Global Differential Geometry

February 26 – March 2, 2018

Titles and Abstracts

1. Mauricio BUSTAMANTE, University of Augsburg

Title: Bundles with fiberwise negatively curved metrics

Abstract: A smooth M-bundle is said to be negatively curved if its fibers are equipped with Riemannian metrics of negative sectional curvature, varying continuously from fiber to fiber. The difference between negatively curved M-bundles and smooth M-bundles is measured by the space of all negatively curved metrics on M. In this talk I will show that the latter space has non-trivial rational homotopy groups, provided certain dimension constraints are satisfied. Hence the two bundle theories generally differ. The results extend to other spaces of metrics, e.g. spaces of Riemannian metrics with geodesic flow of Anosov type. This is joint work with F.T. Farrell and Y. Jiang.

2. Lina CHEN, East China Normal University

Title: Gap vanishing volume entropy with lower Ricci curvature bound

Abstract: For a compact Riemannian manifold M, the volume entropy of M which always exists is an asymptotic geometric invariant that measures the exponential growth rate of the volume of metric balls in its universal cover . In this talk, we will show a gap phenomena on the volume entropy: given $n, d > 0$, there exists a constant $\epsilon(n, d) > 0$ such that if a compact n-manifold M has lower Ricci curvature bound $-(n - 1)$ and upper diameter bound d then with some other additional conditions on the fundamental group of M we have that the volume entropy of M, $h(M) > \epsilon(n, d)$ or $h(M) = 0$.

This is a joint work with Xiaochun Rong and Shicheng Xu.

3. Qi DING, Shanghai Center for Mathematical Sciences (Fudan University)

Title: Minimal Sets in Riemannian Manifolds with Ricci Curvature Bounded Below

Abstract: In Euclidean space, minimal sets have been studied intensely several decades before, and the fundamental theory has been well-understood. In this talk, I would like to talk about minimal sets in manifolds with Ricci curvature bounded below, but without sectional curvature condition. As applications, the theory can be used to study minimal graphs in product manifolds.

4. Bernhard HANKE, University of Augsburg

Title: Spaces and moduli spaces of positive scalar curvature metrics

Abstract: We discuss the topology of spaces of Riemannian metrics of positive scalar curvature on smooth manifolds, and of related moduli spaces. Recent results are mainly based on index theory in combination with new insights related to the diffeomorphism groups of smooth manifolds.

5. Bobo HUA, Fudan University

Title: Combinatorial curvature for infinite planar graphs

Abstract: For any planar graph, its ambient space $\mathbb{S}^2$ or $\mathbb{R}^2$ can be endowed with a canonical piecewise flat metric by identifying its faces with regular Euclidean polygons, called the polyhedral surface. The combinatorial curvature of a planar graph is defined as the generalized Gaussian curvature of its polyhedral surface up to the normalization 2π. The total curvature of an infinite planar graph with non-negative combinatorial curvature will be shown to be an integral multiple of $1/12$ and the number of vertices with non-vanishing curvature is at most 132. Moreover, if the total curvature is positive, then the automorphism group of an infinite planar graph with nonnegative combinatorial curvature is finite. This is based on joint works with Yanhui Su (Fuzhou University).

6. Yi HUANG, Yau Mathematical Sciences Center (Tsinghua University)

Title: McShane identities for finite-area convex real projective surfaces

Abstract: The Teichmüller space $\mathcal{T}(S)$ for an orientable surface S is equivalent to the character variety of discrete faithful $\mathrm{SL}_2\mathbb{R}$ representation of the fundamental group $\pi_1(S)$. This approach to Teichmüller theory leads to natural family of generalized Teichmüller spaces given by increasing the rank of $\mathrm{SL}_2\mathbb{R}$ to $\mathrm{SL}_n\mathbb{R}$. For $n = 3$, there is a geometric interpretation of this *higher (rank) Teichmüller theory* as the theory of strictly convex real projective structures on S. We show that there is a generalization of McShane's identity to this context: a type of infinite-sum trigonometric identity which holds for all cusped convex real projective surfaces.

