Titles and Abstracts

1. David Cushing, Newcastle University, United Kingdom

Title: Bakry Emery curvature, semi definite programming and global structures

Abstract: We present a tool for calculating Bakry Emery curvature and discuss various families of graphs and their curvature. We also introduce an infinite graph with positive curvature everywhere and ask other global questions.

2. Xianfeng David Gu, State University of New York at Stony Brook, USA

Title: Discrete Surface Foliation and Applications

Abstract: In this talk, we will introduce a novel method to compute surface foliations on discrete surfaces based on discrete holomorphic quadratic differentials. First, we find a set of simple loops on the surface, then the surface is decomposed into pairs of pants, a metric graph is constructed from the pants decomposition, finally the harmonic map from the surface to the metric graph is carried out. The Hopf differential of the harmonic map gives the holomorphic differential. The foliation plays a fundamental role in the "holy Grid" problem in computer aided geometric design.

3. Martin Kell, University of Tuebingen, Germany

Title: Sectional curvature-like conditions on metric spaces

Abstract: In this talk I present two concavity assumptions on the distance. The first one is the non-negative curvature analogue of Busemann's non-positive curvature condition and resembles a sectional curvature-like condition comparable to the measure contraction property. It holds for certain non-Riemannian Finsler manifolds, but it is not clear whether it is compatible with Ohta's Ricci curvature on Finsler manifolds. Whenever the n-dimensional Hausdorff measure is non-trivial then the measure contraction property holds and the space is a PI-space. Independent of this one always obtains a bi-Lipschitz splitting theorem. The second condition, called uniform smoothness, is dual to uniform convexity of the distance function and gives a convex exhaustion function, a first step towards a soul theorem.

4. Matthias Keller, University of Potsdam, Germany

Title: Sectional curvature of polygonal complexes with planar substructures

Abstract: We study geometric and spectral consequences of curvature bounds on polygonal complexes that have planar substructures. A special case are Euclidean and hyperbolic buildings. (This is joint work with Norbert Peyerimhoff and Felix Pogorzelski)

5. Shiping Liu, University of Science and Technology of China, China

Title: Bakry-Émery curvature functions of graphs

Abstract: Bakry-Émery curvature-dimension inequalities on graphs attracted a lot of attention in recent years. In this talk, we consider the Bakry-Émery curvature as a function of the dimension parameter. Those curvature functions behave very well under taking Cartesian product of graphs. We will explain the local combinatorial information that one can read from the curvature functions. The curvature functions can be estimated, and in many cases can be expressed precisely, in terms of eigenvalues of some new constructed local graphs, which capture the local connectivity of the graphs. We will also discuss the curvature functions of many particular examples. This is a joint work with David Cushing and Norbert Peyerimhoff.

6. Linyuan Lu, University of South Carolina, USA

Title: Ricci-flat graphs with with girth at least five

Abstract: We use Ollivier type definition of curvature of graph to define recciflat graphs. We call a graph is called Ricci-flat if its Ricci curvatures vanish on all edges. We will classify all Ricci-flat connected graphs with girth at least five: they are the infinite path, cycle of at least 6 vertices, the dodecahedral graph, the Petersen graph, and the half-dodecahedral graph. (Joint work with Yong Lin and S.-T. Yau)

7. Feng Luo, Rutgers University, USA

Title: Discrete uniformization for polyhedral surfaces

Abstract: We discuss some of the recent work on discrete conformal geometry of polyhedral surfaces and a discrete uniformization theorem. The key is to introduce the concept of discrete conformal equivalence between polyhedral metrics. We proved that for each compact polyhedral metric on a surface, there exists a constant curvature polyhedral metric, unique up to scaling, which is discrete conformal to the given one. The relationship among discrete conformal geometry, the classical Weyl problem on convex surfaces, and the Koebe circle domain conjecture will be addressed. This is a joint work with D. Gu, J. Sun, S. Tillmann and T. Wu.

8. Florentin Muench, University of Potsdam, Germany

Title: Rigidity Properties of the Hypercube via Bakry Emery Curvature

Abstract: We give rigidity results for discrete Bonnet-Myers diameter bound and Lichnerowicz eigenvalue estimate. Both inequalities are sharp if and only if the underlying graph is a hypercube. The proofs use well-known semigoup methods as well as new direct methods which translate curvature to combinatorial properties. The results can be seen as first known discrete analogues of Cheng's and Obata's rigidity theorems.

