in dimension 3g+1, which we call the Volterra map, which has 2g+1 polynomial integrals in involution with respect to a Poisson bracket. That is, we have a Liouville integrable map but more can be said: the map is actually algebraic completely integrable for each g. This means the generic level set defined by the intersection of the 2g+1 integrals is the affine part of an Abelian variety of dimension g and the restriction of the map to any of these Abelian varieties is a translation. We give the name Volterra map because its solutions also provide genus g solutions of the infinite Volterra lattice equation. A particular case of the Volterra map when g=2 was previously found by Gubbiotti et al in a systematic search for 4D integrable maps.

[1] A N W Hone, J A G Roberts and P Vanhaecke, A family of integrable maps associated with the Volterra lattice, https://arxiv.org/pdf/2309.02336.pdf (in revision for Nonlinearity (2024)).
[2] A N W Hone, J A G Roberts, P Vanhaecke and F Zullo, Integrable maps in 4D and modified Volterra lattices, OCNMP 12491 (February 15, 2024) https://doi.org/10.46298/ocnmp.12491

Delay soliton equations and delay box-ball systems

Kenichi Maruno(丸野健一)
Waseda University

We propose a systematic method for constructing integrable delay-difference and delay-differential analogues of known soliton equations such as the Lotka-Volterra, Toda lattice, and sine-Gordon equations and their multi-soliton solutions. Then we construct delay analogues of the box and ball system (BBS) by the ultra-discretization of the delay discrete Lotka-Volterra equation, which is an integrable delay analogue of the discrete Lotka-Volterra equation. Soliton patterns generated by this delay BBS are classified into normal solitons and abnormal solitons. Normal solitons have a clear relationship to the solitons of the BBS with K kinds of balls. On the other hand, abnormal solitons show various types of novel soliton patterns, which have not been observed in almost all known BBSs. We obtain them by numerical experiments, and then construct tau-functions of them analytically in 1-soliton cases.

Universal characters, soliton graphs and rigid surface operators

Chuanzhong Li(李传忠)
Shandong University of Science and Technology

Partitions are important subjects in representation theory, combinatorics, mathematical physics. We will review our studies in three different directions of the partitions.
1. The partitions have one to one correspondence to Young diagrams which produces schur function as the tau function of KP systems. In this direction, we considered the double partitions and did a series of studies on the twisted Jacobi-Trudi formula which is called universal character.
2. The partitions connect with permutation group, chord diagram, pipedream, Le-diagrams which all connect with soliton graph of KP systems. In this direction, we considered the combinatorics inside the line-soliton solutions of KP type systems.
3. The symbol of partitions is used to describe the Springer correspondence for the classical groups by Lusztig. We refine the explanation that the S-duality maps of the rigid surface operators are symbol preserving maps. We clear up cause of the mismatch problem of the total number of the rigid surface operators between the B type and C type Lie algebraic theories. And we construct all the B/C rigid surface operators which can not have a dual. A classification of the problematic surface operators is made. Also we prove the conjecture that the symbol invariant of partitions is equivalent to the fingerprint invariant of partitions for the rigid surface operators. Our this work was published in Commun. Math. Phys. (2024).

Space of initial values and symmetries of four dimensional Painlev\'e systems

Tomoyuki Takenawa(竹绳知之)
Tokyo University of Marine Science and Technology

The discrete Painlev\'e equations are two-dimensional non-autonomous dynamical systems obtained by deautonomizing difference equations on elliptic surfaces preserving a singular fiber in the space of their initial values. Even in the four-dimensional case, the discrete Painlev\'e systems can be obtained from dynamical systems with two conserved quantities by deautnomizing them preserving the effectiveness of an anticanonical divisor in the space of initial values.

Q-operators for higher spin eight vertex models

Takashi Takebe(武部尚志)
BIMSA

A Q-operator is a linear operator first introduced by Baxter to solve the eight vertex model. Recently it attracts attention from representation theoretical viewpoint. In fact, Baxter’s constructions of his

Q-operators, which fully exploit the structure of the elliptic R-matrix, seem to be quite technical. However, as will be reported in this talk, they can be generalised to higher spin models defined by representations of the Sklyanin algebra. This suggests existence of mathematical background of the constructions.

Elliptic direct linearisation scheme of the lattice Kadomtsev-Petviashvili equation and the elliptic Nth root of unity

Yingying Sun(孙莹莹)
University of Shanghai for Science and Technology

In this talk I will introduce the elliptic Nth root of unity and show the elliptic direct linearisation scheme of the lattice Kadomtsev-Petviashvili equation. As a particular concrete application, the class of elliptic multi-soliton solutions of the lattice Boussinesq systems. Furthermore, we interpret the lattice Boussinesq system as a nonlinear superposition formula of the Bäcklund transformation of the continuous Boussinesq equation, and obtain the elliptic soliton solution of the continuous Boussinesq equation.

KP solitons under the Gelfand-Dickey reductions and the vertex operators

Yuji Kodama(児玉裕治)
Shandong University of Science and Technology

We classify all the regular soliton solutions of the KP equation under the Gelfand-Dickey reductions. We also give a method to construct these solitons using the vertex operators.

Elliptic solitons related to the Lamé functions

Dajun Zhang(张大军)
Shanghai University

In this talk, I will report recent progress on the elliptic solitons related to the Lamé functions. Apart from the classical solitons that are composed by usual exponential type plane wave factors, there exist “elliptic solitons” which are composed by the Lamé-type plane wave factors and expressed using Weierstrass functions. Recently, we found vertex operators to generate tau functions for such type of solitons. We also established an elliptic scheme of direct linearization approach.