This is work in collaboration with Zhe Sun, YMSC (Tsinghua University).

7. Huihong JIANG, Shanghai Jiao Tong University

Title: Bounded topology of complete manifolds with nonnegative Ricci curvature and quadratically nonnegatively curved infinity

Abstract: A manifold is said to be of finite topological type if it is homeomorphic to the interior of a compact manifold with boundary. In this talk, I will give a brief introduction to the main results of manifolds with nonnegative Ricci curvature and quadratically nonnegatively curved infinity about the finite topological type. This includes some finite results under certain conditions of diameter growth (resp. volume growth) and some counterexamples of infinite topology with positive Ricci curvature.

8. Yi JIANG, Yau Mathematical Sciences Center (Tsinghua University)

Title: Involution on pseudoisotopy spaces and the space of nonnegatively curved metrics

Abstract: Let $\mathfrak{R}_{K\geq 0}(V)$ denote the space of complete Riemannian metrics of nonnegative sectional curvature on a connected manifold V, equipped with the smooth compact-open topology. It is shown by Belegradek, Farrell and Kapovitch that for many open manifolds V the space $\mathfrak{R}_{K\geq 0}(V)$ has nontrivial rational higher homotopy groups. For example, when U is the total space of the tangent bundle to the $2d$ dimensional sphere S^{2d} for $d \geq 2$, they find explicit integers $i \geq 2$ such that there are sufficiently large integers m for which

$$\pi_i \mathfrak{R}_{K\geq 0}(U \times S^m) \otimes \mathbb{Q} \neq 0.$$

A remaining question is to specify these m for given integers i. In this talk, we present a way to answer this question by studying the involution on pseudoisotopy space. This is joint work with Mauricio Bustamante and F. Thomas Farrell.

9. Stephan KLAUS, Mathematisches Forschungsinstitut Oberwolfach and Mainz University

Title: On a combinatorial Gauss-Bonnet Theorem for Euclidean simplicial complexes

Abstract: We present a very short proof of a combinatorial Gauss-Bonnet Theorem which holds for general Euclidean simplicial complexes, not only for combinatorial manifolds. A simplicial complex is called Euclidean if all simplices carry a flat Euclidean metric. This additional information is equivalently given by edge lengths in an abstract simplicial complex which satisfy a realizability condition. Then we can define the defect of normalized higher dihedral angles associated to a simplex. For combinatorial manifolds the defect measures the deviation from flatness in the link of a simplex. The simplex-version of the theorem states that the sum of angle defects over all simplices equals the Euler characteristics. For the vertex-version of the theorem we define a local combinatorial curvature which is a kind of logarithm of the angle defects of all simplices adjacent to the vertex. Again, the sum of this curvature over all vertices gives the Euler characteristics. The theorem can be applied not only to closed triangulated manifolds, but also to manifolds with boundary or even with corners and other singularities. We will also discuss the relation to other curvature measures.

10. Nan LI, the City University of New York

Title: Quantitative Estimates on the Singular Sets of Alexandrov Spaces

Abstract: The notion of quantitative singular sets for spaces with lower Ricci curvature bounds was initiated by Cheeger and Naber. Volume estimates were proved for these singular sets in a non-collapsing setting. For Alexandrov spaces, we obtain stronger and volume-free estimates. We also show that the (k, ϵ)-singular sets are k-rectifiable and such structure is sharp in some sense. This is a joint work with Aaron Naber.

11. Pedro ONTANEDA, Binghamton University

Title: Baunslag-Solitar and Riemannian nonpositive curvature

Abstract: We study the question of whether there are nonpositively curved Riemannian manifolds with Baumslag-Solitar fundamental groups.

12. Xiaochun RONG, Capital Normal University and Rutgers University

Title: Collapsed manifolds with Ricci local bounded covering geometry

Abstract: We will report a recent progress in extending nilpotent structures discovered by Cheeger-Fukaya-Gromov on collapsed manifolds with sectional curvature local bounded covering geometry to collapsed manifolds with Ricci local bounded covering geometry. Our construction of local nilpotent structures does not rely on the work of Cheeger-Fukaya-Gromov, which in particular gives a new approach when restricting to collapsed manifolds with bounded sectional curvature.