9. Shin-ichi Ohta, Kyoto University, Japan

Title: Rigidity for the spectral gap inequality on $\mathrm{RCD}(K, \infty)$-spaces

Abstract: We consider a rigidity problem for the spectral gap of Laplacian on an $\mathrm{RCD}(K,\infty)$-space (a metric measure space satisfying the Riemannian curvature-dimension condition) for positive K. For a weighted Riemannian manifold, Cheng–Zhou [CZ] showed that the sharp spectral gap is achieved only when a 1-dimensional Gaussian space is split off. Generalization to RCD-spaces is not straightforward due to the lack of smooth structure. We employ the theory of regular Lagrangian flows recently developed by Ambrosio–Trevisan [AT] to overcome this difficulty, and partly mimic Gigli's proof of the splitting theorem [Gi]. This is a joint work in progress with Nicola Gigli, Christian Ketterer and Kazumasa Kuwada.

References

[AT] L. Ambrosio and D. Trevisan, Well-posedness of Lagrangian flows and continuity equations in metric measure spaces. Anal. PDE **7** (2014), 1179–1234.

[CZ] X. Cheng and D. Zhou, Eigenvalues of the drifted Laplacian on complete metric measure spaces. Commun. Contemp. Math. (to appear).

[Gi] N. Gigli, The splitting theorem in non-smooth context. Preprint (2013). Available at arXiv:1302.5555

10. Norbert Peyerimhoff, Durham University, United Kingdom

Title: Geometric and spectral consequences of nonpositive combinatorial corner curvature

Abstract: In this talk, I will recall combinatorial curvature notions of planar tessellations and present global geometric and spectral results. The presented material is based on joint work with Oliver Baues, Steffen Klassert, Matthias Keller, Daniel Lenz, and Peter Stollmann.

11. Jacobus Willem Portegies, Eindhoven University of Technology, Netherlands

Title: Curvature bounds and continuity of eigenvalues under intrinsic flat convergence

Abstract: In this talk we investigate the continuity properties of eigenvalues of the Laplace-Beltrami operator on oriented Riemannian manifolds, as these manifolds converge in the intrinsic flat distance to a limit space. We contrast situations with and without a uniform lower bound on Ricci curvature.

12. Pascal Romon, Universit Paris-Est-Marne-la-Valle, France

Title: Discrete minimal and constant mean curvature surfaces and integrable systems combinatorial corner curvature

Abstract: Minimal and constant mean surfaces have played a key role in classical (smooth) Riemannian geometry, but finding analogous discrete objects turns out to be difficult. Various definitions of these compete, e.g. critical points of the area functional, though that one remains unsatisfactory and breaks the maximum principle (as is obvious from the cotan Laplace operator). We will present here one based on circular quad-based nets and explain how it relates to the minimal/CMC PDE. We will also show that it does have an interpretation in terms of Lax pair, much like the

smooth PDE has. As a consequence it offers a (partial) constructive approach. This is a joint work with Alexander Bobenko (TU Berlin).

13. Emil Saucan, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Title: Forman curvature for networks: From polyhedral complexes to graphs and back again

Abstract: Traditionally, network analysis is based on local properties of vertices, like their degree or clustering coefficient, and their statistical behavior across the network in question, thus concentrating on the elements of the network (nodes), rather than on their interrelations (edges), that define, in effect, the network. We propose an alternative edge-based approach. The geometric tool that enables us to do this is Forman's discretization of Ricci curvature, initially devised for quite general weighted CW complexes. We show that in the limit case of 1-dimensional complexes, i.e. networks (or graphs) this notion is still powerful and expressive enough to allow us to capture not only local, but also global properties of networks, both weighted and unweighted, directed as well as undirected. We show the robustness of this notion and compare it to other, more classical, graph invariants and network descriptors, both on standard model networks and on a variety of real-life networks. Furthermore, we develop a fitting Ricci flow, and we apply it in the analysis of dynamic networks, and employ it to such tasks as change detection, denoising and clustering of experimental data, as well as to the extrapolation of network evolvement. Moreover, we consider not only the pairwise correlations in networks, but also the higher order ones, that are especially important in biological and social networks, and apply Forman's original notion to the resulting complexes (hyper-networks) together with an adapted Ricci flow. This approach allows us, by applying Bloch's extension of Forman's curvature, to give a theoretical sound notion of Euler characteristic for (hyper-)networks, that in turn allows us to predict their long time behavior and suggest their classification in terms of their limits under the flow.