13. Saskia ROOS, Max-Planck Institute for Mathematics

Title: The Dirac operator under collapse to a smooth limit space

Abstract: Let $(M_i, g_i)_{i\in\mathbb{N}}$ be a sequence of spin manifolds with uniform bounded curvature and diameter that converges to a lower dimensional Riemannian manifold (B, h) in the Gromov-Hausdorff topology. Lott showed that the spectrum converges to the spectrum of a certain first order elliptic differential operator $\mathcal{D}$ on B. In this article we give an explicit description of $\mathcal{D}$. We conclude that $\mathcal{D}$ is self-adjoint and characterize the special case where $\mathcal{D}$ is the Dirac operator on B.

14. Takashi SHIOYA, Tohoku University

Title: Isoperimetric rigidity and distributions of 1-Lipschitz functions

Abstract: In this talk, we consider the relation between the isoperimetric profile and the observable variance of a metric measure space, where the observable variance is defined to be the supremum of the variance of 1-Lipschitz functions. One of our main theorem states that, if the isoperimetric profile is not greater than that of a one-dimensional model and the observable variance coincides with that of the model, then the space is foliated by minimal geodesics. Our result can be considered as a variant of Cheeger-Gromoll's splitting theorem and also of Cheng's maximal diameter theorem. As an application, we obtain a new type of splitting theorem for a complete Riemannian manifold with positive Bakry-Emery Ricci curvature. This is a joint work with Hiroki Nakajima.

15. Michael WIEMELER, Mathematisches Institut (University of Münster)

Title: Non-negatively curved GKM orbifolds

Abstract: A GKM_4 orbifold is an orbifold with an action of a torus of dimension at least four, whose cohomology can be computed in a simple way from a graph associated to the torus action.

In this talk we study non-negatively curved and rationally elliptic GKM_4 manifolds and orbifolds. We show that their rational cohomology rings are isomorphic to the rational cohomology of certain model orbifolds. These models are quotients of isometric actions of finite groups on non-negatively curved torus orbifolds. Here

a torus orbifold is a 2n-dimensional orbifold with an effective action of a torus of dimension n such that the fixed point set of the action is non-empty.

This is joint work with Oliver Goertsches.

16. Masoumeh ZAREI, Beijing International Center for Mathematical Research (Peking University)

Title: Cohomogeneity one Alexandrov spaces with positive curvature

Abstract: Alexandrov spaces are a synthetic generalization of Riemannian manifolds with a lower curvature bound. It is then a natural question that to what extent one can generalize the basic results of Riemannian manifolds with a lower curvature bound to Alexandrov spaces. In this talk, I will explore this question in relation to the cohomogeneity one action of a compact Lie group on an Alexandrov space with $\mathrm{curv} \geq 1$, and explain some obstructions to the existence of an invariant metric of positive curvature on a cohomogeneity one Alexandrov space. In the end, I will give a classification of closed simply-connected cohomogeneity one Alexandrov spaces with positive curvature in dimensions at most 6. It is an ongoing project with F. Galaz-García.

17. Shu-Cheng ZHANG, National Taiwan University

Title: CR Cheeger-Gromov Compactness of Pseudo-Einstein Manifolds and Its Applications

Abstract: In this talk, we first survey the basic properties such as CR volume doubling property and Sobolev inequality and then we obtain the CR analogue of Gromov precompactness theorem. Secondly, we have Cheeger-Gromov compactness theorem for the space of normalized compact pseudo-Einstein manifolds with pseudohermitian curvature integral bounds. With its applications, we have the smooth compactness on the space of normalized compact Sasakian-Einstein manifolds. Moreover, we will have the compactness property for solutions of the torsion flow on a CR three manifold as well. Parts of talk is based on jointed works with Y.-X. Dong and Y.-B. Ren, also D.-C. Chang, Y.-B. Han and J. Tie.