This represents joint work with Jürgen Jost, Melanie Weber and Areejit Samal (and others).

14. Gerardo Garciamarin Sosa, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Title: Metric measure spaces and automorphism groups

Abstract: We present a condition on a *class* of metric measure spaces which, if met, guarantees the *smoothness* of the *automorphism* group of the space. We show as well that this is also a necessary condition. As a consequence we recover the results in Riemannian and metric geometry concerning *smooth* isometry groups and are able to find new examples of spaces with such property.
Particularly, a large set of these examples is found on a *class* of spaces fulfilling curvature-dimension conditions in the Lott-Sturm-Villani sense. We look into these and other examples.

In the talk we develop on the meaning of some of the intentionally vaguely stated terms like *classes*, *smoothness*, and *automorphisms* and, depending on time and interest, cover other related problems.

15. Karl-Theodor Sturm, University of Bonn, Germany

Title: Heat flow on time-dependent metric measure spaces and super Ricci flows

Abstract: We study the heat equation on time-dependent metric measure spaces (being a dynamic forward gradient flow for the energy) and its dual (being a dynamic backward gradient flow for the Boltzmann entropy). Monotonicity estimates for transportation distances and for gradients will be shown to be equivalent to the so-called dynamical convexity of the Boltzmann entropy on the Wasserstein space. For time-dependent families of Riemannian manifolds the latter is equivalent to be a super-Ricci flow. This includes all static manifolds of nonnegative Ricci curvature as well as all solutions to the Ricci flow equation. The latter will also be characterized in terms of coupled pairs of Brownian motions.

16. Toshikazu Sunada, Meiji University, Japan

Title: Generalized Riemann Sums and Certain Arithmetic Quasicrystals

Abstract: The primary aim of this talk is to explain how the idea of Riemann sum is linked to other branches of mathematics; for instance, some counting problems in elementary number theory and the theory of quasicrystals, the former having a long history and the latter being an active field still in a state of flux.

17. Guofang Wang, Freiburg University, Germany

Title: The Gauss-Bonnet-Chern curvature surfaces

Abstract: In this talk, we will introduce a generalized scalar curvature, the Gauss-Bonnet-Chern curvature, and discuss its related masses and geometric inequalities. This talk is based on the joint work with Yuxin Ge, Jie Wu and also Chao Xia.

18. Chao Xia, Xiamen University, China

Title: Bochner technique for a family of affine connections

Abstract: In this talk, we introduce a 2-parameter family of affine connections and its Ricci curvature on Riemannian manifolds. Special choice of the parameters leads to the conformal Ricci, the static Ricci and the 1-Bakry-Emery Ricci curvature. We will talk about the associate Bochner technique for this connection. The integral Bochner formula enable us to prove a family of geometric inequalities. This is joint work with Junfang Li.

19. Huichun Zhang, Sun Yat-sen University, China

Title: Lipschitz Continuity of Harmonic Maps between Alexandrov Spaces

Abstract: Alexandrov spaces are singular metric spaces with generalized lower (or upper) bounds of sectional curvature via Toponogovs triangle comparison theorem.

In 1992, to study the p-adic superrigidity for lattices in groups of rank one, M. Gromov and R. Schoen developed a theory of harmonic maps from smooth manifolds into nonpositively curved (NPC) singular metric spaces. In 1997, J. Jost and F. H. Lin proved, independently, that every harmonic map from an Alexandrov space with curvature bounded from below to an NPC metric space is locally Hölder continuous. Based on Gromov-Schoens and Korevaar-Schoens results, F. H. Lin also conjectured that the H?lder continuity can be improved to Lipschitz continuity. J. Jost also asked a similar problem. In this talk, we will introduce a resolution to this Lins problem. This is a joint work with Prof. Xi-Ping Zhu.